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AUTOMORPHISM GROUPS OF DENUMERABLE BOOLEAN ALGEBRAS

RALPH McKENZIE

We are concerned with the extent to which the structure of a Boolean algebra $\mathfrak{A}$ (or *BA*, for brevity) is reflected in its group of automorphisms, Aut $\mathfrak{A}$. In particular, for which algebras can one conclude that if Aut $\mathfrak{A} \cong$ Aut $\mathfrak{B}$, then $\mathfrak{A} \cong \mathfrak{B}$? Monk has conjectured [**3**] that this implication holds for denumerable *BA*'s with at least one atom. We shall refute his conjecture, but show that the implication does hold if $\mathfrak{A}$ and $\mathfrak{B}$ are denumerable, if each has at least one atom, and if the sum of the atoms exists in $\mathfrak{A}$. In fact, under those assumptions the algebra $\mathfrak{A}$ can be rather neatly recovered from its abstract automorphism group.

The assumption of denumerability is important. It is well-known that the automorphism group of any denumerable *BA* has the power of the continuum. S. Shelah has recently constructed in every uncountable cardinal κ, a *BA* of power κ having only one automorphism. (This very significant result that concludes a long chain of investigations by de Groot, Jónsson, Lozier, Monk, Balcar and Štěpánek, and others, is yet unpublished.)

M. Rubin [**4**] has shown how to recover $\mathfrak{A}$ from Aut $\mathfrak{A}$ if $\mathfrak{A} = \mathfrak{A}_\lambda$ is the atomic saturated *BA* of uncountable power $\lambda = \lambda^\lambda$. (His result is actually much stronger than this implies.)

Shelah independently discovered our example refuting Monk's conjecture.

Our notation is the same as in [**2**], or [**3**]. $\mathfrak{A}$, $\mathfrak{B}$ denote Boolean algebras of cardinality $\aleph_0$. The universe of $\mathfrak{A}$ is denoted A, its set of atoms is denoted by At $\mathfrak{A}$. The principal ideal algebra determined by $a \in A$ is denoted $\mathfrak{A} \upharpoonright a$.

Throughout, we denote by $\mathfrak{Q}$ a denumerable atomless *BA* (any two such are isomorphic), and $\mathfrak{F} =$ Aut $\mathfrak{Q}$. If κ is any cardinal number, Sym κ denotes the group of all permutations of a κ-element set. Monk proved that Aut $\mathfrak{A} \cong \mathfrak{F}$ if and only if $\mathfrak{A} \cong \mathfrak{Q}$ or $\mathfrak{A}$ is isomorphic to the direct product of $\mathfrak{Q}$ and a 2-element *BA*. And he proved that if $m \geqq 2$ is an integer, then Aut $\mathfrak{A} \cong$ Sym $m \times \mathfrak{F}$ if and only if $\mathfrak{A}$ has exactly m atoms ($\mathfrak{A}$ is isomorphic to the product of $\mathfrak{Q}$ with m copies of a 2-element algebra). Hence we can restrict our attention to algebras with denumerably many atoms.

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By Fin $\mathfrak{A}$ we mean the subgroup of Aut $\mathfrak{A}$ consisting of automorphisms that move only finitely many atoms, and fix every element disjoint from all the atoms moved. Each finite permutation of At $\mathfrak{A}$ is the restriction of a unique member of Fin $\mathfrak{A}$. We focus attention on the members of Fin $\mathfrak{A}$ which move exactly two atoms. Let us call these *atomic transpositions*.

LEMMA 1. *Let* $\pi \in$ Aut $\mathfrak{A}$, *where* $|A| = |\text{At } \mathfrak{A}| = \aleph_0$. *Then* π *is an atomic transposition if and only if it satisfies in* Aut $\mathfrak{A}$ *the following formula* $\theta(\pi)$:

$$\pi^2 = 1 \quad \textit{and} \quad \sim\pi = 1 \quad \textit{and} \quad (\forall x)((x\pi x^{-1}\pi^{-1})^6 = 1).$$

Proof. Let π be an atomic transposition. Clearly $\pi^2 = 1$. Let $\sigma \in$ Aut $\mathfrak{A}$ and put $\sigma\pi\sigma^{-1}\pi^{-1} = \gamma$. Now, $\sigma\pi\sigma^{-1}$ is an atomic transposition. There are two cases, depending on whether $\sigma\pi\sigma^{-1}$ moves some atom moved by π, or does not. In the first case, γ is essentially only acting in an interval $\mathfrak{A} \upharpoonright c$ that is finite with at most 3 atoms. Hence $\gamma^6 = 1$. In the second case, $\sigma\pi\sigma^{-1}$ commutes with π, so $\gamma^2 = (\sigma\pi\sigma^{-1})^2(\pi^{-1})^2 = 1$.

