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ALGEBRAIC CHARACTERIZATIONS OF LOCALLY COMPACT GROUPS

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Abstract

Let G_1, G_2 be locally compact real-compact spaces. A linear map T defined from $C(G_1)$ into $C(G_2)$ is said to be *separating or disjointness preserving* if $f \cdot g \equiv 0$ implies $Tf \cdot Tg \equiv 0$ for all $f, g \in C(G_1)$. In this paper we prove that both a separating map which preserves non-vanishing functions and a separating bijection which satisfies condition (M) (see Definition 4) are *automatically continuous* and can be written as weighted composition maps. We also study the effect of separating surjections (respectively injections) on the underlying spaces G_1 and G_2.

Next we apply the above results to give an algebraic characterization of locally compact Abelian groups, similar to the one given in [7] for compact Abelian groups in the presence of ring isomorphisms.

Finally, locally compact (not necessarily Abelian) groups are considered. We provide a sharpening of a result of Edwards and study the effect of onto (respectively injective) weighted composition maps on the groups G_1 and G_2.

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1. Introduction

Let G_1, G_2 be completely regular Hausdorff spaces. $C(G_i)$ $(i = 1, 2)$ denotes the algebra of all complex-valued continuous functions on G_i equipped with the compact-open topology. $C^*(G_i)$ denotes the subset of all bounded functions of $C(G_i)$. If G_i is also locally compact, then let $C_0(G_i)$ (respectively $C_{00}(G_i)$) be the Banach algebra of all complex-valued continuous functions on G_i which are zero at infinity (respectively the normed algebra of all continuous functions with compact support).

The deduction of topological links between G_1 and G_2 from certain algebraic relationships between $C(G_1)$ and $C(G_2)$ has been widely studied in the literature.

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The first result of this type is the well-known Banach-Stone theorem. However, we follow the direction of the following classical result: if a linear map T is an algebra isomorphism between $C(G_1)$ and $C(G_2)$, then the real-compactifications of G_1 and G_2 must be homeomorphic ([14, pp. 115-118]). Moreover, as in the Banach-Stone theorem, T is a weighted composition map, that is, $Tf = a \cdot (f \circ h)$ where 'the weight' a belongs to $C(G_2)$ and $h : G_2 \to G_1$ is a continuous map. Indeed, $|a| \equiv 1$ whenever T is an onto linear isometry and $a \equiv 1$ provided T is an algebra isomorphism. In this paper we deal with a weaker algebraic connection between $C(G_1)$ and $C(G_2)$.

A linear map T defined from $C(G_1)$ into $C(G_2)$ is said to be *separating or disjointness preserving* (also called a *d-homomorphism*) if $f \cdot g \equiv 0$ implies $Tf \cdot Tg \equiv 0$ for all $f, g \in C(G_1)$. Throughout this paper, T will denote a separating map unless otherwise specified. Algebra homomorphisms, lattice homomorphisms, onto isometries, and bipositive or weighted composition operators, are all separating maps.

Disjointness preserving maps between general vector lattices were considered first by several authors; see for example [1, 2, 5, 10, 15, 21]. A thorough study of operators preserving disjointness in the context of $C(K)$-modules can be found in [3].

Disjointness preserving maps were later considered in [6] for spaces of real or complex-valued continuous functions defined on a compact Hausdorff space with the name of separating maps. The main goal of these two papers and [20, 4, 11, 12] is to prove automatic continuity results for separating maps when G_i (i=1,2) is compact, real-compact, locally compact and a locally compact Abelian group respectively. As a consequence, certain topological links between the underlying spaces are deduced and weighted composition type representations for separating maps are obtained. Similar representations have also been obtained in, for example, [1, 5, 19], though in the presence of a continuity type assumption about T. Automatic continuity for separating maps in the context of vector-valued continuous functions is considered in [16]. It is important to remark that a separating map need not be continuous; indeed, Jarosz proved in [20] that, given two compact spaces G_1 (infinite) and G_2, there always exists a discontinuous separating map defined from $C(G_1)$ onto $C(G_2)$.

In Section 3, we prove that both a separating map which preserves non-vanishing functions and a separating bijection which satisfies condition (M) (see Definition 4) are automatically continuous and can be written as weighted composition maps (Theorem 1 and Theorem 2, (1)). We also study the effect of separating injections, surjections and bijections on the underlying spaces G_1 and G_2 (Theorem 1 and Theorem 2, (2)-(4)).

In Section 4, we apply the results of Section 3 to give an algebraic characterization of locally compact Abelian groups, similar to the one given in [7] for compact Abelian groups in the presence of ring isomorphisms.

In Section 5, locally compact (not necessarily Abelian) groups are considered and $C_{00}(G_i)$ is regarded as an algebra under both the pointwise multiplication and the

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convolution product. We provide (Theorem 3) a sharpening of a result of Edwards ([8, Theorem 2]) since isometric and bipositive bijections are nothing but separating maps. Finally, we study the effect of onto (respectively injective) weighted composition maps on G_1 and G_2 (Theorems 4 and 5).

2. Preliminaries

$\mathbb{N}$ (respectively $\mathbb{R}$, $\mathbb{C}$) stands for the set of all natural numbers (respectively real, complex numbers). If G_i is locally compact, then G_i^* denotes its Alexandroff compactification; that is, $G_i^* = G_i \cup \{\infty\}$, ∞ being an ideal point with a neighbourhood consisting of all sets in G_i^* whose complement is compact in G_i. Let cG_i denote any Hausdorff compactification of G_i. If $f \in C(G_i)$, the co-zero of f is the set $\mathrm{coz}(f) = \{t \in G_i : f(t) \neq 0\}$ and $\mathrm{supp}(f)$ will denote the closure of $\mathrm{coz}(f)$. When U is any subset of G_i, we denote by $\mathrm{int}(U)$ the interior of U and by $\mathrm{cl}(U)$ the closure of U in G_i. We let $f_{|U}$ stand for the restriction of f to U, for any $f \in C(G_i)$, and 1 denotes the unit of $C(G_i)$; that is, $1(s) = 1$ for every $s \in G_i$.

