1
The Standard Model

The 1970s witnessed the emergence of what has become the Standard Model (SM) of particle physics. The SM describes the interactions of quarks and leptons that are the constituents of all matter that we know about. The strong interactions are described by quantum chromodynamics (QCD) while the electromagnetic and the weak interactions have been synthesized into a single electroweak framework. This theory has proven to be extremely successful in describing a tremendous variety of experimental data ranging over many decades of energy. The discovery of neutral currents in the 1970s followed by the direct observation of the W and Z bosons at the CERN S$p\bar{p}$S collider in the early 1980s spectacularly confirmed the ideas underlying the electroweak framework. Since then, precision measurements of the properties of the W and Z bosons at both e^+e^- and hadron colliders have allowed a test of electroweak theory at the 10^{-3} level. QCD has been tested in the perturbative regime in hard collision processes that result in the breakup of the colliding hadrons. In addition, lattice gauge calculations allow physicists to test non-perturbative QCD via predictions for the observed properties of hadrons for which there is a wealth of experimental information.

1.1 Gauge invariance

One of the most important lessons that we have learned from the SM is that dynamics arises from a symmetry principle. If we require the Lagrangian density to be invariant under *local* gauge transformations, we are *forced* to introduce a set of gauge potentials with couplings to elementary scalar and fermion matter fields that, apart from an overall scale, are completely determined by symmetry principles. The most familiar example of such a field theory is the electrodynamics of Dirac fermions or complex scalars, where the invariance of the Lagrangian under

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spacetime-dependent phase transformations,

$$\psi(x) \to e^{iq_\psi \alpha(x)}\psi(x),$$

or

$$\phi(x) \to e^{iq_\phi \alpha(x)}\phi(x),$$

requires us to introduce the vector potential A_μ, with a coupling given by,

$$\mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu \psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{1.1a}$$

or

$$\mathcal{L} = (D^\mu \phi)^*(D_\mu \phi) - m^2\phi^*\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}. \tag{1.1b}$$

Here, D_μ is the gauge covariant derivative given by $D_\mu = \partial_\mu + iq_{\psi/\phi}A_\mu(x)$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $q_{\psi/\phi}$ is any real number identified with the charge of the field. It is easy to check that if, in addition to the local phase transformation of the fields ψ and ϕ, the vector potential transforms inhomogeneously as

$$A_\mu(x) \to A'_\mu(x) = A_\mu(x) - \partial_\mu \alpha(x),$$

the Lagrangians of Eq. (1.1a) and Eq. (1.1b) will be invariant under the set of local gauge transformations. The phase transformations of the fermion or scalar "matter" fields form the group $U(1)$. We will thus regard electrodynamics as a gauge theory based on the group $U(1)$, which is an Abelian group – i.e. its elements commute with one another. We stress two features of these Lagrangians.

- The coupling of the vector potential (identified with the photon field when the theory is quantized) to matter fields is given by the "minimal coupling principle" where the ordinary derivative is replaced by the gauge-covariant derivative. For fermionic matter, this gives the familiar fermion–antifermion–photon "vector" coupling (proportional to the charge q_ψ), while in the case of scalar matter, we have both a three-point derivative coupling to the photon proportional to the charge q_ϕ *and* a four-point non-derivative scalar–scalar–photon–photon coupling proportional to q_ϕ^2. The point to be made is that *the form of the interactions of the photons with matter is completely fixed by the requirement of local gauge invariance.*
- The photon field is massless because a mass term $\frac{1}{2}m_\gamma^2 A_\mu A^\mu$ would not be locally gauge invariant. The matter fields may, however, be massive.

Yang and Mills, and independently Shaw (and later Utiyama), generalized this idea to more complicated transformations of matter fields that form a non-Abelian group rather than the group $U(1)$. The construction of these Yang–Mills gauge theories is given in many texts and will not be repeated here. Instead of a single photon field, we now have several gauge fields (equal to the number of generators) in the adjoint representation of the gauge group. Matter and gauge fields ($V_{A\mu}$) are again "minimally coupled" via the prescription

$$\partial_\mu \to D_\mu = \partial_\mu + \mathrm{i} g t_A V_{A\mu},$$

where t_A is the matrix representation of the group generator in the representation to which the matter field belongs (for example, if the gauge group is $SU(2)$ with matter forming $SU(2)$ doublets, $t_A = \frac{1}{2}\sigma_A$, where σ_A ($A = 1, 2, 3$) are the Pauli matrices), and g is a universal (gauge) coupling constant. Again, as before, there can be no mass term for the gauge potentials, and the interaction of matter and gauge fields is fixed by the local gauge symmetry. There are some important distinctions from the Abelian case.

