therefore, as $v' < v$, we have each term of the series for $A_{x+n} - A_x$ greater than the corresponding term of the series for $A'_{x+n} - A'_x$. But in the event of some of the terms being negative, which might easily be shown to be the case according to various mortality tables, we could not state which series is the greater. If the value of $A_{x+n} - A_x$ be written

$$v(q_{x+n} - q_x) + v^2(p_{x+n}q_{x+n+1} - p_x q_{x+1}) + \ldots\ldots$$

it becomes apparent that, when the mortality increases with the age,

$$A_{x+n} - A_x > A'_{x+n} - A'_x,$$

and consequently $$\frac{A_{x+n} - A_x}{A'_{x+n} - A'_x} > \text{unity}.$$

But as $$A_x > A'_x, \quad \frac{1 - A_x}{1 - A'_x} < \text{unity}, \quad \therefore {}_nV_x > {}_nV'_x.$$

In both of the demonstrations above referred to, the proof is founded on the assumption that $a_x > a_{x+1}$, and $>$ the annuity at all succeeding ages. Mr. Sprague, in vol. xxi, p. 94, referring to the theorem, investigates under what conditions $a_x < a_{x+1}$, and there proves that when such is the case we must have

$$vq_x > \varpi_{x+1}.$$

That is, the premium for a single year greater than the whole-life premium at the next higher age.

I am, Sir,
Your obedient servant,

Scottish Amicable Life Society, WM. G. WALTON.
Glasgow, 16 *May* 1879.

MR. GRAY'S METHODS OF CONSTRUCTING LIFE TABLES.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The example appended to my letter in your last number contained some errors (for which I am to blame), and it was not, as I now find, sufficiently explained.

Believing, as I do, that the process which I then sought to elucidate deserves more attention than it has hitherto gained, I should feel obliged by your permitting me to give a further illustration of it. For this purpose I will use the Institute H^M Table, which is accessible to all readers of the *Journal*.

The problem is to construct $\log N_x$, the log of D_x being given. This is Problem XXII, page 124 of Gray's *Tables and Formulæ*. Mr. Gray showed that the work brought out $\log a_x$ and $\log (1 + a_x)$ as well as $\log N_x$; but he did not specially notice the remarkable fact that, excepting the datum $\log D_x$, not a figure besides appears in the process, and he did not mention any further uses of these quantities.

On inspection of the following example it will be seen that there are two working columns, the respective contents of which will be best shown by a typical representation, thus:—

$\log \mathrm{N}_x$	
$\log \mathrm{D}_x$	$\log a_x$
$\log (1+a_x)$	a_x
$\log \mathrm{N}_{x-1})$	
$\log \mathrm{D}_{x-1}$	$\log a_{x-1}$
$\log (1+a_{x-1})$	a_{x-1}
$\log \mathrm{N}_{x-2}$	

Two complete steps are here exhibited.

The first portion of the work is the insertion of the initial value of $\log \mathrm{N}_x$, which at the oldest age is the same as $\log \mathrm{D}_{x+1}$, and of the successive values of $\log \mathrm{D}_x$ in their proper places, in reversed order, on paper ruled in the form above shown.

The subsequent working consists of, first, the subtraction of $\log \mathrm{D}_x$ from $\log \mathrm{N}_x$, setting the remainder (which is $\log a_x$), in the adjoining column in line with $\log \mathrm{D}_x$; second, the entering of the table of $\log (1+x)$ with this remainder, setting the result (which is $\log \overline{1+a_x}$), under $\log \mathrm{D}_x$; and third, the addition of this result to $\log \mathrm{D}_x$. The sum is $\log \mathrm{N}_{x-1}$, and so on.

Of the 27 values of $\log \mathrm{N}_x$ here formed with *six* figure logarithms, seven values differ by ± 1 from the corresponding values on page 28 of the Institute volume, which were found with *seven* figure logarithms. The annuities are all true in the last figure.

If $\log a_x$ only were wanted, $\log vp_x$ would take the place of $\log \mathrm{D}_x$ and the work would be shorter, the several quantities being additive. But the extra trouble in finding $\log \mathrm{N}_x$ is well repaid. Each value on the working sheet may be turned to good account in other processes. For example, $\log \mathrm{N}_{x+n} - \log \mathrm{D}_x$ gives the logarithm of a deferred annuity; and since the logarithms of a_x and $(1+a_x)$ are both in hand, the very useful result $\varpi_x = v - \frac{a_x}{1+a_x}$ can be found with peculiar facility.

I am indeed of opinion that Gray's Problem XXII is a memorable contribution to the practical valuation of life contingencies, and I think that Mr. Gray did not sufficiently appreciate his own incomparable process.

I am, Sir,

Yours faithfully,

London, 10 *November* 1879. J. HANNYNGTON.

Calculation of log N_x, H^M *Table, at* 4 *per-cent.*

96	$\bar{1}$·302009		87	409532		78	545008	
	0·054996 ·070630 2	$\bar{1}$·247013 (·1766)		042236 522335 67	367296 (2·3297)		934467 705735 33	610541 (4·0789)
95	·125628		86	564638		77	640235	
	512167 149398 18	613461 (·4106)		166982 543825 40	397656 (2·4984)		004403 726168 26	635832 (4·3235)
94	661583		85	710847		76	730597	
	836617 222246 26	824966 (·6683)		286326 563204 15	424521 (2·6578)		070280 746170 14	660317 (4·5742)
93	058889		84	849545		75	816464	
	087072 287159 8	971817 (·9372)		399660 581732 63	449885 (2·8176)		132281 765795 69	684183 (4·8326)
92	374239		83	981455		74	898145	
	292071 344017 37	082168 (1·2083)		505991 600770 48	475464 (2·9886)		190843 785104 2	707302 (5·0969)
91	636125		82	106809		73	975949	
	471982 390785 25	164143 (1·4593)		604654 620927 42	502155 (3·1780)		245441 804583 7	730508 (5·3766)
90	862792		81	225623		72	050031	
	631352 431963 25	231440 (1·7039)		696394 641693 22	529229 (3·3824)		296305 824225 22	753726 (5·6719)
89	063340		80	338109		71	120552	
	775180 468544 40	288160 (1·9416)		781284 663147 20	556825 (3·6043)		343945 843769 6	776607 (5·9787)
88	243764		79	444451		70	187720	
	910999 498489 44	332765 (2·1516)		860328 684662 18	584123 (3·8382)		388865 862856 48	798855 (6·2930)
87	409532		78	545008		69	251769	

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