Finally, if $s \in G_2$, let $T's' : C(G_1) \to \mathbb{C}$ be defined as $T's'(f) = Tf(s)$ for all $f \in C(G_1)$.

3. Automatic continuity and representation of separating maps

Throughout this section G_1, G_2 will be locally compact real-compact spaces. The following two definitions and Propositions 1, 2 and 3 follow the pattern given by Abramovich in [1] for similar results in the context of general vector lattices.

DEFINITION 1. Given $s \in G_2$, we denote by $\mathrm{supp}\, T's'$ the set $\{t \in cG_1$: for any cG_1-neighbourhood U of t, there exists $f \in C(G_1)$ such that $Tf(s) \neq 0$ and $\mathrm{coz}(f) \subseteq U \cap G_1\}$.

LEMMA 1. *For every $s \in G_2$,* $\mathrm{supp}\, T's'$ *has only one element.*

PROOF. This can be found essentially in [1, Proposition 3.1]. (See also [6] or [20].)

DEFINITION 2. The lemma above lets us define a mapping $h : G_2 \to cG_1$, such that $h(s) = \mathrm{supp}\, T's'$. We call h the *support map* of T.

PROPOSITION 1. *Let U be an open subset of cG_1 and suppose that $f \in C(G_1)$. Then the following conditions hold:*

(1) *The support map h of T is continuous.*

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(2) $f_{|U\cap G_1} \equiv 0$ *implies that* $Tf_{|h^{-1}(U)} \equiv 0$.
(3) $h(\mathrm{coz}(Tf)) \subset cl_{cG_1}(\mathrm{coz}(f))$.
(4) *If* T *is injective, then* $h(G_2)$ *is a dense subset of* cG_1.

PROOF. This can be essentially found in [1, Proposition 3.1]. (See also [6] or [20].)

REMARK. Let $G_{20} \subseteq G_2$ be the subset of all points s such that there exists $f_s \in C(G_1)$ with $Tf_s(s) \neq 0$. Throughout this article we will assume that $G_{20} = G_2$. Note that if T is onto or preserves non-vanishing functions, then $G_{20} = G_2$.

PROPOSITION 2. *If* $s \in G_2$ *and* $T^t s^t$ *is a continuous map, then* $h(s) \in G_1$.

PROOF. Let us suppose that $T^t s^t$ is continuous and $h(s) \in (cG_1 \backslash G_1)$. Let $\{K_a : a \in A\}$ be the family of all compact subsets of G_1. Since $h(s) \notin G_1$, let us consider, for all $a \in A$, an open neighbourhood V_a of $h(s)$ such that $cl_{cG}(V_a) \cap K_a = \emptyset$. By the definition of $h(s)$, there exists $f_a \in C(G_1)$ such that $\mathrm{coz}(f_a) \subset V_a$ and $Tf_a(s) = 1$.

On the other hand, we can order A by taking, for $a, b \in A$, $a \leq b$ if $K_a \subset K_b$, so that $(f_a)_{a\in A}$ form a net in $C(G_1)$. It is clear that, for all $a \in A$, $f_{a|K_a} \equiv 0$ and $f_{b|K_a} \equiv 0$ if $b \geq a$, that is, the net $(f_a)_{a\in A}$ converges to 0 in the compact-open topology of $C(G_1)$. This contradicts the continuity of $T^t s^t$ since $T^t s^t(f_a) = Tf_a(s) = 1$ for all $a \in A$.

DEFINITION 3. We denote by G_{2c} the subset of G_2 consisting of all $s \in G_2$ such that $T^t s^t$ is continuous, and by G_{2d} the complement of G_{2c} in G_2.

PROPOSITION 3. *The following statements are equivalent.*
(1) *The map* $T^t s^t$ *is continuous for every* $s \in G_2$, *that is,* $G_2 = G_{2c}$.
(2) T *is a weighted composition map. Indeed,* $Tf(s) = T1(s) \cdot f(h(s))$ *for every* $s \in G_2$ *and* $f \in C(G_1)$.
(3) T *is a continuous map.*

PROOF. (1) implies (2). This is in [6, Theorem 2.2].

(2) implies (3). Let us consider a net (f_α) in $C(G_1)$ converging to $f \in C(G_1)$ in the compact-open topology. Given a compact subset K of G_2, it suffices to prove that the net (Tf_α) converges to Tf on K.

Let $\epsilon > 0$. As $h(K)$ is a compact subset of G_1, there exists α' such that $\sup\{|T1(s)| \cdot |f_\alpha(h(s)) - f(h(s))| : s \in K\} < \epsilon$ for every $\alpha \geq \alpha'$ because the map $T1$ is bounded on K. Thus, $\sup\{|Tf_\alpha(s) - Tf(s)| : s \in K\} < \epsilon$ and this completes the proof.

(3) implies (1) is clear.

PROPOSITION 4. *For any separating map T, the subsets of Definition 3 have the following properties:*

(1) *G_{2c} is closed in $h^{-1}(G_1)$.*

(2) *$h(G_{2d})$ is a subset of limit points of cG_1.*

PROOF. (1) From Proposition 2 we have $G_{2c} \subseteq h^{-1}(G_1)$. Let us consider a net (s_α) in G_{2c} which converges to $s \in h^{-1}(G_1)$. By Proposition 3, $Tf(s_\alpha) = T1(s_\alpha) \cdot f(h(s_\alpha))$ for every α and every $f \in C(G_1)$. Since $T1$, $f \circ h$, and Tf are continuous mappings, it is clear that $Tf(s) = T1(s) \cdot f(h(s))$ for every $f \in C(G_1)$; that is, $s \in G_{2c}$.