- The gauge field strength $F_{A\mu\nu} = \partial_\mu V_{A\nu} - \partial_\nu V_{A\mu} - g f_{ABC} V_{B\mu} V_{C\nu}$, where f_{ABC} are the structure constants of the gauge group, contains a new term in addition to the curl that is present in electromagnetism. This results in self-interactions of the non-Abelian gauge fields, and has important physical consequences such as the well-known asymptotic freedom of QCD.
- The "charge" factor in the minimal coupling principle for the non-Abelian case is replaced by $g \times t_A$. As a result, for simple groups, the coupling of matter to gauge bosons is determined to be the universal coupling g times a determined group theory factor. Thus the gauge boson couplings to matter are considerably more restrictive than in the $U(1)$ case where the charge q was any real number. In particular, the ratio of charges in the $U(1)$ case need not be a rational number.

1.2 Spontaneous symmetry breaking

We are familiar with the fact that the symmetries of the Hamiltonian (or the symmetries of the equations of motion) do not coincide with the symmetries of the solutions of these equations. For instance, Newton's laws governing the gravitational force between the Earth and the Sun are rotationally symmetric, yet the motion of the Earth around the Sun (i.e. a solution to the rotationally invariant equations of motion) is confined to a plane. Moreover, the orbit of the Earth, in general, is elliptical, and not even invariant under rotations about a single axis. This is also true in quantum theory. The $p, d, f \ldots$ orbitals of the hydrogen atom are rotationally variant solutions of the rotationally invariant Schrödinger equation.

What then does it mean for a Hamiltonian to be invariant under some symmetry transformation? These symmetries do not reflect themselves as symmetries of the solutions to the corresponding equations of motion. What is true, however, is that given a solution to the equations of motion, then we can find other solutions with the same energy by acting on the known solution by the symmetry transformation: for the example of the Earth's orbit that we considered, orbits where this ellipse is differently oriented (but with the Sun still at the focus) correspond to allowed motions and have the same total energy as the original orbit. If, however, the solution that we found itself happens to be invariant under the symmetry transformation, new solutions cannot be generated in this way.

In quantum field theory, we are especially interested in the symmetries of the ground state of the system, since it is the excitations of the ground state that are identified as particles. If, however, a ground state is not invariant under a symmetry transformation, we know there must be another solution with the same energy; i.e. the ground state must be degenerate. If the symmetry transformation that leaves the equations of motion invariant is labeled by a continuous parameter, in general there will be a continuous infinity of ground states. Which one should we choose to build the spectrum of excitations upon? The answer is that it does not matter. It is, however, important to note that once we make this choice, and express the Hamiltonian (or the Lagrangian) in terms of fields whose quanta correspond to excitations about any *one particular* ground state, the original symmetry of the action is no longer manifest. The underlying symmetry is hidden, and is (perhaps misleadingly) generally referred to as being spontaneously broken.

Although the symmetry is not really broken, it will not be obvious to an observer doing experiments with particles that are excitations of one of the many ground states of the theory. This is not to say that the underlying symmetry has no experimental implications. For instance, in a renormalizable theory, all coupling constant relationships for dimension four operators implied by the symmetry are preserved even when this symmetry may be spontaneously broken. It is this feature that gives us the universality of gauge interactions even though the gauge symmetry is spontaneously broken. Relationships between lower dimensional operators can, however, be modified by spontaneous symmetry breaking. A familiar example of this is the fact that gauge bosons may acquire mass via the Higgs mechanism even though, as we have seen, the explicit inclusion of such a mass term is forbidden by gauge invariance. Indeed our interest in gauge theories with spontaneous symmetry breaking stems mainly from this single observation, which allows the construction of gauge theories where (some of) the gauge bosons acquire mass, resulting in short-range forces as required by phenomenology.

