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A COUNTEREXAMPLE IN THE PERTURBATION THEORY OF C^*-ALGEBRAS

BY

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ABSTRACT. The strongest positive results in the stability theory of C^*-algebras assert that if $\mathfrak{A}, \mathfrak{B}$ are sufficiently close C^*-subalgebras of $\mathfrak{L}(H)$ of certain kinds, then there is a unitary operator U on H near I, such that $U^*\mathfrak{B}U = \mathfrak{A}$. We give examples of C^*-algebras $\mathfrak{A}, \mathfrak{B}$, both isomorphic to the algebra of continuous functions from $[0, 1]$ to the algebra of compact operators on Hilbert space, which can be as close as we like, yet for which there is no isomorphism $\alpha: \mathfrak{B} \to \mathfrak{A}$ with $\|b - \alpha b\| \le 1/70 \|b\| \, (b \in \mathfrak{B})$. Thus the results mentioned do not extend to these C^*-algebras.

We shall describe, for each $\varepsilon' > 0$, two C^*-subalgebras $\mathfrak{A}$ and $\mathfrak{B}$ of $\mathscr{L}(K)$, the algebra of bounded operators on a Hilbert space K whose Hausdorff distance

$$d(\mathfrak{A}, \mathfrak{B}) = \max\left(\sup_{a \in \mathfrak{A}_1} \inf_{b \in \mathfrak{B}_1} \|a - b\|, \sup_{b \in \mathfrak{B}_1} \inf_{a \in \mathfrak{A}_1} \|a - b\|\right)$$

satisfies $d(\mathfrak{A}, \mathfrak{B}) < \varepsilon'$ yet for which there is no isomorphism $\alpha : \mathfrak{B} \to \mathfrak{A}$ with $\|b - \alpha(b)\| \le 1/70 \|b\| \, (b \in \mathfrak{B})$. ($\mathfrak{A}_1, \mathfrak{B}_1$ denote the unit balls of the respective algebras.) The algebras $\mathfrak{A}$ and $\mathfrak{B}$ are both isomorphic to the algebra of continuous functions from $[0, 1]$ into $\mathscr{LC}(L)$, the algebra of compact operators on the Hilbert space L. In fact we show that $\mathfrak{B}$ has a subalgebra $\mathfrak{C}$ isomorphic with $\mathscr{LC}(L)$ so that $\mathfrak{C} \overset{\varepsilon'}{\subseteq} \mathfrak{A}$ in the notation of [2; Definition 2.1] yet there is no homomorphism $\beta : \mathfrak{C} \to \mathfrak{A}$ with $\|c - \beta(c)\| \le 1/70 \|c\| \, (c \in \mathfrak{C})$. Replacing 70^{-1} by 1000^{-1} we get the same result for a subalgebra $\mathfrak{C}_0$ of $\mathfrak{C}$ isomorphic with c_0. Phillips and Raeburn have shown ([7] Theorem 4.22) that there are $s, t > 0$ such that if $\mathfrak{A}$ is a unital continuous trace C^*-algebra and $d(\mathfrak{A}, \mathfrak{B}) < \varepsilon < s$ then there is a isomorphism $\alpha : \mathfrak{B} \to \mathfrak{A}$ with $\|b - \alpha(b)\| \le t\varepsilon^{1/2} \|b\| \, (b \in \mathfrak{B})$. Thus our example shows that their theorem cannot be extended to non-unital continuous trace C^*-algebras. Christensen [2; Corollary 6.3] has shown that if $\mathfrak{C}$ is a finite dimensional abelian C^*-algebra and $\mathfrak{C} \overset{\varepsilon}{\subseteq} \mathfrak{A}$ then there is a * homomorphism $\alpha : \mathfrak{C} \to \mathfrak{A}$ with $\|c - \alpha(c)\| < 22\varepsilon^{1/2} \|c\| \, (c \in \mathfrak{C})$. Thus our example also shows that this result does not extend to the case of an AF algebra $\mathfrak{C}$.