(2) Let us see that if $h(s) \in G_1$ is isolated in G_1, for some $s \in G_2$, then $T^t s^t$ is a continuous map; that is, $s \in G_{2c}$. Given $f \in C(G_1)$, let us define the map $g = f(h(s)) \cdot 1$. As $f_{|\{h(s)\}} \equiv g_{|\{h(s)\}}$, we have that $Tf(s) = Tg(s)$ by Proposition 1. Hence, $T^t s^t(f) = T1(s) \cdot f(h(s))$ for every $f \in C(G_1)$, which implies that $T^t s^t$ is continuous.

DEFINITION 4. We say that T satisfies condition (M) if $T(C_{00}(G_1)) \subset C^*(G_2)$.

PROPOSITION 5. *If the separating map T satisfies condition (M), then $h(G_{2d}) \cap \operatorname{int}(K)$ is finite for every compact subset K of G_1.*

PROOF. To prove that $h(G_{2d}) \cap \operatorname{int}(K)$ is finite for every compact subset K of G_1, let us suppose that there exists a sequence $(h(s_n))$ of distinct elements of $\operatorname{int}(K)$ such that $s_n \in G_{2d}$ for every $n \in \mathbb{N}$. As K is a normal space, by taking a subsequence if necessary, we can assume that $\{U_n\}$ is a pairwise disjoint sequence of open subsets of K such that $h(s_n) \in U_n \subseteq K$ for every $n \in \mathbb{N}$. Let V_n be a closed neighbourhood of $h(s_n)$ with $V_n \subset U_n$. Thus, there exists a map $K_n \in C_{00}(G_1)$ such that $0 \leq K_n \leq 1$, $K_{n_{|V_n}} \equiv 1$ and $\operatorname{coz}(K_n) \subset U_n$ for each $n \in \mathbb{N}$.

On the other hand, since $T^t s_n^t$ is discontinuous, there exists a map $f_n \in C(G_1)$ with $sup\{|f_n(t)| : t \in K\} \leq 1$ and such that $|T^t s_n^t(f_n)| = |Tf_n(s_n)| > n^3$ for all $n \in \mathbb{N}$. Let us define the map

$$g_n = \frac{1}{n^2} \cdot f_n \cdot K_n.$$

Since $K_n \equiv 1$ on V_n, we have

$$|Tg_n(s_n)| = \frac{1}{n^2} \cdot |Tf_n(s_n)| > n.$$

Consequently, $|Tg_n(s_n)| > n$ for each $n \in \mathbb{N}$.

It is clear that $\|g_n\| \leq 1/n^2$ since $K_{n_{|T \setminus K}} \equiv 0$. Thus, we can define the map $g := \sum_{n \in \mathbb{N}} g_n$. Given that $\operatorname{supp}(g) \subseteq \operatorname{cl}(\bigcup(\operatorname{supp}(g_n))) \subset K$, we deduce that $g \in C_{00}(G_1)$. On the other hand, as the family $\{U_n\}$ is pairwise disjoint and $\operatorname{coz}(g_n) \subset U_n$ for all

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$n \in \mathbb{N}$, then we have that $Tg_n|_{h^{-1}(U_m)} \equiv 0$ for $n \neq m$. Thus, $|Tg(s_n)| = |Tg_n(s_n)| > n$ for every $n \in \mathbb{N}$, which is a contradiction since T satisfies condition (M).

PROPOSITION 6. *Let T be a separating bijection from $C(G_1)$ onto $C(G_2)$ which satisfies condition (M). Then:*

(1) *G_{2c} is dense in G_2; indeed $h^{-1}(G_1) = G_{2c}$.*

(2) *$h(G_2) \subseteq G_1$.*

PROOF. (1) As $G_{2c} \cup G_{2d} = G_2$, then $h(G_{2c}) \cup h(G_{2d}) = h(G_2)$. Since T is injective, $h(G_2)$ is dense in cG_1 by Proposition 1(4). Hence, given $t \in G_1$ and a compact neighbourhood U of t, we have, by Propositions 4(2) and 5, $h(G_{2c}) \cap U \neq \emptyset$. That is, $h(G_{2c})$ is dense in cG_1.

Let $s \in h^{-1}(G_1)$ such that $s \notin G_{2c}$. By Proposition 4, there exists a closed subset C of G_2 such that $G_{2c} = C \cap (h^{-1}(G_1))$. Clearly, $s \notin C$. Since T is onto, let $f \in C(G_1)$ such that $f \not\equiv 0$, $Tf(s) = 1$ and $Tf_{|C} \equiv 0$. This implies that $Tf_{|G_{2c}} \equiv 0$. Consequently, $f \equiv 0$ since, from the onto-ness of T, $T1(s) \neq 0$ for all $s \in G_{2c}$ (see Proposition 3(2)) and $h(G_{2c})$ is dense in cG_1. This contradiction proves that $G_{2c} = h^{-1}(G_1)$. In like manner, we prove that $G_{2c} = h^{-1}(G_1)$ is dense in G_2.

(2) Let $s_0 \in G_2$ be such that $h(s_0) \in (cG_1 \backslash G_1)$. From (1), there exists a net (s_α) in G_{2c} such that (s_α) converges to s_0. Thus, by the continuity of h, we have that the net $(h(s_\alpha))$ converges to $h(s_0)$. Let K be a compact subset of G_2 such that $\{(s_\alpha) \bigcup \{s_0\}\} \subseteq K$. Let us suppose first that $T1(s_0) = 0$. Then $(|T1(s_\alpha)| \cdot |f(h(s_\alpha))|) = (|Tf(s_\alpha)|)$ converges to $|Tf(s_0)|$ for all $f \in C(G_1)$. Since $(f \circ h)$ is bounded on K, $(|T1(s_\alpha)| \cdot |f(h(s_\alpha))|)$ converges to 0. This implies that $Tf(s_0) = 0$ for all $f \in C(G_1)$, which contradicts the onto-ness of T.

