

## Problems with the Pulsation Mode Selection Mechanism in the Lower Instability Strip (Observations and Theory)

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**Abstract.** We examine the severe disagreement between the number of predicted and observed pulsation modes for  $\delta$  Scuti stars. The selection of nonradial modes trapped in the outer envelope is considered on the basis of kinetic energy arguments. The trapped  $\ell = 1$  modes for the star 4 CVn are in good, but not perfect agreement with the observations. The trapping of the  $\ell = 2$  modes is weaker, so that this simple rule of mode selection may apply to  $\ell = 1$ , and possibly not to  $\ell = 2$  modes.

### 1. Introduction

Delta Scuti star models predict pulsational instability in many radial and non-radial modes. The observed number of low-degree modes is much lower than the predicted number. The problem of mode selection is most severe for post-main-sequence  $\delta$  Scuti stars, which comprise about 40 percent of the observed  $\delta$  Scuti stars. The theoretical frequency spectrum of unstable modes is very dense. Most modes are of mixed character: they behave like p-modes in the envelope and like g-modes in the interior. For example, for a model of 4 CVn we predict 554 unstable modes of  $\ell = 0$  to 2: 6 for  $\ell = 0$ , 168 for  $\ell = 1$ , and 380 for  $\ell = 2$ . However, only 18 (and an additional 16 combination frequencies) were observed (Breger et al., 1999). This is demonstrated in Fig. 1. The problem for other  $\delta$  Scuti stars, such as BI CMi, is similar.

Consequently, either the theoretical predictions or the observational techniques are imperfect. On the observational side, only modes with photometric amplitudes in excess of 0.5 mmag are observed. However, this does not solve the dilemma, since the problem could then be rephrased to why only some favored modes are excited to observable amplitudes, which usually are much higher than the observational limit.

A number of different scenarios ranging from chaotic mode selection to energy transfer between the modes through nonlinear mode coupling can be invoked to explain the disagreement. Some of these hypotheses are very promising and are amenable to observational tests:

4 CVn: Frequency spectrum and probable mode identifications

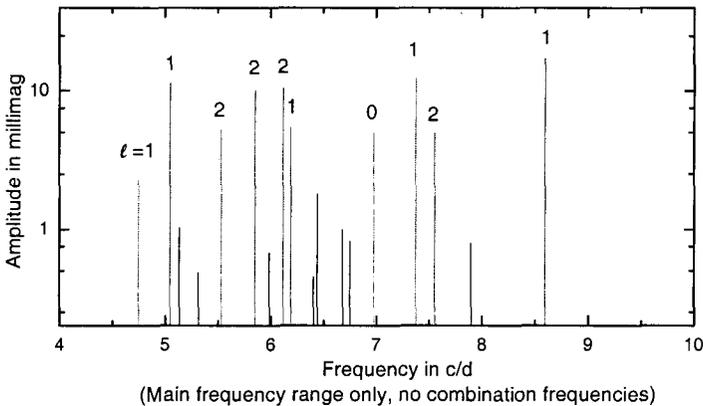


Figure 1. Observed pulsation modes and probable mode identifications for the evolved  $\delta$  Scuti star 4 CVn.

## 2. Scenario: random mode selection

Here the selection of the star's observationally visible modes from the multitudes of possible modes is random. A consequence of this scenario is that we will not be able to make use of nonradial-mode frequencies for seismic probing.

This hypothesis leads to several observational predictions: (i) there are no regular frequency patterns of the photometrically visible modes and, (ii) there is no similarity of the frequencies of the visible modes from star to star with similar masses, temperature and ages.

## 3. Scenario: temporary growth of randomly selected modes to observable amplitudes by mode interaction

In this scenario, most or all of the predicted modes are indeed unstable, but with amplitudes too small to observe photometrically. However, due to mode interaction, power is transferred between the modes. A few modes can temporarily grow to high amplitudes and are observed. After several years, which represent the typical growth cycle of  $\delta$  Scuti stars, these modes will decay again.

The hypothesis is supported by the fact that  $\delta$  Scuti stars show time-variable amplitudes as well as linear mode combinations of the modes with high amplitudes. The amplitude variability may be so large that over a decade the star may look like a completely different star (e.g. compare the power spectra of 4 CVn during the 1966-1970, 1974-1978, 1983-1984 and 1996-1997 time periods). However, a detailed investigation of the data shows that modes do not disappear to be replaced by others, but are still present at millimag amplitudes. This argues against the hypothesis, at least on time scales of decades.