The above remarks show that $\theta(\pi)$ holds in Aut $\mathfrak{A}$.

Let us suppose, conversely, that π is an element of Aut $\mathfrak{A}$ satisfying θ. It must move some element of $\mathfrak{A}$, and we can find $a \in A$ so that $a \cdot \pi(a) = 0 \neq a$. We can even suppose that a is an atom or $\mathfrak{A} \upharpoonright a$ is atomless. It must be that a is an atom, because if $\mathfrak{A} \upharpoonright a$ is atomless it has an automorphism of infinite order, and we can concoct $\gamma \in$ Aut $\mathfrak{A}$ such that $\gamma \upharpoonright \mathfrak{A} \upharpoonright \pi a = \text{id} \upharpoonright \mathfrak{A} \upharpoonright \pi a$, $\gamma(a) = a$, and $\gamma \upharpoonright \mathfrak{A} \upharpoonright a$ is of infinite order—then $\gamma\pi\gamma^{-1}\pi^{-1}$ acts like γ on $\mathfrak{A} \upharpoonright a$ and has infinite order.

So we have an atom a with $\pi a \neq a$. It remains to show that π is the atomic transposition exchanging a and $\pi(a)$. In other words, that π on the interval $\mathfrak{A} \upharpoonright -(a + \pi a)$ is trivial. If not, then the above argument shows that there are additional atoms moved by π. We break the argument into cases: (i) π moves at least four atoms but fixes at least two atoms; (ii) π moves at least eight atoms.

In case (i), say $a_1, \ldots, a_4$ are moved, $\pi a_1 = a_2$, $\pi a_3 = a_4$; and a_5 and a_6 are fixed. Then let $\sigma \in$ Fin $\mathfrak{A}$ be a mapping that includes the cycles $(a_3a_2a_5)(a_4a_6)(a_1)$. One calculates that $\sigma\pi\sigma^{-1}\pi^{-1}$ has the cycle $(a_1a_6a_2a_5)$, hence its order does not divide 6.

In case (ii), say π includes the cycles $(a_1a_2)(a_3a_4)(a_5a_6)(a_7a_8)$. Let $\sigma \in$ Fin $\mathfrak{A}$ be so that $\sigma\pi\sigma^{-1}$ includes $(a_2a_3)(a_4a_6)(a_5a_7)(a_1a_8)$. By computation, $\sigma\pi\sigma^{-1}\pi^{-1}$ includes $(a_1a_3a_6a_7)$ so its sixth power is not 1.

This concludes the proof.

We remark that the formula θ was used in [**1**] to characterize the transpositions in Sym κ, where κ is an infinite cardinal.

Lemma 1 opens the way to recovering $\mathfrak{A}$ from its automorphism group. We define some further group-theoretic formulas. (Here $[x, y]$ is an abbreviation

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for $xyx^{-1}y^{-1}$.)

$$A(x, y, z): \theta(x) \text{ and } \theta(y) \text{ and } \theta(z) \text{ and } [x, y] \neq 1 \text{ and } [x, z] \neq 1 \text{ and } [y, z] \neq 1.$$

$$E(x_1, x_2, x_3; y_1, y_2, y_3): A(x_1, x_2, x_3) \text{ and } A(y_1, y_2, y_3) \text{ and } \bigwedge_{i,j} ([x_i, y_j] = 1 \rightarrow x_i = y_j).$$

$$M(u; x_1, x_2, x_3; y_1, y_2, y_3): A(\tilde{x}) \text{ and } A(\tilde{y}) \text{ and } E(ux_1u^{-1}, ux_2u^{-1}, ux_3u^{-1}, y_1, y_2, y_3).$$