On the other hand, let us suppose that $T1(s_0) \neq 0$. Without loss of generality, we can assume that cG_1 is βG_1, the Stone-Čech compactification of G_1. Since G_1 is real-compact, there exists ([23, p.81]) a function $f_0 \in C(\beta G_1)$ such that $f_0(t) \in \mathbb{R}$ for every $t \in \beta G_1$, $f_0(h(s_0)) = 0$ and $f_0(t) > 0$ for every $t \in G_1$. Let $g_0 := (1/f_0)$. It is clear that $g_0 \in C(G_1)$. Furthermore, $(|g_0(h(s_\alpha))|)$ converges to $(+\infty)$ since the net (s_α) belongs to G_{2c} and $\{(s_\alpha) \bigcup \{s_0\}\} \subseteq K$. This contradicts the fact that the net $(|T1(s_\alpha) \cdot g_0(h(s_\alpha))|) = (|Tg_0(s_\alpha)|)$ converges to $|Tg_0(s_0)| \in \mathbb{R}$.

THEOREM 1. *If there exists a separating bijection T from $C(G_1)$ onto $C(G_2)$ which satisfies condition (M), then T is automatically continuous (indeed, a weighted composition map) and G_1 and G_2 are homeomorphic.*

PROOF. The automatic continuity of T and its multiplicative representation follow from Propositions 3 and 6. This and the onto-ness of T yield easily the injectivity of the map h (note that $T1$ is non-vanishing on G_2).

Let us prove that the inverse of T, T^{-1}, is also a separating bijection. Let g_1 and g_2 be two elements of $C(G_2)$ such that $\text{coz}(g_1) \cap \text{coz}(g_2) = \emptyset$. Let $f_1, f_2 \in C(G_1)$ be such that $Tf_1 = g_1$ and $Tf_2 = g_2$. Since $T1$ is non-vanishing on G_2, we deduce that $\text{coz}(f_1) \cap \text{coz}(f_2) = \emptyset$ on $h(G_2)$, which is dense in G_1. As a consequence, $\text{coz}(f_1) \cap \text{coz}(f_2) = \emptyset$.

We can now define the support map $k : G_1 \to cG_2$ of T^{-1}. Like h, k is continuous and its range is dense in cG_2. Let $f \in C(G_1)$ be such that $Tf \in C_{00}(G_2)$. From the multiplicative representation of T, we deduce that $(f \circ h)$ is bounded on G_2, which implies that f is bounded on G_1. Therefore T^{-1} also satisfies condition (M). Consequently, we can represent T^{-1} as a weighted composition map and $k(G_1) \subseteq G_2$.

If $t \in G_1$ is an element of $h(G_2)$, that is, $h(s) = t$ for some $s \in G_2$, we will show that $k(t) = s$. If $k(h(s)) \neq s$, then there exist disjoint compact neighbourhoods U and V of $k(h(s))$ and s respectively. By the definition of k, there exists a function $f_0 \in C(G_2)$ such that $\text{coz}(f_0) \subseteq U$ and $T^{-1}(f_0)(h(s)) \neq 0$. This implies that $s \notin \text{cl}(\text{coz}(f_0))$ and $k(h(s)) \in \text{cl}(\text{coz}(f_0))$. Let $f_1 \in C(G_2)$ such that $f_1(s) \neq 0$ and $f_{1|\text{coz}(f_0)} \equiv 0$. Consequently, we have that $\text{coz}(f_1) \cap \text{coz}(f_0) = \emptyset$ and, since T^{-1} is separating, $\text{coz}(T^{-1}(f_1)) \cap \text{coz}(T^{-1}(f_0)) = \emptyset$.

On the other hand, $\text{coz}(T^{-1}(f_0))$ is a neighbourhood of $h(s)$. If we take any $g \in C(G_1)$ such that $\text{coz}(g) \subseteq \text{coz}(T^{-1}(f_0))$, then $\text{coz}(f_1) \cap \text{coz}(Tg) = \emptyset$ because T is separating. Consequently, $Tg(s) = 0$ since $f_1(s) \neq 0$. This contradicts the definition of h. In like manner, we obtain that $h^{-1} = k$, that is, h is a homeomorphism of G_2 onto G_1.

REMARKS. Note that if G_2 is a pseudo-compact space, then condition (M) is redundant. Thus, we extend the result proved by Jarosz in [20]: Let G_1 and G_2 be compact spaces. If T is a separating bijection of $C(G_1)$ onto $C(G_2)$, then T is continuous, has a separating inverse and G_1 and G_2 are homeomorphic.

A close result is proved in [4] when G_1 and G_2 are real-compact and either the inverse of T is separating or G_1 is zero-dimensional, though different techniques are used.

THEOREM 2. *Let T be a separating map from $C(G_1)$ into $C(G_2)$ which preserves non-vanishing functions. Then:*

(1) *T is automatically continuous (indeed, a weighted composition map).*
(2) *If T is onto, then there exists a closed subset H_1 of G_1 such that H_1 is homeomorphic to G_2.*
(3) *If T is injective and preserves the functions with compact support, then G_1 is homeomorphic to a quotient of G_2.*
(4) *If T is a bijection, then G_1 and G_2 are homeomorphic.*

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PROOF. (1) Let us first show that $h^{-1}(G_1) = G_{2c}$. Suppose that there exists $s \in G_2$ such that $h(s) \in G_1$ and $T^t s^t$ is not continuous. Then, by Proposition 3, there is some $f \in C(G_1)$ such that $f(h(s)) = 0$ but $Tf(s) \neq 0$. Since $T1(s) \neq 0$, we can choose $z \in C(G_1)$ such that $z \equiv 1$ on a neighbourhood U of $h(s)$, while

$$\sup\{|fz(t)| : t \in G_1\} < |Tf(s)|/|T1(s)|.$$

Let

$$g := 1 - \frac{T1(s) \cdot fz}{T(fz)(s)}.$$

Since $T(fz)(s) = Tf(s)$ by Proposition 1(2), it follows that g is non-vanishing, but $Tg(s) = 0$, which contradicts the non-vanishing-preserving property of T.

