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OBITUARY

WILFRED HALLIDAY COCKCROFT (1923–1999)

Life

Wilfred Halliday Cockcroft only became known as such after he was knighted in 1983, for he was always 'Bill' to everybody. A large Yorkshireman born and bred, son of a proudly skilled plumber, he moved in 1941 from Keighley Grammar School as an Exhibitioner to Balliol College, Oxford. A year later he was called into the Royal Air Force, to become a technical officer in the Far East. He returned to Oxford in 1946 with a group of talented ex-servicemen, and completed his undergraduate work before becoming a research student under Henry Whitehead. Having married Rhona, he needed a job, and so he joined the staff at Aberdeen University, where he completed his doctorate. His first promotion created a vacancy for me, and I personally found it a great general education to be with him, as well as a great luxury to have another topologist to talk to, at a time when topologists (and indeed, people with a 'modern' vocabulary) were rare. He was strongly extrovert, very sociable in Oxford, and got on well with the Aberdeen Professor, E. M. Wright, at a time when relations between junior staff and professors tended to be stiff; his relaxed manner with Henry Whitehead (and indeed everybody else) amazed those of us raised among the conventionally introverted. It was thus that Bill had begun to acquire some of his extensive *savoir faire* and confidence. Besides research, he was naturally drawn to reflect on the problem of teaching the huge 'First Ordinary' class in the (to us) strange traditions of a Scottish university. At that time, G. H. Hardy had been writing about 'Youth' in a way that was to damage women and the over-thirties, and this aroused fears in Bill that he might soon be too old for research in mathematics, so it was interesting to hear him make the surprising remark (for 1952), that he would want to encourage 'old' lecturers to write good school texts if the Hardyesque social climate was making them feel that their pukka research days were over. (In spite of some splendid examples, writing almost any mathematics book in those days was often regarded as creative death, though not by Hardy.) Bill seemed to be fascinated by textbooks (and listed such reading as one of his recreations); he read texts with ease, and thereby widened his mathematical knowledge, in contrast to Cambridge contemporaries of the time, who were brought up on a higher standard of lecturing that left many of them narrow.

In 1952 Bill attended his first conference abroad, in Austria, where he immediately became friends with Ianto Davies of Southampton, who persuaded Bill to join him there in 1957. By then he was the lone topologist in Aberdeen (although he had spent 1954–5 in Chicago with Saunders MacLane), so Bill was delighted to find that the Southampton staff included a core of well-read differential geometers; he became their natural leader at a time when Davies was preoccupied with administration in an old institution that had only in 1952 become a university. It needed new curricula, both for mathematics specialists and for the large number of engineering students, and to design these was a great challenge for Bill and colleagues. He was stimulated also by the presence of Bryan Thwaites, Professor of Theoretical Mechanics, who was soon to begin his work with the School Mathematics Project (SMP). At its inaugural conference in Southampton, Bill gave a plenary talk, and although he did not become involved with writing the SMP texts, he was later a member of the A-level advisory group.

After promotion to a Readership in Southampton, Bill was appointed in 1962 to the Grant Chair of Pure Mathematics in the University of Hull, another of the 'new' universities that had been freed from the old London University External Examination system. He attacked his new duties ferociously and soon became Dean of Science, in addition to running a Hall of Residence where he and Rhona were very popular (and now had two sons). At this time, there was great discussion in many countries about reform of mathematics curricula – in universities (resulting from Bourbaki, increasing student numbers from a wider range of backgrounds, and increasing failure rates), in secondary schools (because of increasing demands for secondary, and especially for comprehensive, education) and in primary schools (because they tended to be blamed for the 'failures' in the secondary schools, who were themselves similarly blamed by the universities). A common remedy was to get funds for a large curriculum project, following the American 'New Math' models that arose from the Cold War panic over the flight of Sputnik in 1957. (Details can be found in Griffiths and Howson ⟨**4**⟩.)