Let $\mathfrak{A}$ be a BA, let $\mathfrak{G} = \langle G, \cdot \rangle$ be its automorphism group, and let M be the predicate defined by:

$$M(\sigma, a, b) \Leftrightarrow \sigma \in G \quad \text{and} \quad a \in \text{At}\,\mathfrak{A} \quad \text{and} \quad \sigma(a) = b.$$

Consider the 2-sorted structure Aut $^*\mathfrak{A} = \langle G, \text{At}\,\mathfrak{A}, \cdot, M \rangle$. The above formulas serve to interpret Aut $^*\mathfrak{A}$ in $\mathfrak{G}$, provided that $\mathfrak{A}$ is denumerable and has infinitely many atoms. More precisely, $\mathfrak{G} \models A(\sigma_1, \sigma_2, \sigma_3)$ if and only if $\sigma_1, \sigma_2, \sigma_3$ are distinct atomic transpositions with just one atom moved by all; $\mathfrak{G} \models E(\tilde{\sigma}, \tilde{\tau})$ if and only if $\tilde{\sigma}$ and $\tilde{\tau}$ are nested over the same common atom. Clearly, E defines an equivalence relation on A and A/E is canonically isomorphic to At $\mathfrak{A}$. If equivalence classes $\tilde{\sigma}/E$ and $\tilde{\tau}/E$ are canonically associated with the atoms a and b, then $\mathfrak{G} \models M(\pi, \tilde{\sigma}, \tilde{\tau})$ if and only if $\pi(a) = b$.

The following should now be obvious.

COROLLARY 2. *Let* $\mathfrak{A}$ *and* $\mathfrak{B}$ *be denumerable BA's having infinitely many atoms. If* Aut $\mathfrak{A} \cong$ Aut $\mathfrak{B}$, *then* Aut $^*\mathfrak{A} \cong$ Aut $^*\mathfrak{B}$.

COROLLARY 3. *For every definable property* Φ *of structures of the type of* Aut $^*\mathfrak{A}$, *there is a corresponding property* Φ' *of groups so that for denumerable* $\mathfrak{A}$ *with denumerably many atoms,* Aut $^*\mathfrak{A} \models \Phi$ *if and only if* Aut $\mathfrak{A} \models \Phi'$.

Monk proved [**3**, Thm. 4] that the property of $\mathfrak{A}$ that it is atomic is reflected faithfully in a property of Aut $\mathfrak{A}$, viz that it is not simple and that it has a smallest non-trivial normal subgroup. We have a similar result for the property: the sum of all atoms exists in $\mathfrak{A}$.

THEOREM 4. *Let* $\mathfrak{A}$ *and* $\mathfrak{B}$ *be denumerable BA's such that* Aut $\mathfrak{A} \cong$ Aut $\mathfrak{B}$. *Then* $\sum$ At $\mathfrak{A}$ *exists in* $\mathfrak{A}$ *if* $\sum$ At $\mathfrak{B}$ *exists in* $\mathfrak{B}$.

Proof. Relying on Monk's results for algebras with finitely many atoms, we can assume that each of $\mathfrak{A}$ and $\mathfrak{B}$ has infinitely many atoms. The formula $(\forall a)(\text{At}(a) \rightarrow M(\pi, a, a))$ defines in Aut $^*\mathfrak{A}$ the subgroup $\mathscr{H}$ of Aut $\mathfrak{A}$ constituted by the automorphisms that fix all atoms. If the sum of atoms exists in $\mathfrak{A}$, then the group $\mathscr{H}$ either is trivial ($\mathfrak{A}$ is atomic) or is simple (isomorphic to $\mathfrak{F}$). If the sum of atoms does not exist, then $\mathscr{H}$ has a large normal subgroup composed of those π such that for some atomless $a \neq 0$ in $\mathfrak{A}$, $\pi \restriction \mathfrak{A} \restriction -a$ is the identity function.

We shall show that this is a proper subgroup of $\mathscr{H}$. Let $\langle a_n : n < \omega \rangle$ be an enumeration of A, and $\langle c_n : n < \omega \rangle$ be an enumeration of the non-zero atomless elements of $\mathfrak{A}$. Since there is no least element above all the atoms of $\mathfrak{A}$, likewise the set of atomless elements has no least upper bound in $\mathfrak{A}$. We construct by induction a sequence $\langle d_n: n < \omega\rangle$ of atomless elements, and a sequence $\langle b_n : n < \omega\rangle$ such that in $\mathfrak{A} \upharpoonright b_n$ the sum of atoms does not exist.