We assume that the reader is sufficiently familiar with the physics of spontaneous symmetry breaking which is discussed in many excellent text books, so that we will not describe the Goldstone and Higgs phenomena here. Instead, we confine our discussion to some very general features of symmetry and spontaneous symmetry breaking. Our purpose is mainly to illustrate that these familiar considerations also apply to supersymmetry.

We begin by considering the action of a symmetry transformation (which, by definition, leaves the equations of motion invariant) on a state. This can be written as,

$$|\psi\rangle \to |\psi'\rangle = e^{i\alpha Q}|\psi\rangle, \tag{1.2}$$

where Q is the generator of the transformation and α is a real parameter. The symmetry in question may be a spacetime symmetry in which case Q would be one of the generators of the Poincaré group, or it may be an internal symmetry. In general, we get a new state. As we have mentioned, the action of this transformation on the ground state is especially important: the symmetry is spontaneously broken, unless

$$e^{i\alpha Q}|0\rangle = |0\rangle, \tag{1.3a}$$

or equivalently,

$$Q|0\rangle = 0. \tag{1.3b}$$

The symmetry transformation changes the dynamical variables (which are operators $\mathcal{O}$ acting on the states) as,

$$\mathcal{O}' = e^{i\alpha Q}\mathcal{O}e^{-i\alpha Q} \approx \mathcal{O} + i\alpha[Q, \mathcal{O}], \tag{1.4}$$

where the last equality holds for an infinitesimal transformation. We thus see that in order for a symmetry not to be spontaneously broken, we must have

$$\langle 0|\delta\mathcal{O}|0\rangle \equiv i\alpha\langle 0|[Q, \mathcal{O}]|0\rangle = 0, \tag{1.5}$$

where $\delta\mathcal{O}$ is the change in $\mathcal{O}$ under the (infinitesimal) symmetry transformation. Of course, $\delta\mathcal{O}$ is itself a dynamical variable.

In quantum field theory,[1] the field operators are the dynamical variables $\mathcal{O}$. In this case, as we have just seen, the vacuum expectation value (VEV) of some (possibly composite) field operator acts as the order parameter for symmetry breaking. In order that Poincaré invariance not be spontaneously broken, only spin zero field operators may acquire a VEV. If this is to result in the spontaneous breaking of a symmetry generated by Q, then the field operator in question must transform non-trivially under this symmetry. In the SM one is led to introduce a weak isodoublet of spin zero fields that acquires a VEV and results in the spontaneous breaking of

[1] In this case, the operator Q is obtained as the space integral of the time component of a Noether current. We will not enter into discussions as to whether the integral is defined or whether we necessarily have to discuss these issues in terms of densities. We will merely state that as long as we refer only to commutator brackets of Q with some dynamical variable, we appear to be safe.

electroweak gauge symmetry. We will see in Chapter 7 that these general considerations apply to supersymmetry, so that at least in this sense supersymmetry is not different from other familiar symmetries.

1.3 Brief review of the Standard Model

The SM is a non-Abelian Yang–Mills type gauge theory based on the group $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$, with $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ spontaneously broken to $U(1)_{\mathrm{em}}$. Color $SU(3)_{\mathrm{C}}$ is assumed to be unbroken.

1.3.1 QCD

The $SU(3)_{\mathrm{C}}$ gauge bosons are the gluons and the resulting gauge theory is QCD. Quarks are assigned to the fundamental **3** representation. Thus antiquarks are assigned to the conjugate $\mathbf{3}^*$ representation. All other particles are $SU(3)_{\mathrm{C}}$ singlets, and do not directly couple to the gluons. The QCD Lagrangian is given by

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G_{A\mu\nu} G_A^{\mu\nu} + \sum_{i=\text{flavors}} \bar{q}_i (\mathrm{i} \not{D} - m_i) q_i \tag{1.6}$$

where $G_{\mu\nu A} = \partial_\mu G_{A\nu} - \partial_\nu G_{A\mu} - g f_{ABC} G_{B\mu} G_{C\nu}$, $D_\mu = \partial_\mu + \mathrm{i} g_s \frac{\lambda_A}{2} G_{A\mu}$ and q_i contains a color triplet of quarks of flavor i. Quantization of the theory is possible if appropriate gauge fixing terms are added to the QCD Lagrangian.

