RELATIVISTIC EFFECTS INCLUDED IN THE APPARENT POSITIONS OF FUNDAMENTAL STARS (APFS)

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ABSTRACT. The relativistic effects included in the APFS from 1984 onwards are described. They are discussed in comparison with the accuracy available with modern techniques of observation.

1. INTRODUCTION

The development of new techniques of observation such as VLBI or the planned astrometry satellite HIPPARCOS has increased the observational accuracy considerably in the last about 10 years. These developments have made it necessary to increase the accuracy of the ephemerides to be compared with the observations. This involves also to include relativistic effects which could be neglected in the past. The aim of the present paper is mainly to compare the relativistic effects which are included in the volumes of the Apparent Places of Fundamental Stars (APFS) with the accuracy of modern observations and catalogue positions.

2. CHANGES IN THE PROCEDURE FOR COMPUTING APPARENT POSITIONS FROM 1984 ONWARDS

The APFS are still based on the data given in the FK4 (Fricke, Kopff, 1963) until the FK5 will be available. In considering the increasing observational accuracy the following main changes in the computational procedure were performed in the volumes of the APFS from the year 1984 onwards:

(a) A correction

$$\Delta\alpha = +\, 0\overset{s}{.}035 + 0\overset{s}{.}085\ (T - 19.50)$$

to the FK4 right ascensions at the epoch T is applied in order to correct them for the error of the equinox;

(b) The IAU (1976) System of Astronomical Constants is used, in particular the new constants for precession, aberration, and the obliquity of the ecliptic;

(c) The 1980 IAU Theory of Nutation is used;

J. Kovalevsky and V. A. Brumberg (eds.), Relativity in Celestial Mechanics and Astrometry, 223–227.

(d) Stellar aberration is computed from the total velocity of the Earth referred to the barycentre of the Solar System. The so-called E-terms are eliminated from the mean positions and included in the reduction to apparent positions.
(e) Reduction to apparent places are computed rigorously and directly without the use of the mean place for the beginning of the year; the rigorous computations also include relativistic effects.
These changes are in accordance with the resolutions adopted in the IAU Meetings at Grenoble, Montreal and Patras.

3. RELATIVISTIC EFFECTS INCLUDED IN THE APFS

There are many relativistic effects which could in principle be included in a general relativistic treatment of the motions of the stars. Most of these effects, however, are small compared with the observational accuracy available with classical instruments such as transit circles, astrolabes or PZT. Since the APFS provide the ephemerides for comparison with observations performed with this type of instruments only the following two effects are explicitely modelled in the APFS by formulae describing the coordinate transformations according to special and general relativity:

(a) Annual aberration,
(b) Light deflection by the Sun.

Implicitely included in the APFS is the

(c) Geodesic precession, which is incorporated in the constant of precession. Other relativistic effects such as geodesic nutation, gravitational pulsations and contractions, spin-spin interactions or differences between terrestrial and barycentrical time-scales are ignored in the case of the periodic terms because of their smallness, or they are absorbed by appropriate definitions of the scales in the case of the non-periodic terms.

The procedure for computing the APFS is described by Lederle and Schwan (1984). For details concerning the relativistic treatment of the motions of stars (and also of objects within the Solar System), reference is made to the explanatory parts in "The Astronomical Almanac for the year 1984", page B36-B41, and to the Supplement of the "1985 Japanese Ephemeris". The problem of including relativistic effects in the reductions from mean to apparent places were discussed for several decades, but more from a theoretical point of view because the accuracy of observations did not require to take them into account. It should be mentioned that, to our knowledge, Tagaki (1956) was the first who treated both light deflection and relativistic aberration in the same paper. Formulae for the latter have been given for instance already by Gubanov (1973). Formulae which are suited for practical application in the reduction of radar, optical, and radio-interferometric measurements are given by Brumberg (1981).

The most significant relativistic effect is the annual aberration which is of the order of 20".5 for a star in the pole of the ecliptic. Stumpff (1980) has shown that the differences between a treatment of aberration according to Special Relativity and to the classical procedure do not exceed 0".001.

In the case of the gravitational deflection of the star's light by the Sun it has been assumed in the APFS that the star is in an infinite distance. The gravitational deflection angle Θ for a star at the angular distance D from the Sun's centre is then given by

$$\Theta = 0''.004072\ (1 + \cos D)/\sin D.$$

Murray (1981) has shown that this formula has to replace the widely used Equ. (1) in Wade's (1976) paper which is based on Brandt's (1975) work. A small term 0.25 sin (2D) is erroneously included in Wade's paper. Although the deflection angle Θ decreases rapidly with increasing angular distance D from the Sun its value is still Θ = 0".01 for D = 45°. The influence of the gravitational deflection on the coordinates α, δ is illustrated in Table 1. Given are the coordinate differences Δα, Δδ (upper and lower row, respectively) between the deflected and undeflected positions of fictitious stars with coordinates α_*, δ_*; the Sun is assumed to be at $\alpha_\odot = \delta_\odot = 0°$. Light deflection caused by the gravitational field of planets has been ignored in the APFS.