We denote the set of strictly positive integers by $\mathbb{Z}^+$, $L = \ell^2(\mathbb{Z}^+)$ and $\xi_1, \xi_2, \ldots$ is the standard basis of L. E_n is the orthogonal projection onto the span of

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$\xi_1, \ldots, \xi_n$. Any $A \in \mathscr{L}(L)$ is given by a matrix $A_{ij} = \langle A\xi_j, \xi_i\rangle$. If $d = \{d_1, d_2, \ldots\}$ is a strictly increasing sequence from $\mathbb{Z}^+$ and $A \in \mathscr{L}(L)$ we define A_d by

$$\begin{aligned}(A_d)_{ij} &= 0 \quad \text{if for some } k\\ &\qquad d_{k-1} < i \le d_k \quad \text{and} \quad d_{k-2} < j \le d_{k+1}\\ &= A_{ij} \quad \text{otherwise}\end{aligned}$$

where $d_{-1} = d_0 = 0$. If we partition the basis into blocks of length $d_k - d_{k-1}$ and make a corresponding partition of the matrix for A then A_d is obtained from A by replacing the blocks on the main diagonal and the two adjacent diagonals by zero. Since the diagonal maps

$$A \mapsto \sum_k (E_k - E_{k-1})A(E_{k+l} - E_{k+l-1})$$

have norm 1 ($l \in \mathbb{Z}$ and we put $E_m = 0$ if $m \le 0$) we see $A_d \in \mathscr{L}(L)$ and $\|A_d\| \le 4\,\|A\|$.

Lemma 1. *For each* $\varepsilon > 0$ *and each* $d_1 < d_2 < \cdots$ *there exists a self-adjoint element* A *of* $\mathscr{L}(L)$ *with*

$$\begin{aligned}&\|A\| \le 1\\ &\|A_d\| = 1\\ &\|AE_n - E_nA\| \le \varepsilon \qquad n \in \mathbb{Z}^+.\end{aligned}$$

Proof. Let α_n $(n \in \mathbb{Z})$ be the Fourier coefficients of the function $f(e^{i\theta}) = i\theta/\pi(-\pi < \theta \le \pi)$ in $L^\infty(\mathbb{T})$. Then $\alpha_n = (-1)^{n+1}/n\pi(n \ne 0)$ and $\alpha_0 = 0$. However $\sum \alpha_{|n|}e^{in\theta}$ is the Fourier series of the L^2 function $2\pi^{-1}\log|1+e^{i\theta}|$ which is not in $L^\infty(\mathbb{T})$. Thus [4; p. 135] the matrix $[\alpha_{j-i}]$ represents an operator on $\ell^2(\mathbb{Z})$ of norm 1 but $[\alpha_{|j-i|}]$ does not represent a bounded operator. Thus taking only $i, j > 0$, $[\alpha_{j-i}]$ is an operator on L of norm 1 [4; p. 139] whereas $[\alpha_{|j-i|}]$ is not because, writing $\ell^2(\mathbb{Z}) = L \oplus L^\perp$ divides $[\alpha_{|j-i|}]$ into four blocks of which the off diagonal blocks are the same as the corresponding blocks in $\pm[\alpha_{j-i}]$ and so represent a bounded operator whereas the two blocks on the main diagonal are in fact the same and so must both represent unbounded operators.

By taking m sufficiently large the matrix $C = E_m[\alpha_{|i-j|}]E_m$ represents an element of $\mathscr{L}(L)$ of norm $> \varepsilon^{-1}$. Define $S_d, T_d : \mathscr{L}(L) \to \mathscr{L}(L)$ by

$$\begin{aligned}(S_dB)_{ij} &= B_{d_{2i}d_{2j}}\\ (T_dB)_{ij} &= B_{kl} \qquad \text{if } i = d_{2k}, j = d_{2l}\\ &= 0 \qquad \text{if } (i, j) \text{ is not of the form } (d_{2k}, d_{2l}).\end{aligned}$$

T_d is an isometry, S_d is a contraction and $S_dT_d = $ identity. Put $A = \|C\|^{-1}T_dC$.

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Then $A = A^* = A_d$ and $\|A\| = 1$. Also if $d_{2k} \le n < d_{2k+2}$

$$\begin{aligned}\|AE_n - E_nA\| &= \|S_d(AE_n - E_nA)\|\\ &= \|(S_dA)E_k - E_k(S_dA)\|\\ &= \|C\|^{-1}\,\|CE_k - E_kC\|\\ &= \|C\|^{-1}\max(\|(I-E_k)CE_k\|, \|E_kC(I-E_k)\|)\\ &= \|C\|^{-1}\max(\|(I-E_k)E_m[\alpha_{j-i}]E_mE_k\|, \|E_kE_m[\alpha_{j-i}]E_m(I-E_k)\|\\ &\le \|C\|^{-1} < \varepsilon.\end{aligned}$$

We denote the set of self adjoint operators in $\mathscr{L}(L)$ by $\mathscr{L}(L)_{\text{s.a.}}$.