Put $b_0 = a_0$, or $b_0 = -a_0$ according as the sum of atoms does not, or does exist in $\mathfrak{A} \upharpoonright a_0$. Then there is an atomless $d \leqq b_0$ with $d \cdot c_0 = 0 \neq d$. Put $d_0 = d$. Having obtained b_n, d_n, put $b_{n+1} = b_n \cdot a_{n+1}$ or $b_{n+1} = b_n \cdot - a_{n+1}$ so that the sum of atoms $\leqq b_{n+1}$ does not exist. Then choose for d_{n+1} a non-zero atomless element such that $d_{n+1} \cdot (\sum_{i\leqq n} d_i + \sum_{i\leqq n+1} c_i) = 0$ and $d_{n+1} \leqq b_{n+1}$.

The sequence $\langle d_n : n < \omega\rangle$ has the following properties: (1) for each $x \in A$, either $x \cdot d_n = 0$ for large n, or $x \geqq d_n$ for large n; (2) for each atomless $x \in A$, we have $x \cdot d_n = 0$ for n large.

Now we can define an automorphism π as follows. Fix, for each $n < \omega$, an isomorphism $\sigma_n : \mathfrak{A} \upharpoonright d_{2n} \rightarrowtail \mathfrak{A} \upharpoonright d_{2n+1}$. Then put (where either $x \cdot d_m = 0$ for $m \geqq 2k$, or else $x \geqq d_m$ for $m \geqq 2k$):

$$\pi(x) = x \cdot - \sum_{i<2k} d_i + \sum_{j<k} [\sigma_j(x \cdot d_{2j}) + \sigma_j^{-1}(x \cdot d_{2j+1})].$$

Clearly, $\pi \in \mathscr{H}$ and its action is not bounded by any atomless element.

We have produced a property of Aut $^*\mathfrak{A}$ that determines whether the sum of atoms exists in $\mathfrak{A}$. By Corollary 3, this is equivalent to a definable property of Aut $\mathfrak{A}$.

THEOREM 5. *Let $\mathfrak{A}$ and $\mathfrak{B}$ be denumerable BA's not isomorphic to $\mathfrak{Q}$, with* Aut $\mathfrak{A} \cong$ Aut $\mathfrak{B}$. *If the sum of atoms exists in $\mathfrak{A}$, then $\mathfrak{A} \cong \mathfrak{B}$.*

Proof. By [**3**], we can assume that each algebra has denumerably many atoms. By Theorem 4, the sum of atoms exists in $\mathfrak{B}$. Hence $\mathfrak{A} \cong \mathfrak{A}_1 \times \mathfrak{A}^1$, $\mathfrak{B} \cong \mathfrak{B}_1 \times \mathfrak{B}^1$, where $\mathfrak{A}^1$, $\mathfrak{B}^1$ are either atomless or 1-element algebras, $\mathfrak{A}_1$ and $\mathfrak{B}_1$ are atomic. By Monk's result, $\mathfrak{A}$ is atomic if and only if $\mathfrak{B}$ is, i.e. $\mathfrak{A}^1 \cong \mathfrak{B}^1$. By Corollary 2, Aut $^*\mathfrak{A} \cong$ Aut $^*\mathfrak{B}$. Part of this isomorphism is a bijection j: At $\mathfrak{A} \leftrightarrow$ At $\mathfrak{B}$. Now $\mathfrak{A}_1$ is canonically isomorphic to an algebra of subsets of At $\mathfrak{A}$, whose universe is $\{\bar{a}: a \in A\} = \bar{A}$, where $\bar{a} = \{x \in \text{At } \mathfrak{A}: x \leqq a\}$. And likewise for $\mathfrak{B}_1$. To establish the theorem, it is sufficient to prove that j carries $\bar{A}$ onto $\bar{B}$. Actually, we can define $\bar{A}$ in the structure Aut $^*\mathfrak{A}$, as follows.

P) Let $X \subseteq$ At $\mathfrak{A}$. There exists $a \in A$ with $X = \bar{a}$ if and only if $\{\sigma X: \sigma \in \text{Aut } \mathfrak{A}\}$ is countable.