Table 1. Coordinate differences {Δα in $0^s.0001$, Δδ in 0".001} produced by the light deflection by the Sun for fictitious stars at α_*, δ_*. The Sun is assumed to be at $\alpha_\odot = \delta_\odot = 0°$.

δ_* \ α_*	1°	15°	30°	60°	90°	120°	150°	180°
0°	+311	+ 21	+ 10	+ 5	+ 3	+ 2	+ 1	0
	0	0	0	0	0	0	0	0
+45°	0	+ 3	+ 5	+ 5	+ 4	+ 2	+ 1	0
	+ 10	+ 9	+ 6	+ 2	0	− 1	− 2	− 2
+89°	+ 3	+ 41	+ 79	+136	+156	+134	+ 77	0
	+ 4	+ 4	+ 4	+ 2	0	− 2	− 4	− 4

Geodesic precession is the gravitational analogue to the Thomas precession known from the theory of atomic spectra. According to General Relativity the coordinate system fixed to an object which is accelerated by some force, for instance since it moves in an orbit around a central mass M, performs a slow rotation with respect to an inertial system in which M is at rest. This effect is very small but it is cumulative and in the case of the Earth moving around the Sun it amounts to about 1".9/century. Geodesic precession acts in the opposite direction than the luni-solar precession and it is included implicitely in the constant of precession which is derived directly from the observations. Geodesic precession must therefore not be taken into account explicitely in the computation of ephemerides.

4. COMPARISON WITH THE ACCURACY OF OBSERVATIONS

We have restricted our considerations to the importance of relativistic effects in comparison with the accuracy of star positions. Time delay or related effects which are of importance for measurements within the Solar System are not discussed.

Information on the accuracy of observations of star positions obtained with modern techniques of observation is given in Table 2. The mean accidental errors hold for a catalogue position. Part of these data are estimates which have been obtained within our work on the compilation of the FK5.

Table 2. Accuracy of modern catalogue positions

Cat.	±σ	Reference
FK4	".04	Fricke, Kopff (1963)
Bordeaux 50(PZT stars)	.06	Mazurier et al. (1977)
Wash 5/50	.08	Hughes, Scott (in press)
AGK3	.20	Dieckvoss et al. (1975)
VLBI	.005	Fanselow et al. (1984)
HIPPARCOS	.002	Kovalevsky (1984)
Space Telescope	.002	Fresneau (1984)

The observations performed at Bordeaux differentially with respect to the FK4 and the absolute observations performed at Washington belong to the most accurate observations which have been made with transit instruments. The catalogue positions are based on about 10 single observations in the case of Bordeaux and about 30 observations in the case of Washington.

An accuracy of about 0".20 as valid for the AGK3 positions is typically for modern photographic observations. The VLBI measurements made by Fanselow et al. from 1971 - 1980 have yielded positions for 117 radio sources north of -40° declination; each position is the result of typically 10 sessions of observations.

For the HIPPARCOS mission it is expected to obtain an accuracy of 0".002 for the positions from observations over 2.5 years. The same accuracy is expected for the observations performed with the Space Telescope.

For completeness the average internal mean error of an FK4 position at the mean FK4 epoch is also given. The accidental errors in the FK5 will be of the same order of magnitude, perhaps an reduction by a factor of about 2/3 will be possible.

It should be noted that the values listed in Table 2 belong to more or less different types of observational techniques, ranging from absolute measurements as in the case of Wash 5/50 to differential measurements in very small fields as in the case of the Space Telescope. Systematic errors are not included in the values given in Table 2.

5. CONCLUSIONS

The APFS provide ephemerides to be compared with observations performed with classical instruments such as transit circles, astrolabes or PZT. Light deflection by the Sun is the only effect which can be of any significant influence on such observations in very particular cases. For the computation of the APFS relativistic effects are therefore only of little interest.

This is, however, quite different for VLBI observations and for observations using space techniques. In particular if VLBI observations will be possible in combination with space techniques, then relativistic effects must be carefully discussed.

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DISCUSSION

Pavlov : if you take into consideration quadratic terms of aberration, you should include other terms of the same order of magnitude.

Lederle : we have used the complete precise algorithm with all possible effects. We found that many of these terms were negligible and hence we have chosen the simplified algorithm. Another reason is that we wanted to have an algorithm for pocket calculators.