

an interesting book with its own individual flavour. It was inspired by an honours course at the University of Wisconsin; the author does not say how long that course was but it was evidently not short. In fact the book is in three parts which implicitly concedes that it might well be used in three different courses. Indeed the third part would not be out of place in a graduate program, whether in USA or UK. It is a book about theory—it aims to motivate and to explain theory with as much involvement from the reader as possible. Almost all the exercises are geared to theoretical aspects. The student who wishes a training in analytic techniques for solving specific problems should seek elsewhere.

Part I is intended for an honours calculus course; it covers differentiation and integration in one variable, continuous functions, Taylor's formula and sequence and series. To quote the author: "Part I begins with a half intuitive—half rigorous discussion of applications, chosen to arouse interest and to show the need for a precise and general theory, and then develops this theory for functions of one variable". The student may be forgiven for wondering where half rigour becomes whole rigour, and the author himself does concede the difficulty of encapsulating in a book the "spontaneous, creative disorder that characterizes an exciting course". For such an individual book one may be permitted some individual criticisms. I regret that for differentiation the author has chosen to emphasize quotients instead of linear approximation. I regret that he has expounded the Riemann integral and not the elementary integral obtained via uniform limits of step functions. He does give a solid treatment of Taylor's formula, in particular of the uniqueness theorem, but it is a pity that the elegant derivation of the formula by repeated integration by parts is relegated to a late exercise.

While one may have reservations about Part I, such reservations evaporate in Part II. This gives an excellent and well integrated account of metric spaces (essentially \mathbb{R}^n and function spaces), paths in \mathbb{R}^n , the requisite linear algebra, functions of several variables up to the implicit function theorem, manifolds (very well expounded) and higher derivatives. Part III begins with an efficient account of Lebesgue integration in \mathbb{R}^n , based on outer measures and includes some interesting applications (for example, multiple series and Sard's theorem). The remaining chapters become increasingly technical—differentiation of regular Borel measures, the problem of surface area, Brouwer topological degree, and finally extension theorems for various classes of functions, in particular with a view to applications to partial differential equations.

All in all, despite one's criticisms, the book is something of a tour de force. As usual it has been produced very well by Springer with ample spaces in the margins for the student to make his own annotations!

J. DUNCAN

BLASCHKE, WILHELM, *Gesammelte Werke*, Band 1 ed. by W. BURAU, S. S. CHERN *et al.* (Thales-Verlag, Essen 1982), pp. 365.

Wilhelm Blaschke was born in Graz on 13th September 1885. He died on 17th March 1962, and during his long life made an international reputation as one of the most influential geometers in Germany. From 1906 he studied at Vienna where he was influenced by Wirtinger. He then studied at Bonn, at Pisa with L. Bianchi, and at Göttingen with F. Klein and D. Hilbert who together had perhaps the greatest influence on his subsequent career.

In 1910 he was appointed Privat-Dozent in Bonn, and in 1913 he became professor at the Technische Hochschule in Prague. In 1915 he was appointed professor at the University of Königsberg and in 1919 he was appointed to the University of Tübingen. Later the same year he went to Hamburg to take charge of a new mathematics institute. He devoted much of his life to making Hamburg a centre for mathematical research and teaching. Hamburg was his main location until 1953. Throughout that period he was regarded as the best known German geometer. In 1931/32 he was guest professor at the Universities of Stanford and Chicago. Moreover, he visited India, China, Japan, the Soviet Union, South America and all states in Europe. His favourite place abroad was Pisa. In 1953 and 1955 he was professor at Istanbul.

Blaschke wrote over 200 research papers in different mathematical journals as well as textbooks and monographs. His research interests ranged over the whole field of geometry, including the geometry of webs and integral geometry. He was also fascinated by function theory and the calculus of variations, especially those parts relevant to geometrical problems and questions.

However, his main interest was in differential geometry studied in the light of the Erlangen Programme of F. Klein, which considers the role of geometry as a study of invariants under an appropriate group of transformations. For example, Blaschke considered the group of isometries, affine transformations, conformal transformations, complex projective transformations, etc. Some German mathematicians also regarded him as a substantial contributor to Riemannian geometry, and he certainly did use vector and tensor calculus effectively. However, the reviewer does not regard his work in Riemannian geometry as commensurate with his contributions to other branches of geometry; perhaps this is because Riemannian geometry does not fit naturally within the framework of Klein's Erlangen Programme.

He was fascinated by the isoperimetric problem and its three-dimensional analogues in the study of convex bodies, and his book *Kreis und Kugel* published in 1916 gives an inspiring introduction to these topics.

His three volume work, *Vorlesungen über Differentialgeometrie* has remained one of the standard treatises on differential geometry. In the first volume he gave a summary of classical differential geometry as developed by Gauss, but also included several global questions, especially with regard to isometric embeddings. These global questions also appeared in the study of critical distances along geodesic lines. The second volume considered affine differential geometry in considerable depth, making great use of special properties of ellipses and ellipsoids. The third volume was devoted to a study of the higher "Kreis-und-Kugelgeometrie" of Lie, Möbius and Laguerre.

Cartan's book *Leçons sur la géométrie de Riemann* (1925) showed the importance of topology for differential geometry. That this viewpoint was shared by Blaschke can be seen by his paper (1932) "Topological Questions of Differential Geometry". His treatment of the Cartan calculus was not given with his usual explicit authority, and it seems to the reviewer that this topic was still regarded by Blaschke as mysterious. When K. Leichtweiss re-edited the *Vorlesungen*, it was interesting to see that parts of the book involving differential forms were largely re-written.

