A HERTZSPRUNG-RUSSELL DIAGRAM FOR STELLAR OSCILLATIONS

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ABSTRACT. I present evolutionary tracks and curves of constant central hydrogen abundance in diagrams based on frequencies of high-order, low-degree p modes. For stars with masses between 0.7 and 1.5 $M_{\odot}$, a clean separation is obtained between the effects of varying mass and varying evolutionary state.

1. INTRODUCTION.

In analyses of observations of low-degree oscillations, spacings between the frequencies are particularly useful, since they can be determined without knowledge about the absolute radial order of the modes. From asymptotic theory the spacings have simple interpretations in terms of the structure of the star (*e.g.* Tassoul 1980, Gough 1986). Consequently they have been of great use in the analysis of observations of low-degree solar oscillations, and in the initial study of the corresponding stellar oscillations (*e.g.* Gelly, Grec & Fossat, these proceedings). Christensen-Dalsgaard (1984) introduced a two-dimensional diagram, similar to a Hertzsprung-Russell diagram, in terms of two such average spacings. Here I present preliminary results of a more extensive calculation.

2. COMPUTATIONS.

I have computed evolution sequences for models with masses from 0.7 to 2.0 $M_{\odot}$, extending to the end of central hydrogen burning. The physics of the calculation was as described in Christensen-Dalsgaard (1982). The heavy metal abundance was $Z = 0.02$, and the initial hydrogen abundance and mixing length parameter were adjusted to get a correct model of the present Sun. The subsequent analysis is based on adiabatic oscillations for p modes with $\ell = 0 - 3$, for selected models (typically 7) in each sequence.

The asymptotic relations for the p mode frequencies $\nu_{n,\ell}$ motivate a fit on the form (Scherrer *et al* 1983; Christensen-Dalsgaard 1984)

$$\nu_{n,\ell} \approx \bar{\nu}_{\ell} + \Delta\nu_{\ell} x + \gamma_{\ell} x^2 + \cdots \tag{1}$$

where $x = n + \ell/2 - n_0$ and n_0 is a suitably chosen reference order. Furthermore, to leading order, $\bar{\nu}_{\ell}$ and $\Delta\nu_{\ell}$ are linear functions of $\ell(\ell+1)$, *i.e.*

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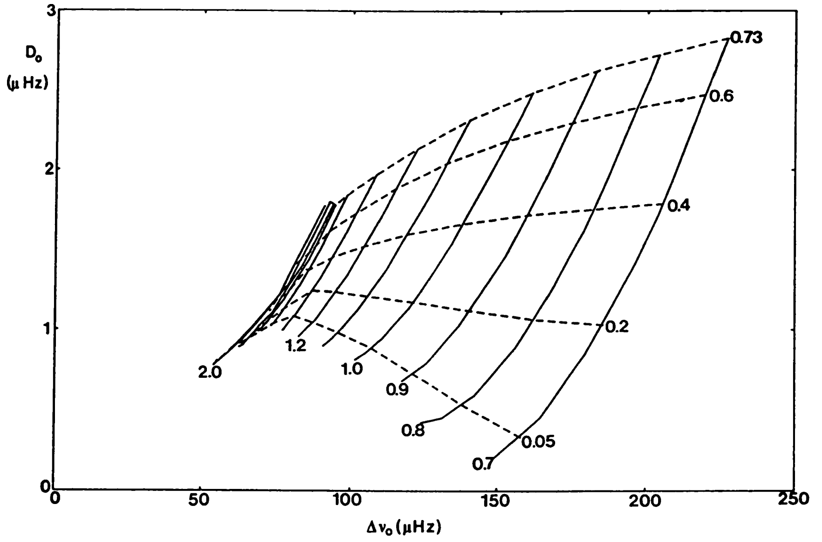

Figure 1. Evolution tracks ———————— and isopleths, - - - - - - - - - in an "oscillation HR-diagram". The masses, in solar units, and the values of the central hydrogen abundance for the isopleths, are indicated.

$$\bar{\nu}_\ell \approx \nu_0 - \ell(\ell+1)D_0, \qquad \Delta\nu_\ell \approx \Delta\nu_0 + \ell(\ell+1)d_0. \tag{2}$$

Here, according to the asymptotic relations,

$$\Delta\nu_0 \approx \left[2\int_0^R \frac{dr}{c}\right]^{-1} \tag{3}$$

and

$$D_0 \approx \frac{1}{4\pi^2(n_0 + \varepsilon)}\left[\frac{c(R)}{R} - \int_0^R \frac{dc}{dr}\,\frac{dr}{r}\right], \tag{4}$$

where ε is determined by conditions near the surface of the model.