The proof of the forward implication in P) is trivial. For the converse, we first prove

Q) Let $X \subseteq$ At $\mathfrak{A}$, $X \notin \bar{A}$. There is a convergent set $Y \subseteq$ At $\mathfrak{A}$ such that $Y \cap X$ and $Y \sim X$ are infinite. (By "convergent" is meant that for all $a \in A$, one of the sets $Y \cap \bar{a}$, $Y \sim \bar{a}$ is finite.)

To prove Q), start with an enumeration $\langle a_n: n < \omega\rangle$ of A. Assume that $X \notin \bar{A}$. Then put $c_0 = a_0$ or $-a_0$ so that $X \cap \bar{c}_0 \notin \bar{A}$. Having generated

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$c_0 \geqq \ldots \geqq c_n$ and $X \cap \bar{c}_n \notin \bar{A}$, put $c_{n+1} = c_n \cdot a_{n+1}$ or $c_n \cdot - a_{n+1}$ so that $X \cap \bar{c}_{n+1} \notin \bar{A}$. Now with the sequence $\langle c_n: n < \omega \rangle$ so constructed, we can choose distinct atoms $a_n, b_n (n < \omega)$ with $a_n \in \bar{c}_n \cap X$, $b_n \in \bar{c}_n \sim X$. (Since $\bar{c}_n \cap X \notin \bar{A}$, also $\bar{c}_n \sim X \notin \bar{A}$, so both sets are infinite.) Put $Y = \{a_n: n < \omega\} \cup \{b_n: n < \omega\}$. This set has the desired properties.

To finish the proof of P), let $X \notin \bar{A}$, and let Y be as in Q). By [**3**, Lemma 1.1], every permutation of Y extends to an automorphism of $\mathfrak{A}$. This gives continuum many automorphic images of X in $\mathfrak{A}$.

The proof of Theorem 5 is complete.

Remarks. The Cantor-Bendixson derivative of $\mathfrak{A}$ is the factor algebra $\mathfrak{A}^{(1)} = \mathfrak{A}/I$ where I is the ideal generated by the atoms in $\mathfrak{A}$. Restricted to isomorphism types of atomic denumerable algebras, the Cantor-Bendixson derivative gives a one-to-one map onto all isomorphism types of countable algebras excepting that of the 1-element algebra. If $\mathfrak{A}^{(-1)}$ denotes the unique atomic denumerable algebra whose derivative is $\mathfrak{A}$, then by Theorem 5, the map $\mathfrak{A} \rightarrow \text{Aut } \mathfrak{A}^{(-1)}$ provides a one-to-one map of isomorphism types of countable BA's with at least two elements into isomorphism types of groups of permutations of a denumerable set, each group having continuum many elements.

THEOREM 6. *There exist denumerable BA's $\mathfrak{A}$ and $\mathfrak{B}$, each having denumerably many atoms, such that* Aut $\mathfrak{A} \cong$ Aut $\mathfrak{B}$ *and $\mathfrak{A}$ is not isomorphic to $\mathfrak{B}$.*

Proof. Let $\mathfrak{A}_f$ be the algebra of finite and co-finite subsets of ω. Let D be an ultrafilter in $\mathfrak{Q}$ (the denumerable atomless algebra). Let $\mathfrak{A}_{f'}$ be the subalgebra of $\mathfrak{A}_f \times \mathfrak{Q}$ whose universe is

$$\{(a, b): a \text{ co-finite} \leftrightarrow b \in D\}.$$

Set $\mathfrak{A}_0 = \mathfrak{A}_f^{(-1)}$, $\mathfrak{A}_1 = \mathfrak{A}_{f'}^{(-1)}$, $\mathfrak{C} = \mathfrak{A}_0 \times \mathfrak{A}_1$. Now $\mathfrak{A}_0$ has an ultrafilter D_0 which is the inverse image under the projection $\mathfrak{A}_0 \twoheadrightarrow \mathfrak{A}_f$ of the filter of cofinite subsets of ω. In $\mathfrak{A}_1$ we have an ultrafilter D_1 which is the inverse image under $\mathfrak{A}_1 \twoheadrightarrow \mathfrak{A}_{f'}$ of the filter

$$\{(a, b): a \text{ is cofinite and } b \in D\}.$$

These filters have unique extensions to ultrafilters in $\mathfrak{C}$, which we also denote by D_0, D_1.