LEMMA 2. *For each* $\varepsilon > 0$ *there is a function* $A:[0,1] \to \mathscr{L}(L)_{\text{s.a.}}$ *and functions* $A_n:[0,1]\to\mathscr{L}(L)_{\text{s.a.}}$ $n \in \mathbb{Z}^+$ *such that*

1. $\|A_n(x) - A(x)\| \le \varepsilon$ $(n \in \mathbb{Z}^+)$
2. $\|A(x)\| \le 1$ $(x \in [0,1])$
3. *A is continuous in the weak operator topology and* $x \mapsto A_n(x)\xi_i$ $i = 1, \ldots, n$ *are norm continuous.*
4. *There is no function* $A_\infty:[0,1]\to\mathscr{L}(L)$, *continuous in the strong * operator topology, for which*

$$\|A_\infty(x) - A(x)\| \le \tfrac{1}{5} \quad (x \in [0,1]).$$

*The strong * operator topology is that determined by the semi norms* $\|B\xi\|, \|B^*\xi\|, \xi \in L$.

Proof. Consider the set

$$\mathscr{A}_\varepsilon = \{A\,;\, A \in \mathscr{L}(L)_{\text{s.a.}}, \|A\| \le 1, \|AE_n - E_nA\| \le \varepsilon \ (n \in \mathbb{Z}^+)\}$$

with the weak operator topology. $\mathscr{A}_\varepsilon$ is a weak operator closed bounded convex subset of $\mathscr{L}(L)$ and so is compact. It is also metrisable and so if $X \subseteq [0,1]$ is the Cantor set there is a continuous surjection $A_0: X \to \mathscr{A}_\varepsilon$ [6, p. 166]. We extend A_0 to a continuous surjection $A:[0,1]\to\mathscr{A}_\varepsilon$ by linear interpolation on each interval of $[0,1]\backslash X$ whose endpoints are in X. For each n put $A_n(x) = E_nA(x)E_n + (I-E_n)A(x)(I-E_n)$. We have

$$\begin{aligned}\|A_n(x) - A(x)\| &= \|-E_nA(x)(I-E_n) - (I-E_n)A(x)E_n\|\\ &= \max(\|E_nA(x)(I-E_n)\|, \|(I-E_n)A(x)E_n\|)\\ &= \|A(x)E_n - E_nA(x)\| \le \varepsilon,\end{aligned}$$

giving 1. 2 is obvious and A is weak operator continuous so $x \mapsto A_n(x)\xi_i$ $(i \le n)$ is weakly continuous. As its range is in the finite dimensional space E_nL, on which the weak and norm topologies coincide, it is norm continuous.

Suppose a function A_∞ as in 4 existed. For each i the sets $\{A_\infty(x)\xi_i\,;\, x \in [0,1]\}$ and $\{A_\infty(x)^*\xi_i\,;\, x\in[0,1]\}$ are norm compact. Thus we can define inductively a

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sequence $0<d_1<d_2<d_3\cdots$ of integers such that for each i,

$$\|E_{d_i}A_\infty(x)(I-E_{d_{i+1}})\|<(5.2^{i+2})^{-1} \quad\text{and}\quad \|(I-E_{d_{i+1}})A_\infty(x)E_{d_i}\|<(5.2^{i+2})^{-1}.$$

We then have

$$A_\infty(x)_d=\sum(E_{d_i}-E_{d_{i-1}})A_\infty(x)(I-E_{d_{i+1}})+\sum(I-E_{d_{i+1}})A_\infty(x)(E_{d_i}-E_{d_{i-1}})$$

so that $\|A_\infty(x)_d\|<\frac{1}{5}$. As the map $A\mapsto A_d$ has norm ≤ 4 we see $\|A_\infty(x)_d-A(x)_d\|\leq\frac{4}{5}$ so $\|A(x)_d\|<1$. However by Lemma 1 there are values of x with $\|A(x)_d\|=1$.