The QCD couplings of matter fermions with the gluons can now be extracted by expanding the QCD Lagrangian. The self-interactions of the gluons are completely fixed by gauge invariance. The interaction Lagrangian reads,

$$\begin{aligned} \mathcal{L}_{\mathrm{QCD}} \ni {} & -g_s \textstyle\sum_i \bar{q}_i \gamma^\mu \frac{\lambda_A}{2} G_{A\mu} q_i + \frac{1}{2} g_s f_{ABC} (\partial_\mu G_{A\nu} - \partial_\nu G_{A\mu}) G_B^\mu G_C^\nu \\ & - \tfrac{1}{4} g_s^2 f_{ABC} f_{AB'C'} G_{B\mu} G_{C\nu} G_{B'}^\mu G_{C'}^\nu . \end{aligned} \tag{1.7}$$

A summation over the repeated color indices $A, B \ldots$ is implied, and the sum in the first term is again over all quark flavors.

1.3.2 The electroweak model

In order to allow a chiral structure for the weak interactions,[2] the left- and right-handed components of quark and lepton fields are assigned to different representations of the electroweak gauge group $SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$. The $SU(3)_{\mathrm{C}} \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$ assignment for the matter fields of the first generation of quarks and leptons

[2] The QCD and QED couplings of fermions are vectorial because their left- and right-chiral components are assumed to have the same charge.

Table 1.1 *The matter and Higgs boson field content of the Standard Model along with the gauge quantum numbers.*

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	**1**	**2**	-1
e_R	**1**	**1**	-2
$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	**3**	**2**	$\frac{1}{3}$
u_R	**3**	**1**	$\frac{4}{3}$
d_R	**3**	**1**	$-\frac{2}{3}$
$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	**1**	**2**	1

is shown in Table 1.1. The other generations are copies of this in that they have the same pattern of quantum numbers.

We should mention that we could equally well have written the SM field content solely in terms of left-handed fermion fields. In that case, instead of the right-handed e_R, u_R and d_R, we can work with their charge conjugates, $(e_R)^c$, $(u_R)^c$, and $(d_R)^c$, which are left-handed fields that have opposite hypercharge assignments from those shown in Table 1.1. Needless to say, these charge-conjugated quark fields would transform according to the $\mathbf{3}^*$ representation of $SU(3)_C$. This way of writing the field content of the SM will be useful when we consider the supersymmetrization of the SM.

The electroweak Lagrangian is given by

$$\mathcal{L}_{EW} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}, \tag{1.8}$$

where

$$\mathcal{L}_{gauge} = -\frac{1}{4} W_{A\mu\nu} W_A^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \tag{1.9}$$

$$\mathcal{L}_{matter} = \sum_{\text{generations}} \left[\mathrm{i}\bar{L}\, \not{D} L + \mathrm{i}\bar{Q}\, \not{D} Q + \mathrm{i}\bar{u}_R\, \not{D} u_R + \mathrm{i}\bar{d}_R\, \not{D} d_R + \mathrm{i}\bar{e}_R\, \not{D} e_R \right], \tag{1.10}$$

$$\mathcal{L}_{Higgs} = |D_\mu \phi|^2 + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2, \tag{1.11}$$

and

$$\mathcal{L}_{\text{Yukawa}} = \sum_{\text{generations}} \left[-\lambda_e \bar{L} \cdot \phi e_{\text{R}} - \lambda_d \bar{Q} \cdot \phi d_{\text{R}} - \lambda_u \epsilon^{ab} \bar{Q}_a \phi_b^\dagger u_{\text{R}} + \quad \text{h.c.} \right], \tag{1.12}$$

where the Ds are appropriate covariant derivatives for each matter multiplet, and ϵ^{ab} is the completely antisymmetric $SU(2)$ tensor with $\epsilon^{12} = 1$.