Besides the SMP in England, another of the larger projects was the Nuffield Project for primary schools; Bill was invited to become its first chairman (1963–71), and then to join the School Council's Mathematics panel. From this, Bill became surely the only Professor of Mathematics to write a book [**14**] for *parents*, to explain something of the mysteries of the new things their children were learning, which were totally outside their own experience. In 1968 he was Chairman of the Commonwealth Conference on Mathematics in Schools, Trinidad, where he made all the delegates feel equally welcome and important. (Developing countries needed new curricula, which had to reflect local constraints; to import materials from 'advanced' countries, without modification, could be disastrous.) When he came back from the conference, Bill remarked that Miss Biggs, HMI in London, had strongly stressed the necessity for education to be relevant to the children. Yet at a party in the evening, the participants from all over the world gathered round a piano to sing happily 'On Ilkley moor baht 'at'! Bill was acquiring a taste for committee work, and with his unusually strong streak of common sense he clearly excelled in the art of chairmanship, being good at obtaining consensus, at sensing when to close a discussion, and at extracting ideas without imposing his own.

With such skills, then, Bill had become a member, and later Chairman (1969–73), of the Mathematics sub-committee of the Science Research Council (SRC), in a period when he persuaded the SRC to allocate funds for British mathematicians to visit the IHES in Bures-sur-Yvette, long before such co-operation with Europe became routine. From 1973 until 1976 he served on the Council of the LMS. During this period also, Bill was on various sub-committees of the University Grants Committee; and perhaps it was here that he came to the notice of those personages who recommend such things – so that Bill succeeded the first Vice-Chancellor of the new University of Coleraine in the troubled Province of Northern Ireland, and served from 1976 until 1982. He thus became a member of 'the Establishment', and so was at risk of attack from terrorists, but he and Rhona did much to humanise and warm the social life; he himself gave a course of evening lectures on calculus in nearby Derry at the college where the unconsulted locals would have preferred to see their local university established. This was just another example of his commitment to mathematics education at a time when other academic mathematicians often thought it sufficient to write a letter to *The Times*.

Meanwhile Bill was still serving on the SRC, and a host of other committees (listed in his *Who's Who* entry), but in 1978 he was chosen to chair what became known as the Cockcroft Inquiry, with its famous 'Cockcroft Report' entitled *Mathematics Counts* [**19**]. This was surely Bill's most complex and important mathematical work. The Inquiry had been set up by the then Prime Minister (James Callaghan) as part of his response ('the Great Debate') to increasingly noisy protestations about educational matters, usually aimed at opposing the moves toward Comprehensive Education, but wrapped in allegations (whipped up by the press) from employers that new recruits to the work-force were not as good at arithmetic as in some former 'Golden Age'. At the time, the curriculum was a matter for the teaching profession and the examining boards, and the Government traditionally kept aloof (see Howson ⟨**7**⟩). The worldwide discussion about the 'New Math' was apparently unknown to the critics, and had led to several very 'British' versions (including those mentioned above). For ten years or more, these were tried out by decentralised bodies, in keeping with a tradition that goes back to the founding of the Mathematical Association. For various reasons, these trials often did not live up to the hopes of their designers, and were frequently blamed for the poor performance of children who had never been near them. Both within and without the mathematical community there was much argument of largely poor quality, with people of 'higher' levels expecting those at 'lower' levels to accept blame and (often unsound) advice.

Consequently, any Chairman for the proposed Inquiry would need to be widely trusted by all the factions, and Bill was just the right person. His Committee sat for four years, and its brief was 'to inquire into the teaching of mathematics in schools in England and Wales'. It contained representatives from academia, industry, and other special-interest groups such as the immigrant community; advertisements in the press invited anybody with relevant evidence to come forward. Disappointingly few employers replied to back up their earlier complaints, so the committee appointed mathematicians to go into workplaces to find out what mathematics the work-people – nurses, tool-makers, van-drivers, and so on – actually used. Some of their reports appeared in pamphlets, or in later lectures by Bill, and make interesting reading. Frequently, such was the general fear of mathematics by the work-people (who came largely from the age-group that had been taught in traditional ways) that the mathematicians had to investigate incognito; one of their important conclusions was that people often do not use methods taught at school, but invent surprisingly effective procedures when needed. (Just as engineering students are known to call 'mathematics' what they find mysterious, and 'common-sense' what they can understand.)

