TIME-DEPENDENT CONVECTIVE COLLAPSE OF FLUX TUBES

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ABSTRACT

The time-dependent collapse of a slender flux tube extending
vertically in the convection zone of the Sun is modelled. Starting
from an initial state in which the flux tube is in hydrostatic equili-
brium, the non-linear MHD equations are used to examine its temporal
evolution. A detailed study of the flow variables and magnetic field
within the tube is presented. It is seen that asymptotically in time
a unique state of dynamic equilibrium is established, irrespective of
the value of β_0 (the ratio of the thermal to magnetic pressure at
the initial epoch).

Observations of the solar photosphere have revealed the existence
of intense magnetic fields (1-2 kG) concentrated into isolated flux
tubes (see review by Stenflo, 1976 and references therein). Parker
(1978) suggested that these large fields are produced due to a convective
instability. This paper aims to quantitatively study the temporal
development of the collapse of a slender flux tube as a result of
convective instability.

Consider a flux tube extending vertically into the convection
zone. We use the ideal MHD equations for a slender flux tube as given
in Roberts and Webb (1978). Noting that these equations are hyperbolic,
we solve them by the method of characteristics (for details see Hasan
and Venkatakrishnan, 1980).

We assume an initial state in hydrostatic equilibrium. Furthermore,
the temperatures inside and outside the tube at each depth are taken to
be the same. The convection zone model of Spruit (1977) along with the
model atmosphere of Vernazza et al (1976)are used to specify the
thermodynamic state of the fluid outside the tube. The pressure and
density inside the tube are lower by a constant factor. It can be seen
that the plasma parameter $\beta_0 = 8\pi p_0/B_0^2$, where p_0 and B_0 denote the pressure
and magnetic field strength respectively at the initial time, is
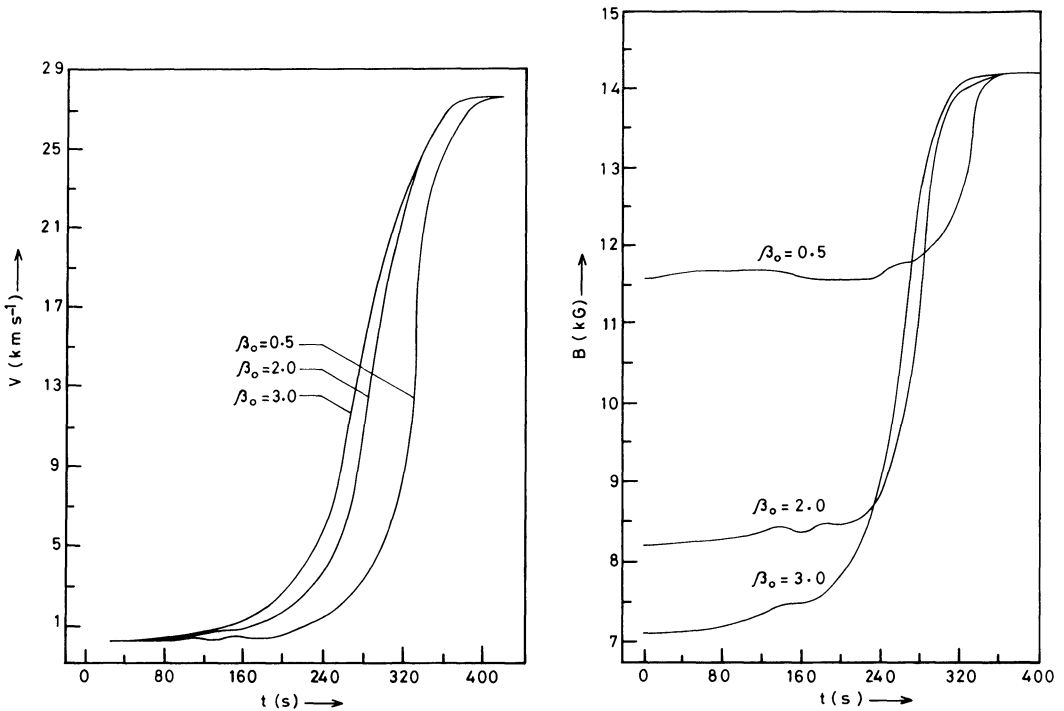independent of depth.