In view of the above summary of Blaschke's work, it comes as no surprise that the Blaschke-Gedächtnis-Stiftung decided several years ago to publish his complete work in six volumes. The first volume of this set is reviewed here.

Opposite a photograph of Blaschke on the frontispiece is a foreword by W. Benz, Secretary of the Blaschke Stiftung. This is followed by a reprint of an 8-page appreciation of the work of Blaschke given in 1962 by Emanuel Sperner in Hamburg. There follows a brief curriculum vitae, a list of prizes and academic distinctions awarded, and a detailed list of 244 items of research papers and books. Pages 33–47 contain a very detailed description by Karl Strubecker of Blaschke's work, especially the papers included in the first volume. Space does not admit a detailed description of these, so we list the titles and references below, the numbers referring to the order in the complete list of publications.

1. Bemerkungen über allgemeine Schraubenlinien, *Mh. Math. u. Physik (Wein)* **19** (1908), 188–204.
2. Über einige unendliche Gruppen von Transformationen orientierter Ebenen im Euklidischen Raume, *Arch. Math. u. Physik* (3) **16** (1910), 182–189.
3. Über einige unendliche Gruppen von Berührungstransformationen in der Ebene, *Math. Ann.* **69** (1910), 204–217.
4. Untersuchungen über die Geometrie der Speere in der Euklidischen Ebene, *Mh. Math. u. Physik (Wein)* **21** (1910), 3–60.
5. Zur Geometrie der Speere in Euklidischen Raum, *Mh. Math. u. Physik (Wein)* **21** (1910), 201–308.
7. Über die Laguerresche Geometrie orientierter Geraden in der Ebene I, *Arch. Math. u. Phys.* (3) **18** (1911), 132–140.
10. Ein Beitrag zur Liniengeometrie, *Rend. Circ. Mat. Palermo* **33** (1912), 247–253.
163. Sulla geometria di Hermite, *Rend. Sem. Milano* **13** (1939), 58–65.

164. (mit H. Terheggen) Trigonometria Hermitiana, *Rend. di Mat. Roma* (4) 3 (1939), 153–161.
166. Über die Massbestimmungen von Hermite, *Atti Fond. Volta* 9 (1940) (20p).
167. Contributi alla geometria analitica degli spazi di Hermite, *Atti Accad. Ital.* (7) 1 (1940), 224–227.
168. Contributi alla geometria proiettiva complessa, *Bol. dell'Unione Mat. Ital.* (2) 2 (1940), 309–314.
170. Zur analytischen Geometrie in der Eben von Hermite, *Mitteil. Math. Ges. Hamburg* 8 II (1940), 3–30.
188. Isotrope Vierflache, *Archiv f. Math.* 1 (1948), 182–189.
191. Kinematische Begründung von Lies, Geraden-Kugel-Abbildung. *Münch. Sitz.-Ber.* (1948), 291–297.
205. Sulla geometria cinematica e descrittiva, *Archimede* 4 (1952), 45–49.

The book ends with a list of students and their dissertations, supervised by Blaschke from 1920 to 1953. It is interesting to see the entry: 15 Febr. 1936, Chern, Shing-shen, Eine Invariantentheorie der Dreigewebe aus r -dimensionalen Mannigfaltigkeiten in R_{2r} (*Abh. math. Sem. Hamburg* 11, (1936), 333–358). It is also interesting that Chern is returning in the 1980s to study afresh the theory of webs which he first met as a member of the Blaschke school.

TOM WILLMORE

DIESTEL, J., *Sequences and series in Banach spaces* (Graduate Texts in Mathematics Vol. 92, Springer-Verlag, Berlin-Heidelberg-New York, 1984) xiii + 261 pp., DM 108.

There are several good research monographs and surveys on various aspects of Banach space theory but these are mainly intended for experts. The theory of Banach spaces has advanced tremendously in the last twenty years with the clarification of Grothendieck's fundamental results in his São Paulo paper by J. Lindenstrauss and A. Pelczynski in 1968, with the deep studies of the classical Banach spaces in their own right and as subspaces and quotient spaces of other Banach spaces, with the investigations of series and sequences in Banach spaces, and with the introduction of factorization methods, p -summing operators, type and cotype. Some of this theory is well covered in J. Lindenstrauss and L. Tzafriri's monographs *Classical Banach Spaces I* and II [Springer-Verlag 1977 and 1979], and in other surveys and books; however, these are not introductory text books.

Joe Diestel's book on sequences and series in Banach spaces covers many of the standard current tools and results in the subject. The book assumes a standard first course in functional analysis as background and develops things from there even defining the weak and weak $*$ -topologies in a Banach space and its dual space, and proving such classical results as the Krein–Milman and Banach–Alaoglu Theorems. The theory is then steadily and carefully built up via chapters on the Eberlein–Šmulian and Orlicz–Pettis Theorems through the Dvoretzky–Rogers Theorem and Grothendieck's inequality to Rosenthal's l_1 -theorem. The tools required are introduced, illustrated, and developed to a useful stage even when this forces a diversion to study structures other than Banach spaces as the author does in the section on Ramsey's Theorem.

Each chapter ends with several interesting exercises, extensive notes and remarks, and a bibliography for the chapter. The exercises on the whole extend and link together the chapters; the notes give a good idea of how the subject developed, where to look for further information, and what has been omitted; the bibliography gives the main references and papers in the area. The style is lively and informal, the explanations are clear, and altogether this is a book in the best tradition of mathematical texts. I recommend it as the place to start any study of Banach spaces at present; it provides more than adequate foundations to read papers on Banach spaces. As in any book there are minor little things that will bother some readers, but in my opinion they are too trivial to mention, and just make the book more human and individual.

A. M. SINCLAIR