Hence $c - b = m + n - \sqrt{4mn} = (\sqrt{m} - \sqrt{n})^2$
$=$ (diff. of odd and even numbers)^2 = (odd number)^2 = odd square.

So in all cases c exceeds a or b by odd square.

LAWRENCE CRAWFORD.

New Clock Problems.—The following problem was suggested by a difficulty in distinguishing between the two hands of a clock.

"The hour and minute hands of one clock A are parallel respectively to the minute and hour hands of another clock B. The time on A is between 7 and 8 o'clock, that on B between 10 and 11 o'clock; find the time on each clock."

I. *Algebraic Solution.*—Let the times registered on the clocks be x minutes past seven, and y minutes past ten. We thus have

$$\left.\begin{array}{ll}(1) & x/12 + 35 = y \\ (2) & y/12 + 50 = x\end{array}\right\}$$

Multiplying (1) by $^1/_{12}$ and adding (2), we have

$$\tfrac{143}{144}x = 50 + \tfrac{35}{12} \text{ or } 143x = 7620,$$

Similarly $$\tfrac{143}{144}y = 35 + \tfrac{50}{12} \text{ or } 143y = 5640,$$

II. *Arithmetical Solution.*—Imagine two clocks one at 7 o'clock and the other at 10.35, and imagine the first clock to go twelve times as fast as the second. The hour hand of the first clock thus keeps parallel to the minute hand of the second, while the minute hand of the first goes 144 times as fast as the hour hand of the second. Hence we have at once

$$\tfrac{143}{144}x = 50 + \tfrac{35}{12}. \quad i.e., \ 143x = 7620.$$

Similarly we find y.

It is to be noticed that the idea involved in making the one clock go twelve times as fast as the other is identical with that used in eliminating one of the variables from equations (1) and (2) above.

If we are given only that the hour and minute hands of the one clock are parallel respectively to the minute and hour hands of the other, the problem of finding the time on each clock is indeterminate, although the number of solutions is finite. We can easily find the number of solutions by supposing two clocks to start from 12 o'clock and the one to go twelve times as fast as the other. It is easy to see that while the slower clock goes 12 hours the two clocks have the conditions satisfied 143 times. But all these combinations of readings are not different. Thus we have

A 7 hr. $53\frac{41}{143}$ min., B 10 hr. $39\frac{63}{143}$ min.; and A 10 hr. $39\frac{63}{143}$ min., B 7 hr. $53\frac{41}{143}$ min. On the other hand, however, solutions of the type A 1 hr. $5\frac{5}{11}$ min., B 1 hr. $5\frac{5}{11}$ min., when all four hands point in the same direction, only occur once. We therefore have in all 77 (*i.e.*, $66 + 11$) different combinations.

Problems can be made involving any relation between the directions of the hands in the two clocks.

Another type of clock problem is indicated by

Find the first time after 1 o'clock when the minute hand of a clock bisects the angle between the hour and seconds hands.

J. JACKSON.

Multiplication and Division of Vulgar Fractions.— The difficulty in teaching Multiplication and Division of Fractions consists solely in the fact that the ideas formerly associated with these operations when whole numbers were concerned no longer fit the case when we come to deal with fractions, *e.g.*, 9×7 can be intelligibly interpreted as meaning the number of articles in 9 bundles, each containing 7 articles. In this respect, multiplication is simply contracted addition. When we come to a case like $\frac{3}{5} \times \frac{2}{7}$, we cannot frame a question on the lines of the former. But if we present the problem in a concrete form we are more likely to be successful in teaching young pupils the method of multiplying and dividing vulgar fractions.

8 tons at 7/- per ton cost (8×7)/-.

So we may say that £$(8 \times \frac{3}{10})$ represents the cost of 8 tons at £$\frac{3}{10}$ per ton.

But £$\frac{3}{10}$ is 3 florins.

Therefore 8 tons at £$\frac{3}{10}$ per ton cost (8×3) florins, *i.e.*, £$\frac{8 \times 3}{10}$.

Therefore we now have that £$(8 \times \frac{3}{10})$ = £$\frac{8 \times 3}{10}$.

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