Fig.1. The variation with time of (a) velocity and (b)magnetic
       field for different β_0 at z = 1400 km.

The initial equilibrium state is perturbed by a velocity
perturbation of the form $v_0 = V_\alpha \sin \{ \pi(z-z_i)/(z-z_f) \}$, where V_α
denotes the amplitude of the downflow velocity and z_i and z_f refer
to the depths of the boundaries. For purposes of computational
economy, we chose z_i = -200km and z_f = 3000km (z is positive into the
Sun). In the computations a fixed value of 200 m s^{-1} for V_α was used.

Figure 1a shows the variation with time t of the fluid velocity
v near the central depth for various β_0. For β_0= 0.5, v initially
exhibits oscillatory behaviour, but subsequently increases monotonically
with t. In contrast, for β_0= 3.0, v increases monotonically with t from
the initial instant. For the intermediate value β_0 =2.0 there is an
initial gradual increase followed by a plateau-like region and then
a rapid increase of v. All the curves asymptotically approach a state
of constant v. From a least squares analysis, it is possible to calculate
a linear growth rate (for β_0 =3.0) of about 200s which is in broad agree-
ment with the value of about 220s one infers from fig.1 of Spruit and
Zweibel (1979).

Figure 1b depicts the magnetic field strength B as a function
of t (at the same depth). The temporal behaviour in this case again
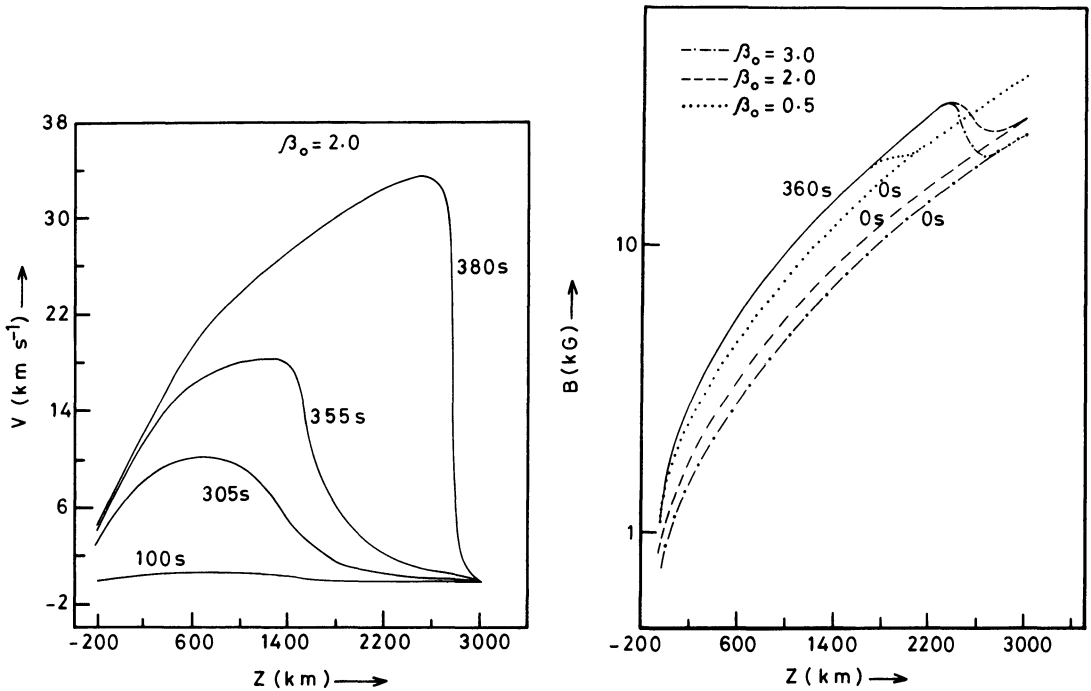exhibits an initial gradual variation followed by a rapid transient

Fig.2 (a) v versus z at different times for β_0=2.0. (b) B versus
      z for various β_0 at t = 0s and t = 360s. The solid line is
      used for the portion where the curves overlap.

phase and an eventual saturation to a constant value of B, which is
independent of β_0.

In figures 2a and 2b, the spatial variation of v and B is shown.
In figure 2a the steep gradient in v near the boundary at z = 3000 km
is a consequence of assuming zero velocity there. From figure 2b, we
see that curves of different β_0 after sufficient time merge into one.