We next show that $h(G_2) \subseteq G_1$. Let us suppose that $h(s_0) \in (\beta G_1 \backslash G_1)$ for some $s_0 \in G_2$. Since G_1 is real-compact, there exists ([23, p.81]) a function $f_0 \in C(\beta G_1)$ such that $f_0(h(s_0)) = 0$ and f_0 never vanishes on G_1. Hence, by hypothesis, $Tf_0(s_0) \neq 0$. Let $z_0 \in C(\beta G_1)$ be such that $z_0 \equiv 1$ on $U \cap G_1$, where U is a neighbourhood of $h(s_0)$ in βG_1 and

$$\sup\{|f_0 z_0(t)| : t \in G_1\} < |Tf_0(s_0)|/|T1(s_0)|.$$

If we take a function g_0 like g above, we get the same contradiction.

By combining the preceding two paragraphs, we have $G_2 = G_{2c}$ and, therefore, T is continuous from Proposition 3.

(2) From Propositions 2 and 3, we know that the range of h is in G_1 and $Tf(s) = T1(s) \cdot f(h(s))$ for all $f \in C(G_1)$ and all $s \in G_2$. In addition, h is injective since T is onto.

We will first prove that if K is a compact subset of G_1, then $h^{-1}(K)$ is a compact subset of G_2. Otherwise, $h^{-1}(K)$ cannot be pseudo-compact since it is real-compact. Consequently, if $T1$ is bounded on $h^{-1}(K)$, there is a sequence (s_n) in $h^{-1}(K)$ and a function $g \in C(G_2)$ such that $|g(s_n)| > n \cdot |T1(s_n)|$ for all $n \in \mathbb{N}$. On the other hand, if $T1$ is not bounded on $h^{-1}(K)$, there is a sequence (s_n) in $h^{-1}(K)$ such that $|T1(s_n)| > n$. Thus, by taking $g = (T1)^2$, we also have $|g(s_n)| > n \cdot |T1(s_n)|$ for all $n \in \mathbb{N}$. Then, whether $T1$ is bounded on $h^{-1}(K)$ or not, take $f \in C(G_1)$ such that $Tf = g$. Since $T1$ is non-vanishing, we deduce that $n \cdot |T1(s_n)| < |Tf(s_n)| = |T1(s_n)| \cdot |f(h(s_n))|$. Hence, f is not bounded on K, which is a contradiction.

Let us show now that $h(G_2)$ is a closed subset of G_1. Assume that $t \in cl_{G_1}(h(G_2))$ and let U be any compact neighbourhood of t. Thus, $h^{-1}(U)$ is a compact subset of G_2. Since $h^{-1}(U) = h^{-1}(U \cap h(G_2))$, we obtain that $(U \cap h(G_2))$ is a compact subset of G_1. That is, $t \in (U \cap h(G_2))$.

Let $h^* : G_2^* \to (h(G_2))^*$ be a map defined by the requirement that $h^*(\infty)$ be ∞ and $(h^*)_{|G_2} \equiv h$. To prove the continuity of h^*, it suffices to check that h^* is continuous at

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∞. Let V be a neighbourhood of ∞ in $(h(G_2))^*$. Hence there exists a compact subset K of $h(G_2)$ such that $(h(G_2)\backslash K) \subseteq$V. Since $h^{-1}(K)$ is compact in G_1, $(G_2\backslash h^{-1}(K))$ is a neighbourhood of ∞ and $h(G_2\backslash h^{-1}(K)) \subseteq (h(G_2)\backslash K) \subseteq$V.

As a consequence, h^* is a continuous bijection between G_2^* and $(h(G_2))^*$, which implies that h is a homeomorphism of G_2 onto $h(G_2)$. It is now clear that $H_1 := h(G_2)$ satisfies the required conditions.

(3) By (1), we know that T is automatically continuous, and thus we can write $Tf(s) = T1(s) \cdot f(h(s))$ for every $f \in C(G_1)$ and every $s \in G_2$.

Now let us prove that $h^{-1}(K)$ is compact for every compact subset K of G_1. Take $f \in C_{00}(G_1)$ with $K \subseteq \text{coz}(f)$. Then $h^{-1}(K) \subseteq h^{-1}(G_1)$. Furthermore, if $s \in h^{-1}(\text{coz}(f)) \subset h^{-1}(G_1)$, then $f(h(s)) \neq 0$, which implies that $Tf(s) \neq 0$. That is, $h^{-1}(K)$ is a closed subset of the compact set $\text{supp}(Tf)$.

As in the proof of (3), we obtain that the map $h^* : G_2^* \to G_1^*$ is continuous and, consequently, closed. In addition, we know that the range of h is dense in G_1 since T is injective. Hence, it is clear that h^* is onto, closed, and continuous and so is h.

Therefore, by [9, Proposition 2.4.3], it is now evident that G_1 is homeomorphic to a quotient of G_2.

(4) We first show that $h(G_2)$ is C-embedded (see [14] for the definition) in G_1. Let $g_0 \in C(h(G_2))$. Let us consider the map $G : C(h(G_2)) \to C(G_2)$ defined by the requirement that $G(g)$ be $T1 \cdot (g \circ h)$ for every $g \in C(h(G_2))$. Since T is onto, there exists a function $f_0 \in C(G_1)$ such that $Tf_0 = G(g_0)$. Hence, $T1(s) \cdot f_0(h(s)) = T1(s) \cdot g_0(h(s))$ for every $s \in G_2$. Since $T1$ is non-vanishing, we have $f_0 \circ h \equiv g_0 \circ h$; that is, $f_{0|h(G_2)} \equiv g_0$ and, thus, the function $f_0 \in C(G_1)$ is the required extension.

Since T is injective, we know the range of h is dense in G_1. By combining this assertion, [14, Theorem 8.6], and the preceding paragraph, we deduce that $G_1 = \upsilon(h(G_2))$, where $\upsilon(h(G_2))$ stands for the real-compactification of $h(G_2)$.