The final Report of the Inquiry disappointed the politicians, because it did not support their alarums, nor did it recommend one magic way of teaching mathematics. Instead, it used careful analysis to place the changes in mathematics teaching within the contemporary social context and patterns of change. One of its many interesting (and, for that time, radical) conclusions was endorsed by the LMS, recommending that approaches to reform should be 'bottom up', rather than the traditional 'top down'. There were also recommendations as to 'good practice', including the famous Paragraph 243, asserting that good teaching should involve exposition, discussion, appropriate practical work, problem solving, investigation, consolidation and practice. Few of these could be found simultaneously in existing classrooms, and never in the universities (which had been declared off-limits to the Inquiry). Incidentally, the Report emphasised that it could not endorse any particular prescription of 'good practice' (but see Ruthven ⟨**10**, p. 204⟩).

The Report appeared after the General Election of 1979, when the Thatcher Government came into power with the intention of drastically reducing Government spending. The Cockcroft committee was thanked for its report, and certain teachers were designated as 'missionaries' to carry the message (but only for three years); anything else – like employing more teachers – would 'cost too much'. The new (and rather eccentric) Minister of Education, Keith Joseph, did fund three small projects based on the Report, because he was concerned about the large proportion of children who left school with no paper qualification at all – in contrast with the situation in Germany, for example. One project was to assess a programme (already initiated by the SMP) of materials and grading for low-attainers. More radical was the Low Attainers Mathematics Project (LAMP), directed by Afzal Ahmed, a member of Bill's committee. This aimed to investigate 'good practice' in six counties, and was later expanded as RAMP (Raising of Achievement in Mathematics Project) in 35 counties (see Ahmed and Williams ⟨**1**⟩), and used an unusual model of in-service training, based on 'teacher-bonding' in contrast to the standard part-time, hierarchical, model with its well-known, short-lived effects. Keith Joseph was eventually replaced by Kenneth Baker, who was keen on managerial techniques and hence to introduce a National Curriculum; thereafter, while lip-service was paid to the Cockcroft philosophy, in reality it was officially dumped. It lingers among older teachers and those in other parts of the world who heard lectures from Bill and his associates, but it was taken more seriously in many countries by professionals, and has thus been absorbed into the practice of many who reflect on their work in mathematics education. See also Howson ⟨**7**, pp. 210–211⟩.

The careful analysis used by the Cockcroft Report sets a standard of rigour for public arguments on mathematics, and it is only a pity that Bill did not attempt to formalise the discipline, which to him was just honesty and common sense. He received honorary doctorates from the Open University, Kent, and Southampton.

Bill left Coleraine in 1982 to become Chairman and Chief Executive of the Secondary Examinations Council (1983–88), the body charged with replacing the old O-Level examination for 16-year-olds, by the General Certificate of Secondary Education (GCSE). He complained about the illogicality of such a reform without reference to A-level, but there was nothing he could do: A-levels were Mrs Thatcher's 'flagship'. By then Rhona had died, and he had remarried, to Viv, whom he met at the SRC. He retired aged 65, but lectured and travelled widely until his death, after a lifetime of dedicating his exceptional energies to the furtherance of mathematics.

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Mathematical work

Cockcroft's mathematical work falls basically into three areas: low-dimensional topology, the algebraic topology of fibrations, and algebraic K-theory. While his last paper was published in 1975, his contribution is still seen as significant, with renewed interest in some of the problems he studied. This shows his good taste in mathematics.

For example, his papers [**1**, **3**] are on what are now called *crossed modules*, whose initial development is largely due to J. H. C. Whitehead. There is growing interest in their being candidates for '2-dimensional groups', and for being level 2 in an approach to what can be called *nonabelian algebraic topology* (see ⟨**2**⟩).