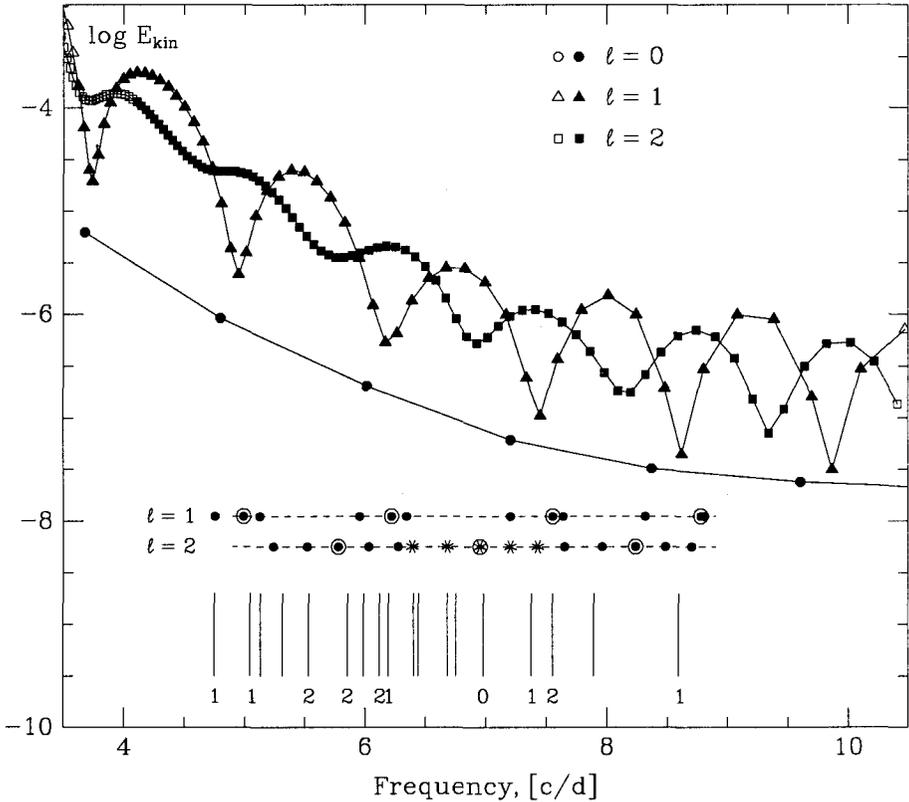


Figure 2. Kinetic energy values associated with the unstable modes of the evolved  $\delta$  Scuti star 4 CVn. The trapped modes are the modes with low kinetic energy values. Only the  $m = 0$  modes are shown. Empty symbols denote stable modes. For the most trapped modes all components of the multiplets are shown separately and compared to the observed frequencies with probable mode identifications.

4. Scenario: trapped envelope modes

In this scenario, the mode selection mechanism is related to modes trapped in the envelope (Dziembowski & Królikowska, 1990). It may be easier to excite these modes with lower kinetic energy to a given amplitude on the surface. Such a mode selection rule would be very simple and the computations rely only on linear theory. However, the strong proof must come from the nonlinear theory.

Figure 2 illustrates the complexity of the oscillation frequency spectrum for a model of 4 CVn, which oscillates with at least 18 frequencies in the range 4.7 - 9.7 c/d. The parameters of the model are within the range allowed by observational data:  $2.4 M_{\odot}$ ,  $\log L/L_{\odot} = 1.76$ ,  $T_{eff} = 6800K$ ,  $\log g = 3.32$ ,  $V_{rot} = 82$  km/s. The chemical composition  $X = 0.70$ ,  $Z = 0.02$  was assumed. The ordinate gives the oscillation kinetic energy, which is evaluated assuming

the same radial displacement at the surface for each mode. The high density of the oscillation spectrum of nonradial modes is caused by very large values of the Brunt-Väisälä frequency in the deep interior. This diagram is similar to one given by Dziembowski (1997), with the observational mode identification and selected rotational splitting added.

The most trapped modes are analogs of pure acoustic modes. It can be seen that the trapping is much more effective for dipole ( $\ell = 1$ ) than for quadrupole modes ( $\ell = 2$ ). The trapping effect for  $\ell = 2$  modes is especially weak at low frequencies and may not apply here. Therefore, the selection rule might work for only  $\ell = 1$  modes (see also Dziembowski & Królikowska, 1990).

The observed spacing between identified  $\ell = 1$  modes is about 1.2  $c/d$ , corresponding to the theoretical spacing between trapped  $\ell = 1$  modes. Also, the rotational splitting for the trapped  $\ell = 1$  and also for  $\ell = 2$  modes is similar to the observed splitting for identified modes. Note, however, that the conditions for the excitation of modes of different azimuthal order  $m$  may differ. Therefore, only some components of multiplets may be excited to observed amplitudes.

For the model considered the theoretical frequencies of the best trapped modes of  $\ell = 1$  fit the identified observed frequencies quite well. However, we still consider this model to be only an illustrative one. To fit theoretical and observed frequencies quantitatively, we must also take the effect of the rotational coupling of modes of different  $\ell$  values into account. Due to this effect, close modes with the same  $m$  and whose  $\ell$  degrees differ by 2 are pushed away in frequency. This was demonstrated for a model of XX Pyx (Pamyatnykh et al. 1998).

In evolved  $\delta$  Scuti models the trapping of nonradial modes in the envelope can be more effective than in RR Lyrae models. For example, for the most-trapped modes of  $\ell = 1$  in 4 CVn up to 67 percent of kinetic energy can be contributed by the envelope – in comparison with less than 20 percent in the RR Lyrae variables (see Dziembowski & Cassisi, 1999).

It was shown that there is a high probability of a resonant excitation of nonradial modes in radially pulsating RR Lyrae star models (see Nowakowski, these proceedings and references therein). Very recently, nonradial modes were detected in these stars (see Kovács, these proceedings and references therein). The theoretical frequency spectra of evolved  $\delta$  Scuti stars are similar to those of RR Lyrae stars, so resonances may be important for these stars too.

## References

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