Since T is onto, we know that h is injective. Let us consider $h^{-1} : h(G_2) \to G_2$, the inverse of h. On the other hand, it is clear that G is injective. Therefore, we can consider $G^{-1} : C(G_2) \to C(h(G_2))$, the inverse of G. It is also easy to verify that $G^{-1}(g) = g \circ h^{-1}/T1$ for every $g \in C(G_2)$. Hence, G^{-1} is also a separating map and its support map is h^{-1}. From Proposition 1 we have h^{-1} is continuous. By [14, Theorem 8.6], h^{-1} has a continuous extension $(h')^{-1} : G_1 \to G_2$. As a consequence, $(h \circ (h')^{-1})$ and the identity of G_1 coincide on $h(G_2)$ and, therefore on G_1, by the density of $h(G_2)$ in G_1. The proof of (4) is now complete.

REMARKS. (1) Theorem 2 is also valid when G_1 and G_2 are only real-compact spaces. However, if G_1 and G_2 are not real-compact, then Theorems 1 and 2 may fail. As an example, consider G_1 a pseudo-compact, not real-compact space and υG_1 its real-compactification, which is compact. It is easy to see that the natural isomorphism of $C(G_1)$ onto $C(\upsilon G_1)$ is a discontinuous separating bijection which preserves non-vanishing functions and satisfies condition (M).

However, let us recall (see [11]) that a separating bijection from $C_0(G_1)$ (respectively $C_{00}(G_1)$) onto $C_0(G_2)$ (respectively $C_{00}(G_2)$) is automatically continuous and induces a homeomorphism between the locally compact (not necessarily real-compact) spaces G_1 and G_2.

There remains open a complete answer to the following question: When is the inverse of a separating bijection defined from $C(X)$ onto $C(Y)$ (X, Y completely regular Hausdorff spaces) also separating?. Partial answers to this problem are then provided by Theorem 1 and Theorem 2 (4). In fact, such question was first proposed by Abramovich in the more general context of vector lattices. The first affirmative answer in this direction is the result (op. cit.) by Jarosz ([20]). Recently, Huijsmans and de Pagter ([18]) have proved the following: An invertible disjointness preserving operator from a Banach lattice onto a normed vector lattice has a disjointness preserving inverse and is norm bounded (=continuous).

(2) The following example shows that preserving functions with compact support is not a redundant hypothesis in Theorem 2(3): let us consider the map $T : C(\beta G_2) \to C(G_2)$, where βG_2 denotes the Stone-Cech compactification of G_2, and such that $Tf = f_{|G_2}$ for every $f \in C(\beta G_2)$. It is easy to verify that T is a separating injection which preserves non-vanishing functions. However, βG_2 is not homeomorphic to a quotient of G_2.

(3) Finally, let us show, with an example due to Beckenstein and Narici, that there exist discontinuous separating injections: Let $G_1 = [0, 1]$ and $G_2 = [0, 1] \cup \{2\}$. For any $f \in C(G_1)$, let $Tf(s) = (1 - s) \cdot f(s)$ if $s \in [0, 1]$ and $Tf(2) = g(f)$, where g is a discontinuous separating functional defined like Example 3.6 in [6].

4. Algebraic characterization of locally compact Abelian groups

Throughout this section, G_1 and G_2 will be locally compact Abelian (LCA) groups. Let us recall that every locally compact group is real-compact. We say that a separating map T of $C(G_1)$ into $C(G_2)$ is *character preserving* if given a character χ of G_1, that is, a complex-valued continuous homomorphism on G_1, then $T\chi$ is a character of G_2.

COROLLARY 1. *If there exists a character preserving separating bijection T from $C(G_1)$ onto $C(G_2)$ which either satisfies condition (M) or preserves non-vanishing functions, then the LCA groups G_1 and G_2 are topologically isomorphic.*

PROOF. By Theorems 1 and 2, we know that T is a weighted composition map and G_1 and G_2 are homeomorphic. Hence, it suffices to check that the support map h of T is a homomorphism.

Take $s_1, s_2 \in G_2$ and let χ be any character of G_1. Hence, both $T\chi(s_1 \cdot s_2) = T1(s_1 \cdot s_2) \cdot \chi(h(s_1 \cdot s_2))$ and $T\chi(s_1 \cdot s_2) = T\chi(s_1) \cdot T\chi(s_2) = T1(s_1) \cdot \chi(h(s_1)) \cdot T1(s_2) \cdot \chi(h(s_2)) = T1(s_1 \cdot s_2) \cdot \chi(h(s_1)) \cdot \chi(h(s_2))$. Since $T1$ is non-vanishing, we have $\chi(h(s_1 \cdot s_2)) = \chi(h(s_1)) \cdot \chi(h(s_2)) = \chi(h(s_1) \cdot h(s_2))$ for every character χ of G_1, which implies that $h(s_1 \cdot s_2) = h(s_1) \cdot h(s_2)$ and we are done.

COROLLARY 2. *Let T be a separating map of $C(G_1)$ into $C(G_2)$ which preserves non-vanishing functions and characters.*

(1) *If T is onto, then there exists a closed subgroup H_1 of G_1 such that H_1 is topologically isomorphic to G_2.*
(2) *If T is injective and preserves functions with compact support, then there exists a closed subgroup H_2 of G_2 such that G_1 is topologically isomorphic to G_2/H_2.*

PROOF. This follows from Theorem 2 (2) and Corollary 1.
(2) This follows from Theorem 2 (3) and Corollary 1, H_2 being the kernel of the homomorphism h.

REMARK. If we consider the Bohr compactification bG_2 of G_2 instead of its Stone-Cech compactification in the remark following Theorem 2, then T is also character preserving. Despite this, it is also evident that preserving functions with compact support is still not redundant.

5. Algebraic characterization of locally compact groups

Throughout this section, G_1 and G_2 are locally compact (not necessarily Abelian) groups. Let us also regard $C_{00}(G_i)$ $(i = 1, 2)$ as an algebra under the convolution product $(f * g)(s) = \int_{G_i} f(z) \cdot g(z^{-1} \cdot s)dz$, integration being with respect to a fixed left Haar measure denoted by dz. Let $C_{00}^+(G_i) = \{f \in C_{00}(G_i) : f \geq 0\}$.