The interaction Lagrangian for the electroweak theory is more complicated since the $SU(2)_{\text{L}} \times U(1)_{\text{Y}}$ symmetry is assumed to be spontaneously broken to $U(1)_{\text{em}}$. The electroweak symmetry breaking sector of the SM is particularly simple, and consists of a single complex $SU(2)_{\text{L}}$ doublet ϕ of spin zero fields with gauge quantum numbers shown in Table 1.1. The field ϕ acquires a VEV signaling the spontaneous breakdown of electroweak symmetry. This VEV is left invariant by one combination of $SU(2)_{\text{L}}$ and $U(1)_{\text{Y}}$ generators which generates a different $U(1)$ group identified as $U(1)_{\text{em}}$. The corresponding linear combination of gauge fields remains massless and is identified as the photon,

$$A_\mu = \sin\theta_{\text{W}} W_{3\mu} + \cos\theta_{\text{W}} B_\mu \tag{1.13}$$

with $\sin\theta_{\text{W}} = g'/\sqrt{g^2 + g'^2}$ and $\cos\theta_{\text{W}} = g/\sqrt{g^2 + g'^2}$, while all other gauge fields acquire mass via the Higgs mechanism. The physical particles in the bosonic sector of the SM are: the photon, a pair of charged massive spin 1 bosons $W^\pm$

$$W_\mu^\pm = (W_{1\mu} \mp \mathrm{i} W_{2\mu})/\sqrt{2}, \tag{1.14}$$

a massive spin 1 neutral boson Z^0

$$Z_\mu^0 = -\cos\theta_{\text{W}} W_{3\mu} + \sin\theta_{\text{W}} B_\mu, \tag{1.15}$$

and finally, one neutral scalar boson H_{SM}, the Higgs boson, which is left over as the relic of spontaneous symmetry breaking. In order to establish our notation, and also for the convenience of the reader, we list the interactions of the physical particles of the SM that we will use later when we discuss phenomenological issues.

The interactions of quarks and leptons with gauge bosons can, as usual, be worked out from the minimal coupling prescription discussed above, and simply rewriting the $SU(2)_{\text{L}} \times U(1)_{\text{Y}}$ gauge fields in terms of the mass eigenstate photon, $W^\pm$, and Z^0 fields. For the electroweak gauge couplings of matter we find,

$$\mathcal{L}_{\text{neutral}} = -e \sum_f q_f \bar{f} \gamma^\mu f A_\mu + e \sum_f \bar{f} \gamma^\mu (\alpha_f + \beta_f \gamma_5) f Z_\mu, \tag{1.16a}$$

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Table 1.2 *The constants α_f and β_f that appear in Eq. (1.16a). The couplings are independent of the fermion generation. Here $t \equiv \tan\theta_W$ and $c \equiv \cot\theta_W$.*

f	q_f	α_f	β_f
ℓ	-1	$\frac{1}{4}(3t - c)$	$\frac{1}{4}(t + c)$
ν_ℓ	0	$\frac{1}{4}(t + c)$	$-\frac{1}{4}(t + c)$
u	$\frac{2}{3}$	$-\frac{5}{12}t + \frac{1}{4}c$	$-\frac{1}{4}(t + c)$
d	$-\frac{1}{3}$	$\frac{1}{12}t - \frac{1}{4}c$	$\frac{1}{4}(t + c)$

and

$$\mathcal{L}_{\text{charged}} = -\frac{g}{\sqrt{2}}\left(\bar{u}\gamma^\mu \frac{1-\gamma_5}{2} V_{\text{KM}} d W_\mu^+ + \bar{\nu}\gamma^\mu \frac{1-\gamma_5}{2}\ell W_\mu^+ + \quad \text{h.c.}\right). \quad (1.16b)$$

Here g is the $SU(2)_L$ gauge coupling, and $e \equiv g\sin\theta_W$ is the electromagnetic coupling. The weak mixing angle θ_W is given in terms of g and the weak hypercharge coupling g' by $g' \equiv g\tan\theta_W$. The constants α_f and β_f that appear in Eq. (1.16a) are listed in Table 1.2. In Eq. (1.16b), V_{KM} is the Kobayashi–Maskawa matrix that arises because the weak interaction quark eigenstates and the corresponding mass eigenstates do not coincide. It should also be understood that u and d in Eq. (1.16b) contain all three generations of up- and down-type quarks, respectively, with matrix multiplication implied over the generation indices.

