Adding, we get,

$$P'_{n+1}(z) - P'_{n-1}(z) = (2n+1)P_n(z). \quad \text{......................(c)}$$

Changing n to $n-1$ in (a) and solving for $P'_n(z)$ and $P'_{n-1}(z)$, we get

$$(z^2-1)P'_n(z) = nzP_n(z) - nP_{n-1}(z), \quad \text{.................... (d)}$$

and

$$(z^2-1)P'_{n-1}(z) = nP_n(z) - nzP_{n-1}(z). \quad \text{....................(e)}$$

Changing n to $n+1$ in (e) and subtracting (d), we have

$$(n+1)P_{n+1}(z) - (2n+1)zP_n(z) + nP_{n-1}(z) = 0. \quad \text{.................(f)}$$

Differentiating (d) with respect to z, we get

$$(z^2-1)P''_n(z) + 2zP'_n(z) = nzP'_n(z) + nP_n(z) - nP'_{n-1}(z)$$
$$= n(n+1)P_n(z) \quad \text{using (b).}$$

Hence

$$(1-z^2)P''_n(z) - 2zP'_n(z) + n(n+1)P_n(z) = 0,$$

which is the well-known Legendre's differential equation.

Thus it seems that (a) and (b) are really the fundamental recurrence formulae from which all others, viz. (c), (d), (e) and (f) are obtained by mere algebraic processes (involving no differentiation). In fact, from any two of the six formulae (a), (b), (c), (d), (e) and (f) we can derive the other four by algebraic manipulations.

2. Whittaker and Watson have suggested alternative methods of proof applicable to the case when n is a positive integer, only for the identities (b) and (f) above. We wish to point out that the formulae (a), (c) and (d) can be derived directly by equating coefficients of powers of h from the following obvious identities :

$$\frac{1-zh}{h}\frac{\partial V}{\partial z} - h\frac{\partial V}{\partial h} = V,$$

$$\frac{1-h^2}{h}\frac{\partial V}{\partial z} - 2h\frac{\partial V}{\partial h} = V,$$

$$\frac{1-z^2}{h}\frac{\partial V}{\partial z} + (z-h)\frac{\partial V}{\partial h} = V,$$

where $V = (1-2zh+h^2)^{-\frac{1}{2}}$. A. K. SRINIVASAN.

CORRESPONDENCE.

W. K. CLIFFORD.

To the Editor of the *Mathematical Gazette*.

SIR,—I protest most strongly against your suggestion (*Math. Gazette*, No. 294, p. 120) that Mallock's " portrait " in *The New Republic* is only a distortion and not a falsification of Clifford's attitude. Anyone who compares the *Lectures and Essays* with *The New Republic* can easily see the fundamental difference between Clifford and Mr. Saunders. Yours, etc., F. E. CAVE.

[The precise point at which distortion becomes falsification is not easy to determine, but I cannot see in Mallock's book anything more than that distortion which is permissible and perhaps essential in a satire. If my remark and Miss Cave's comment will send readers to the *Lectures and Essays* and to *The New Republic*, as well as to the new edition of *The Common Sense of the Exact Sciences*, I shall be content, whatever views they take of " Mr. Saunders ".—T. A. A. B.]

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