DEFINITION 5. We say that a linear operator T of $C_{00}(G_1)$ into $C_{00}(G_2)$ satisfies condition (P) if given $s \in G_2$ and $f, g \in C_{00}^+(G_1)$ such that $T(f * g)(s) = 0$, then $(Tf * Tg)(s) = 0$.

It is evident that every convolution algebra homomorphism satisfies condition (P).

LEMMA 2. *Let $T : C_{00}(G_1) \to C_{00}(G_2)$ be a map defined by the requirement that $Tf = X(f \circ h)$ where X is a non-vanishing scalar-valued continuous function defined on G_2 and h is a continuous mapping from G_2 into G_1. Then the following statements are equivalent:*

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(1) *h is a closed group homomorphism.*
(2) *T satisfies condition (P).*

PROOF. (1) implies (2) Let us suppose that $f, g \in C_{00}^{+}(G_1)$ and $s \in G_2$ are such that $T(f*g)(s) = 0$. Then, since X does not vanish on G_2, we infer that $(f*g)(h(s)) = 0$; that is, $\int_{G_1} f(z)g(z^{-1} \cdot h(s))dz = 0$. Hence, from the continuity of f and g, we have that $f(z)g(z^{-1} \cdot h(s)) = 0$ for all $z \in G_1$.

On the other hand,

$$\begin{aligned}(Tf * Tg)(s) &= \int_{G_2} Tf(y)Tg(y^{-1} \cdot s)dy \\ &= \int_{G_2} X(y)X(y^{-1} \cdot s)(f \circ h)(y)(g \circ h)(y^{-1} \cdot s)dy.\end{aligned}$$

Since h is a homomorphism, we deduce that

$$(f \circ h)(y)(g \circ h)(y^{-1} \cdot s) = (f(h(y))(g((h(y))^{-1} \cdot h(s)) = 0$$

for all $y \in G_2$. As a consequence, it is now clear that $(Tf * Tg)(s) = 0$, as was to be proved.

(2) implies (1). Firstly let us see that h is a closed mapping. As in the proof of Theorem 2 (3), we have that $h^{-1}(K)$ is compact for every compact subset K of G_1. Then the map $h^* : G_2^* \to G_1^*$, defined as in Theorem 2 (2), is continuous. Consequently, it is closed and so is h.

Next suppose that $h(s \cdot t) \neq h(s) \cdot h(t)$ for some $s, t \in G_2$. Then we can find open neighbourhoods U and V of $h(s)$ and $h(t)$ respectively such that $h(s \cdot t) \notin U \cdot V$. Let $f, g \in C_{00}^{+}(G_1)$ be such that $f(h(s)) > 0$, $g(h(t)) > 0$, f vanishes outside U and g vanishes outside V.

Suppose firstly that X is real-valued and let us assume, without loss of generality, that $X(s) > 0$ and $X(t) < 0$. Hence there exists a compact neighbourhood W_1 of the unit of G_2 such that $X_{|sW_1} > 0$ and $X_{|W_1 t} < 0$. Let us take a compact neighbourhood W of the unit of G_2 such that $W \subset \mathrm{int}(W_1)$. If $g_{h(s\cdot t)}$ stands for a function on G_1 defined by the requirement that $g_{h(s\cdot t)}(z) = g(z \cdot h(s \cdot t))$ for all $z \in G_1$, then it is easy to check that

$$\int_{sW} X(y)X(y^{-1} \cdot s \cdot t)(f \circ h)(y)(g_{h(s\cdot t)} \circ h)(y^{-1})dy = c < 0.$$

Take now $k_1 \in C_{00}^{+}(G_1)$ such that $0 \leq k_1 \leq 1$, $k_1(h(s)) = 1$ and $k_1 = 0$ outside $h(sW)$. Similarly, let us consider $k_2 \in C_{00}^{+}(G_1)$ such that $0 \leq k_2 \leq 1$, $k_2(h(t)) = 1$ and $k_2 = 0$ outside $h(Wt)$. Thus we have $((k_1 f) * (k_2 g))(h(s \cdot t)) = 0$. Since T is a weighted composition map, $T((k_1 f) * (k_2 g))(s \cdot t) = 0$.

On the other hand and reasoning as above, we have

$$\begin{aligned}(T(k_1 f) * T(k_2 g))(s\cdot t) &= \int_{G_2} X(y)X(y^{-1}\cdot s\cdot t)(k_1(h(y))(k_2(h(y^{-1}\cdot s\cdot t))(f(h(y))(g_{h(s\cdot t)}(h(y^{-1}))dy \\ &= \int_{sW} X(y)X(y^{-1}\cdot s\cdot t)(k_1(h(y))(k_2(h(y^{-1}\cdot s\cdot t))(f(h(y))(g_{h(s\cdot t)}(h(y^{-1}))dy \\ &\neq 0,\end{aligned}$$

which contradicts condition (P). Assume now that X is complex-valued, that is, $X = X_1 + iX_2$. Hence

$$\begin{aligned}X(y)X(y^{-1}\cdot s\cdot t) = (X_1(y)X_1(y^{-1}\cdot s\cdot t) - X_2(y)X_2(y^{-1}\cdot s\cdot t)) \\ + i(X_2(y)X_1(y^{-1}\cdot s\cdot t) + X_1(y)X_2(y^{-1}\cdot s\cdot t)).\end{aligned}$$

If we denote $\gamma(y) = X_1(y)X_1(y^{-1}\cdot s\cdot t) - X_2(y)X_2(y^{-1}\cdot s\cdot t)$, we will assume, without loss of generality, that $\gamma(s) > 0$. As a consequence, there is a compact neighbourhood W of the unit of G_2 such that for all $u \in sW$ and all $v \in Wt$, $X_1(u)X_1(v) - X_2(u)X_2(v) > 0$. The remainder of the proof follows from the same arguments as in the real-valued case.