Claim. Let $\pi \in \text{Aut } \mathfrak{C}$. Then $\pi^* D_0 = D_0$ and $\pi^* D_1 = D_1$. In fact, it's easy to see that for $c \in C$, $c \in D_0$ if and only if there exists $t \leqq c$, $\mathfrak{C} \restriction t \cong \mathfrak{A}_0$; and $c \in D_1$ if and only if there exists $t \leqq c$, $\mathfrak{C} \restriction t \cong \mathfrak{A}_1$.

Now we define $\mathfrak{A}$ and $\mathfrak{B}$ as the subalgebras of $\mathfrak{C} \times \mathfrak{Q}$ with respective universes

$$\{(\tilde{c}, q): \tilde{c} \in D_0 \leftrightarrow q \in D\},$$
$$\{(\tilde{c}, q): \tilde{c} \in D_1 \leftrightarrow q \in D\}.$$

In $\mathfrak{B}$ there is an interval $\mathfrak{B} \upharpoonright x$ that is atomic and has derivative isomorphic to $\mathfrak{A}_f$. In $\mathfrak{A}$ there is no such element. Hence $\mathfrak{A} \not\cong \mathfrak{B}$.

Now let $\mathfrak{G}$ be the group of automorphisms of $\mathfrak{Q}$ that fix D. We claim that

$$\text{Aut } \mathfrak{A} \cong (\text{Aut } \mathfrak{C}) \times \mathfrak{G} \cong \text{Aut } \mathfrak{B}.$$

To see it, let $\pi \in \text{Aut } \mathfrak{A}$, $x = (\tilde{c}_1, q_1) \in A$, $y = (\tilde{c}_2, q_2) \in A$, $\pi x = (\tilde{c}_1', q_1')$, $\pi y = (\tilde{c}_2', q_2')$, $x \oplus y$ the symmetric difference of x and y. Now $q_1 = q_2 \Leftrightarrow x \oplus y$ is atomic $\Leftrightarrow \pi x \oplus \pi y$ is atomic $\Leftrightarrow q_1' = q_2'$. Similarly, $\tilde{c}_1 = \tilde{c}_2 \Leftrightarrow x \oplus y$ is atomless $\Leftrightarrow \tilde{c}_1' = \tilde{c}_2'$. It follows that we can write $\pi = (\pi_0, \pi_1)$ where π_0 is a permutation of $\mathfrak{C}$, π_1 a permutation of $\mathfrak{Q}$.

Since very automorphism of $\mathfrak{C}$ fixes D_0, it's very easy to see that the above analysis yields an isomorphism Aut $\mathfrak{A} \cong (\text{Aut } \mathfrak{C}) \times \mathfrak{G}$. The proof for $\mathfrak{B}$ is the same.

Remarks. In view of Theorem 5, it is natural (although a bit naive) to expect that the automorphism group may be a one-to-one function of infinite atomic Boolean algebras. To demolish this possibility, we note that by [**2**, Thm 3.1] there is for every infinite κ, an atomic BA $\mathfrak{A}$ whose automorphism group is naturally isomorphic to the group of finite permutations of κ, while $\mathfrak{A}^{(1)}$ is a free BA of power 2^κ. On the other hand, it is known that there exists a rigid, complete, and atomless BA $\mathfrak{B}$. Let S be the Stone space of such a $\mathfrak{B}$. Let $\mathfrak{K}$ be the algebra of subsets of S generated (by finite operations) from $B \cup \{\{s\}: s \in S\}$. Then $\mathfrak{K}^{(1)} \cong \mathfrak{B}$, while it is not hard to show that Aut $\mathfrak{K}$ is naturally isomorphic to the group of finite permutations of S ($\mathfrak{K}$ is atomic).

I offer two conjectures that ought not be too hard to prove or disprove: Let $\mathfrak{A}$ and $\mathfrak{B}$ be denumerable Boolean algebras not isomorphic to $\mathfrak{Q}$. If Aut $\mathfrak{A} \cong$ Aut $\mathfrak{B}$ (or if these groups are elementarily equivalent), then $\mathfrak{A}$ and $\mathfrak{B}$ are elementarily equivalent—in the sense of first order logic.

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University of California,
Berkeley, California