Exercise *Verify that the gauge interactions in Eq. (1.16a) are reproduced when we replace the right-handed fermions with $(E^c)_L$, $(U^c)_L$, and $(D^c)_L$ in our assignment of quantum numbers for the fundamental fields. We use capital letters to denote these fields only to match the notation that we will use later, but here $(E^c)_L$ is just left-handed $SU(2)$ singlet positron field, $(e_R)^c$, and likewise for $(U^c)_L$ and $(D^c)_L$.*

The couplings of the Higgs boson to the gauge bosons are given by,[3]

$$\mathcal{L}_{HVV} = gM_W H_{\text{SM}}(W_\mu^+ W^{\mu-} + \tfrac{1}{2}\sec^2\theta_W Z_\mu Z^\mu) \quad (1.17a)$$

and

$$\mathcal{L}_{HHVV} = \frac{g^2}{4}(W_\mu^+ W^{\mu-} + \tfrac{1}{2}\sec^2\theta_W Z_\mu Z^\mu)H_{\text{SM}}^2, \quad (1.17b)$$

[3] We write all interactions in the unitary gauge where there are no unphysical fields.

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while the self-interactions of the SM Higgs boson are given in terms of its mass $m_{H_{\rm SM}} = \sqrt{-2\mu^2}$ by,

$$\mathcal{L}_H = -\frac{g m_{H_{\rm SM}}^2}{4M_{\rm W}} H_{\rm SM}^3 - \frac{g^2 m_{H_{\rm SM}}^2}{32 M_{\rm W}^2} H_{\rm SM}^4. \tag{1.18}$$

The electroweak vector bosons also have self-interactions with the couplings given by,

$$\begin{aligned}\mathcal{L}_{WWV} = &-{\rm i}g\left[W^+_{\mu\nu}W^{\mu-} - W^-_{\mu\nu}W^{\mu+}\right](A^\nu \sin\theta_{\rm W} - Z^\nu \cos\theta_{\rm W})\\ &- {\rm i}g W^-_\nu W^+_\mu (A^{\mu\nu}\sin\theta_{\rm W} - Z^{\mu\nu}\cos\theta_{\rm W}),\end{aligned} \tag{1.19a}$$

and

$$\begin{aligned}\mathcal{L}_{WWVV} = -\frac{g^2}{4}\Big\{&\left[2W^+_\mu W^{\mu-} + (A_\mu \sin\theta_{\rm W} - Z_\mu\cos\theta_{\rm W})^2\right]^2\\ &- \left[W^+_\mu W^-_\nu + W^+_\nu W^-_\mu + (A_\mu \sin\theta_{\rm W} - Z_\mu \cos\theta_{\rm W})\right.\\ &\left.\times\ (A_\nu \sin\theta_{\rm W} - Z_\nu\cos\theta_{\rm W})\right]^2\Big\}.\end{aligned} \tag{1.19b}$$

In Eq. (1.19a), $A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and likewise for $Z_{\mu\nu}$ and $W_{\mu\nu}$.

Since the two chiralities of matter fermions belong to different representations of $SU(2)_{\rm L} \times U(1)_{\rm Y}$, it is not possible to include fermion mass terms without explicitly breaking gauge invariance. As for electroweak gauge bosons, these masses are also generated when electroweak symmetry is spontaneously broken. Fortunately, one does not have to introduce additional fields for this purpose. The scalar doublet ϕ in Table 1.1 (or its charge conjugate) has gauge invariant, renormalizable Yukawa interactions of the form $\bar{Q}\phi d_{\rm R}$ or $\bar{L}\phi\ell_{\rm R}$ ($\bar{Q}\phi^c u_{\rm R}$) to down-type (up-type) fermions, which acquire mass when the field acquires a VEV. These Yukawa interactions result in a scalar coupling of the Higgs boson $H_{\rm SM}$ to SM fermions that is proportional to the corresponding fermion mass, and is given by,

$$\mathcal{L}_{\rm Yukawa} = -\sum_i \frac{\lambda_{f_i}}{\sqrt{2}} \bar{f}_i f_i H_{\rm SM}, \tag{1.20}$$

where $\lambda_{f_i} = \frac{g m_{f_i}}{\sqrt{2}M_{\rm W}}$. The sum extends over all flavors of quarks and leptons. Notice that the Yukawa couplings of all but the top quark in Eq. (1.20) are much smaller than all the gauge couplings.