THEOREM 3. *Let T be a separating bijection of $C_{00}(G_1)$ onto $C_{00}(G_2)$. Then the following statements are equivalent:*

(1) *There exists a topological isomorphism h of G_2 onto G_1 such that $Tf = X(f\circ h)$.*
(2) *T satisfies condition (P).*

PROOF. According to Theorem 3 and Corollary in [11], we know that $T(f) = X(f\circ h)$ where X is a non-vanishing continuous map defined on G_2 and h is a homeomorphism of G_2 onto G_1. Everything else is a consequence of Lemma 2.

COROLLARY 3. *Let T be a linear bijection of $C_{00}(G_1)$ onto $C_{00}(G_2)$ which is either bipositive or isometric. Then the following statements are equivalent:*

(1) *There exists a topological isomorphism h of G_2 onto G_1 such that $Tf = X(f\circ h)$.*
(2) *T satisfies condition (P).*

PROOF. From Theorem 3, it suffices to check that T is a separating map in both cases. The isometric one is already proved in [11, Proposition 5].

Let us suppose that T is a bipositive bijection. Given $s \in G_2$, let us define

$$F_s = \{g \in C_{00}^{+}(G_2) : g(s) > 0\}.$$

Fix $s_0 \in G_2$. We shall show that $\bigcap_{Tf \in F_{s_0}} \operatorname{supp}(f) \neq \emptyset$. Since, for every $f \in C_{00}^{+}(G_1)$, $\operatorname{supp}(f)$ is a compact subset of G_1, it is enough to prove that $\bigcap_{Tf \in F} \operatorname{supp}(f) \neq \emptyset$ for every finite subset F of F_{s_0}. There exist an open neighbourhood V of s_0 and a positive constant λ such that $Tf_{|V} > \lambda$ for all $Tf \in F$. Since T is onto, we can choose $f_1 \in C_{00}^{+}(G_1)$ such that, multiplying by a constant if necessary, $\|Tf_1\| \leq \lambda$ and $\operatorname{supp}(Tf_1) \subset V$. The positivity of the inverse of T then implies that $\emptyset \neq \operatorname{supp}(f_1) \subset \bigcap_{Tf \in F} \operatorname{supp}(f)$.

From the above arguments we derive the following: for every $s \in G_2$, there exists a subset $I_s \subset G_1$ such that, if $Tf(s) > 0$ for some $f \in C_{00}^{+}(G_1)$, then $t \in \operatorname{supp}(f)$ for all $t \in I_s$. That is, if for some $f \in C_{00}^{+}(G_1)$ and some open neighbourhood $U(t)$ of any $t \in I_s$ we have $f_{|U} \equiv 0$, then $Tf(s) = 0$.

Again fix $s_0 \in G_2$ and let us take $f_0 \in C_{00}^{+}(G_1)$ such that $f_0(t_0) = 0$ for some $t_0 \in I_{s_0}$. We will show that $Tf_0(s_0) = 0$. Let (f_α) be a net $C_{00}^{+}(G_1)$ such that (f_α) converges to f_0 and $f_\alpha \equiv 0$ on some open neighbourhood V_α of t_0 for all α. Consequently, $T^t s_0^t(f_\alpha) = 0$ for all α.

Since $T^t s_0^t$ is a positive linear functional on $C_{00}(G_1)$, it is well known that, given a compact subset K of G_1, $|T^t s_0^t(f)| \leq C_K \cdot \|f\|$, C_K a constant, for all $f \in C_{00}^{+}(G_1)$ whose support is contained in K. Without loss of generality, we can assume that $\operatorname{supp}(f_\alpha) \subset \operatorname{supp}(f_0)$ for all α. Hence the net $(T^t s_0^t(f_\alpha))$ converges to $T^t s_0^t(f_0)$, that is, $T^t s_0^t(f_0) = Tf_0(s_0) = 0$.

In general, if we choose $f_0 \in C_{00}(G_1)$ such that $f_0(t_0) = 0$ for some $t_0 \in I_{s_0}$, then it is easy to check that $Tf_0(s_0) = 0$.

Finally, let $f_1, f_2 \in C_{00}(G_1)$ be such that $f_1 \cdot f_2 \equiv 0$. Let us suppose that there exists $s_0 \in G_2$ such that both $Tf_1(s_0) \neq 0$ and $Tf_2(s_0) \neq 0$. Then, since either $f_1(t) = 0$ or $f_2(t) = 0$ for any $t \in I_{s_0}$, we have either $Tf_1(s_0) = 0$ or $Tf_2(s_0) = 0$, which contradicts the above assumption and proves that T is separating.

REMARK. In [8] (see also [24]), Edwards proved that if there exists a convolution algebra isomorphism of $C_{00}(G_1)$ onto $C_{00}(G_2)$ which is either bipositive or isometric, then G_1 and G_2 are isomorphic topological groups. Theorem 3 and its corollary above are then extensions of Edwards' results.

THEOREM 4. *Let T be a continuous separating map of $C_{00}(G_1)$ onto $C_{00}(G_2)$. Then the following statements are equivalent:*

(1) *There exists a topological isomorphism $h : G_2 \to G_1$ such that $Tf = X(f \circ h)$ (indeed, G_2 is topologically isomorphic to a closed subgroup of G_1).*
(2) *T satisfies condition (P).*

PROOF. Let us suppose that T satisfies condition (P). Since T is continuous, it can be written as a weighted composition map (see [11]); namely, $Tf = X(f \circ$

h). By applying Lemma 2, we infer that h is a closed injective continuous group homomorphism. Consequently, h is a topological isomorphism.

The converse falls inmediately out of Lemma 2.

THEOREM 5. *Let T be a continuous separating injection of $C_{00}(G_1)$ into $C_{00}(G_2)$. Then the following statements are equivalent:*

(1) *There exists a topological homomorphism h of G_2 onto G_1 such that $Tf = X(f \circ h)$ (indeed, G_1 is topologically isomorphic to a quotient of G_2).*

(2) *T satisfies condition (P).*

PROOF. Let us suppose that T satisfies condition (P). Arguments like those in Corollary 2 and Theorem 4 show that G_1 is topologically isomorphic to $G_2/\ker(h)$. The converse follows from Lemma 2.

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