The computed frequencies, at each degree ℓ from 0 to 3, were fitted to equation (1), including the x^2-term. Modes with $n + \ell/2$ between 14 and 30 were included; n_0 was 22. The resulting $\bar{\nu}_\ell$ and $\Delta\nu_\ell$ were fitted to relations (2), to obtain ν_0, D_0, $\Delta\nu_0$ and d_0. From these I finally calculated $\varepsilon_0 = \nu_0/\Delta\nu_0 - n_0$, as an approximation to ε. Thus, to this accuracy, the oscillations are characterized by the set $\Delta\nu_0$, D_0, ε_0, d_0. The parameter D_0 is related to the small separations between the asymptotically almost coincident frequencies by $\nu_{n,0} - \nu_{n-1,2} \approx 6D_0$, and $\nu_{n,1} - \nu_{n-1,3} \approx 10D_0$.

The results are presented on Fig. 1, in terms of a $(\Delta\nu_0, D_0)$ diagram. Here are shown evolutionary tracks, as well as *isopleths*, connecting stars with the same central hydrogen abundance, but different mass. These have been used, instead of the more familiar isochrones, because they give a better indication of the evolutionary state of the star. Fig. 2 shows the location of the evolutionary tracks and the ZAMS in a $(\Delta\nu_0, \varepsilon_0)$ diagram.

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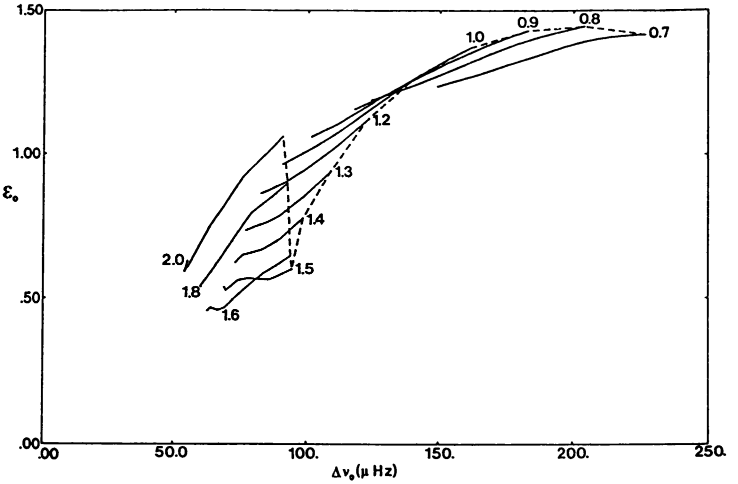

Figure 2. Evolution tracks ———————— and the ZAMS - - - - - - - - - in a ($\Delta\nu_0$, ε_0) diagram. The masses, in solar units, are indicated.

3. DISCUSSION.

The ($\Delta\nu_0$, D_0) diagram provides a clean separation of the effects of changing mass and changing evolutionary state, for masses below about 1.4 $M_\odot$. This indicates that an observational determination of $\Delta\nu_0$ and D_0 can be used to determine stellar masses and ages. For higher masses the evolutionary tracks almost coincide, whereas D_0 still provides a measure of evolutionary state. In fact $\Delta\nu_0$ is largely a measure of mean density (*e.g.* Ulrich 1986), which varies little along the main sequence for stars with masses between 1.5 and 2 $M_\odot$. As indicated by equation (4), D_0 is sensitive to the sound speed near the centre, which in turn is strongly affected by the changes in chemical composition due to evolution. Notice that the ($\Delta\nu_0$, D_0) diagram complements the normal HR diagram, which has difficulties separating mass from age in the low-mass region, the evolution tracks being almost parallel to the ZAMS. The variation with stellar mass and evolutionary state in ε_0 is limited to the range 0.5 to 1.5. This may mean that the mode order can be identified unambiguously from the observed frequencies, given the determination of $\Delta\nu_0$ and ν_0 from the observations.