Paper [**4**] is, I believe, the first to follow up a question of Whitehead: is every subcomplex of a 2-dimensional aspherical complex, aspherical? This question has been regarded as more difficult than the Poincaré conjecture, and there are many results around it. One condition introduced in Cockcroft's work is that the Hurewicz map $\pi_2 X \to H_2 X$ should be trivial, and this is now known as the *Cockcroft property*. A search on MathSciNet in the 'Review text' for 'Cockcroft' yielded 26 matches, the most recent in 2003, and all in group or monoid theory, or low-dimensional topology.

Cockcroft was one of the earliest workers to take up Henry Whitehead's simple homotopy theory, which is the foundation of work in higher algebraic K-theory. Here is a story I heard from Cockcroft, relating to his time at Chicago, and which is included for historical interest. Whitehead gave a lecture there which included his result that for a ring R, $E(R)$ contained the commutator subgroup $[GL(R), GL(R)]$,

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so that $GL(R)/E(R)$ is abelian. Kaplansky suggested that the inclusion should also go the other way. This result appeared later as an appendix to a paper of Whitehead on an entirely different topic!

The paper [**8**] with R. G. Swan gives a nice exposition of relevant machinery of simple homotopy theory, as well as investigating conditions which ensure that two 2-dimensional CW-complexes with isomorphic fundamental groups have the same homotopy type. This paper combines interests in: cancellation problems for modules; simple homotopy types; and Schanuel's lemma. Swan writes that Kaplansky was an important catalyst in all this. There is now a large literature on the homotopy theory of 2-dimensional complexes (see, for example, ⟨**6**⟩). A recent paper by F. E. A. Johnson ⟨**8**⟩ refers to [**8**] as 'pioneering'. These questions were later taken up by Cockcroft in papers with R. M. F. Moss.

The paper [**11**] on the cohomology groups of a fibre space first re-proves a theorem of G. Hirsch ⟨**5**⟩: given a commutative ring with unit, K, and a fibre-space $F \to E \to B$ with $\pi_1(B)$ operating trivially on $H^*(F;K)$, there exists a coboundary operator ∂ on $C^*(B;K) \otimes_K H^*(F;K)$ such that the resulting graded cohomology module is isomorphic to $H^*(E;K)$, provided that $H^*(F;K)$ has a homogeneous K-basis, finite in each dimension. Cockcroft's significant advance is that the theorem is applied to the case where F is a space of type $K(\pi, n)$, and that the boundary is *computed explicitly* in the 'stable range'. Earlier work on twisted tensor products was not able to descend to the homology or cohomology of the fibre. A method for this under assumptions which make the homology of the fibre, with zero differential, a strong deformation retract of the chains, and then tensoring with the chains on the base, was shown in ⟨**2**⟩ (this work was strongly influenced by discussion and correspondence with Michael Barratt in 1959–60). These ideas led to what is now called *homological perturbation theory*, see ⟨**9**⟩. The actual formula achieved by Cockcroft has still not been obtained by these methods, and seems to involve a tricky homotopy twisting cochain.

Paper [**13**] is still an excellent introduction to basic methodology in homotopy theory, and ideas in this direction are now subsumed into abstract homotopy theory.

Cockcroft's progress into administration was partly a dissatisfaction with trends in algebraic topology that did not fascinate him, and also due to his interest in educational matters and issues. He told the following story, which illustrates his style. He asked the Minister of the time if desired improvement in numeracy meant that people should understand that a decrease in the rate of inflation did not mean more money in their pocket. The reply was that this might be too much to expect.

He was energetic, and democratic, in his work for the Department of Pure Mathematics at Hull, and took very seriously his responsibilities as Dean of Science and as Warden of a Hall of Residence. He encouraged his staff to keep up their research, and found ways of enabling visits of overseas mathematicians to Hull.

His appointment to chair the SRC Mathematics Committee shows how much his judgement was respected in UK Mathematics. Such a chairmanship was especially important in the days when subject areas in SRC had a senior academic as chairman, with responsibility for ensuring that important issues were recognised and addressed.

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