On the basis of our time-dependent study it is possible to discern
several interesting features. Firstly, we see that a velocity perturba-
tion in the form of a downflow can trigger a convective instability in
a slender flux tube in hydrostatic equilibrium. This leads to a
collapse of the tube and an intensification of the magnetic field.
Secondly, there does not appear to exist a critical value of β_0
(at least in the range 0.5 - 3.0) below which the instability is
suppressed. This is contrary to the results of Spruit and Zweibel
(1979) who find a critical value for β_0 close to 2. Lastly, we notice
that after large enough time the instability saturates leading to a
halt of the collapse and to the establishment of a new state of dynamic
equilibrium. The physical properties of this state are the same,

irrespective of the choice of β_0.

Let us briefly consider the observational implications of this study. An important finding that emerges is that flux tubes formed as a result of convective collapse should be characterized by a unique value of v and B at each level (for some choice of boundary conditions). From figures 2a and 2b we find that at z=0, $B \simeq 1.5$ kG and $v \simeq 9$ km s^{-1}. In order to actually observe the collapse of a single flux tube, observations with a spatial resolution better than 0.5 arc sec and time resolution of about 50s would be desirable. Since measurements of magnetic fields are comparatively difficult, it might be easier to attempt to detect changes in downflow velocity in magnetic elements. Observations showing a rapid increase of downflow velocity could perhaps be considered as a signature of convective collapse.

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DISCUSSION

SPRUIT: You used a boundary condition allowing free inflow at the top at constant pressure. In my opinion this is the cause of the differences with other calculations. A nonlinear calculation of the possible equilibrium states of a fluxtube (Spruit: 1979, *Solar Phys.* **61**, p. 363) with a no-flow condition at the top shows the existence of equilibria with lower energy and higher field strength for tubes with $\beta > 2$.

HASAN: Initially, when I started this calculation, I used the no-flow condition, but this led to a practical computational problem because the pressure dropped very rapidly at the boundary. In your nonlinear calculation you assumed hydrostatic equilibrium, which may not be valid as observations show the existence of downflows in fluxtubes.

SPRUIT: I still think that a calculation with zero flow at the top would be very interesting, and I would urge you to remove these numerical difficulties.

CRAM: Are magnetic fields "concentrated" only after they emerge through the photosphere, or is it possible that the *subphotospheric* field is already in strong-field form — i.e. stronger than equipartition — before the field can be observed? *If* the field is intensified only after it emerges, then the discussed model is probably not particularly relevant, since it predicts collapse times of about 5 min, and this would not be short compared with the time scale for flux emergence itself.

HASAN: The purpose of my calculation was to study the final states that result due to the convective collapse of a fluxtube. The results show that intense fields ~ 1.5 kG at the solar surface can be produced by this process. To answer your question whether the subphotospheric field is already in strong form when it emerges, one would need a separate calculation studying the dynamical evolution of a fluxtube as it rises up towards the photosphere.

WEISS: Your models imply fields of the order of 14 kG relatively close to the surface — much greater than anything that has ever been observed. These field strengths follow, I believe, from your initial assumption of a uniform β_0. Would the results be significantly affected by allowing a more plausible field distribution?

HASAN: The field strength of 14 kG that you allude to is at a depth of about 1500 km below the surface. At the surface, the field strength is only 1.5 kG, which is within the observed range. My calculations indicate that the final results depend more sensitively upon the boundary conditions, rather than on the initial conditions.

RIBES: I would like to stress the importance of dynamics on the thermal and magnetic structure of fluxtubes. Steady downdrafts in thin magnetic tubes may give various theoretical solutions: A strong field solution ($B_0 \approx 1.3$ kG) is possible with either a strong downdraft ($v_0 \approx 6$ km s^{-1}) and no excess temperature ($\Delta T_0 = 0$), or with a large excess temperature ($\Delta T_0 \approx 2400$ K) (cf. Webb's thesis) with subsonic flow. The latter case is difficult to accept because the radiative cooling time is much shorter than the dynamical time. The former case has to be ruled out if large velocities are not observed. So, the alternative is to reduce the field strength to moderate values (500 G $< B < 1$ kG), or to give up the thin tube approximation.

HASAN: I entirely agree with your comment.