To measure how well the frequencies are represented by the fit, one can consider the departure $\nu_{n,\ell} - \nu_{n,\ell}^{(\mathrm{fit})}$, where

$$\nu_{n,\ell}^{(\mathrm{fit})} = \nu_0 - \ell(\ell+1)D_0 + [\Delta\nu_0 + \ell(\ell+1)d_0](n + \frac{\ell}{2} - n_0). \qquad (5)$$

Fig. 3 shows the departures, for 1 $M_\odot$ and 1.5 $M_\odot$ models on the ZAMS. Being several μHz, they are highly significant in observations where the separation between the individual modes can be resolved. They clearly correspond to the γ_ℓ term and higher-order terms in x in equation (1). It is interesting that they are substantially smaller for 1.5 $M_\odot$ than for 1 $M_\odot$.

I have not yet investigated the sensitivity of the results to other parameters in stellar evolution calculations, such as the chemical composition and the treatment of convection. This would influence their use to measure

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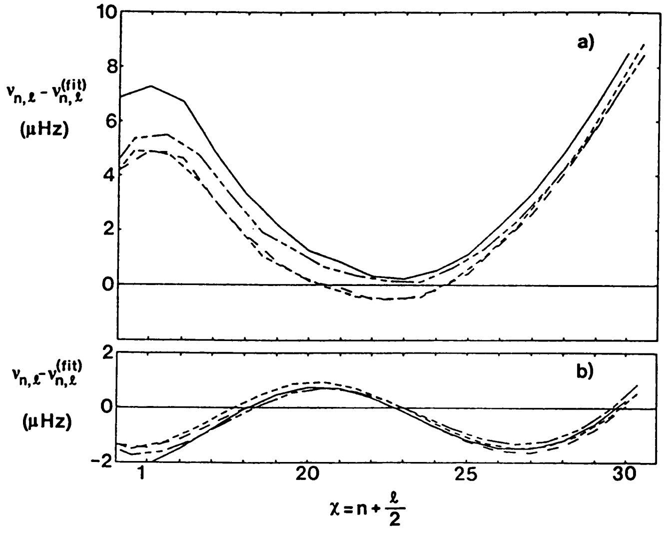

Figure 3. Departure $\nu_{n,\ell} - \nu^{(fit)}_{n,\ell}$ of the actual frequencies from the fit, for a 1 $M_\odot$ model (a) and a 1.5 $M_\odot$ model (b), on the ZAMS. The curves are ————: $\ell = 0$, - - - - - -: $\ell = 1$, — — — — —: $\ell = 2$, - -——- -——: $\ell = 3$.

stellar mass and evolutionary status. Also required is a better understanding of the asymptotic theory. Equation (4) is satisfied fortuitously for models of the present Sun, but for other stellar models the values obtained for D_0 deviate substantially from the asymptotic prediction. A reason may be that dc/dr varies rapidly near the centre of the star, compared with the wavelength of the modes, thus compromising the asymptotic analysis. Finally we must investigate the departure from the fit. *We clearly want to learn more about the stars than their masses and ages.* This requires investigation of the precise information content in a given set of modes, observed to a given accuracy.

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REFERENCES:

Christensen-Dalsgaard, J., 1982. *Mon. Not. R. astr. Soc.*, **199**, 735.

Christensen-Dalsgaard, J., 1984. *Space Research Prospects in Stellar Activity and Variability*, (ed. Mangeney, A. & Praderie, F., Observatoire de Paris), p. 11.

Gough, D. O., 1986. *Highlights in Astronomy*, **7**, 283.

Scherrer, P. H., Wilcox, J. M., Christensen-Dalsgaard, J. & Gough, D. O., 1983. *Solar Phys.*, **82**, 75.

Tassoul, M., 1980. *Astrophys. J. Suppl.*, **43**, 469.

Ulrich, R. K., 1986. *Astrophys. J.*, **306**, L37.