We denote the C^*-algebra of bounded functions $[0,1]\to\mathcal{L}(L)$ by $\mathfrak{D}$ and the subalgebra of norm continuous functions with values in $\mathcal{LC}(H)$ by $\mathfrak{A}$. Given $\varepsilon>0$ let $A\in\mathfrak{D}$ as in Lemma 2 and put $U=\exp i\pi A/8$ and $\alpha=\text{ad } U$ (that is $\alpha(B)=U^*BU$) and $\alpha(x)=\text{ad } U(x)$. We denote the map $C\mapsto CB-BC$ by δB. For $c\in\mathcal{LC}(L)$ let $j(c)\in\mathfrak{A}$ be the constant function with value c, that is $j(c)(x)=c$, $0\leq x\leq 1$.

THEOREM 3. *Let $\varepsilon'>0$. For $\varepsilon<\min\{\frac{1}{2},\frac{1}{2}\varepsilon'\exp-3\pi/4\}$ we have $d(\mathfrak{A},\alpha\mathfrak{A})<\varepsilon'$ and hence $\mathfrak{C}=\alpha j(\mathcal{LC}(L))\overset{\varepsilon'}{\subseteq}\mathfrak{A}$. There is no homomorphism $\beta:\mathfrak{C}\to\mathfrak{A}$ with $\|c-\beta(c)\|\leq\frac{1}{70}\|c\|$ $(c\in\mathfrak{C})$.*

Proof. Let $D=\delta A$, $a\in\mathfrak{A}$. Then $\|D\|\leq 2$ and $E_na(x)E_n\to a(x)$ uniformly for $0\leq x\leq 1$ so for some value of n, $\|E_na(x)E_n-a(x)\|\leq\varepsilon\,\|a\|$ $(x\in[0,1])$. Then $\|Da-(\delta A_n)(E_naE_n)\|\leq\|D(a-E_naE_n)\|+\|(D-\delta A_n)(E_naE_n)\|\leq 4\varepsilon\,\|a\|$. However, $(\delta A_n)(E_naE_n)(x)=E_na(x)E_nA_n(x)-A_n(x)E_na(x)E_n\in\mathfrak{A}$ because E_nA_n and $A_nE_n\in\mathfrak{A}$. Thus for each $a\in\mathfrak{A}$ there is $b\in\mathfrak{A}$ with $\|b-Da\|\leq 4\varepsilon\,\|a\|$. Using this we can show by induction that for each n there is $b_n\in\mathfrak{A}$ with $\|b_n-D^na\|\leq 6^n\varepsilon\,\|a\|$ and hence $\text{dist}(\alpha(a),\mathfrak{A})\leq\varepsilon\,\|a\|\exp 3\pi/4\leq\frac{1}{2}\varepsilon'\,\|a\|$. Similarly $\text{dist}(\alpha^{-1}(a),\mathfrak{A})=\text{dist}(a,\alpha(\mathfrak{A}))\leq\frac{1}{2}\varepsilon'\,\|a\|$ and so $d(\mathfrak{A},\alpha(\mathfrak{A}))\leq\varepsilon'$.

If β is as stated then α and $\beta\alpha$ are homomorphisms $j(\mathcal{LC}(L))\to\mathfrak{D}$ with $\|\alpha-\beta\alpha\mid j(\mathcal{LC}(L))\|\leq 70^{-1}$. For each $x\in[0,1]$, $\gamma(x)(c)=\beta\alpha j(c)x$ defines a homomorphism $\gamma(x);\mathcal{LC}(L)\to\mathcal{LC}(L)$ with $\|\alpha(x)-\gamma(x)\|\leq 70^{-1}$. As $\|\alpha-\text{id}\mathfrak{D}\|\leq 2\sin\pi/8<\frac{7}{9}$ this implies $\|\gamma(x)-\text{id}\mathcal{LC}\|<\frac{4}{5}$ so $\gamma(x)$ is an isomorphism. As $x\mapsto\gamma(x)(c)=\beta\alpha j(c)(x)$ defines an element of $\mathfrak{A}$ the map $x\mapsto\gamma(x)$ is continuous with respect to the point-norm topology (that is the topology defined by the semi-norms $\lambda\mapsto\|\lambda(C)\|$, $C\in\mathcal{LC}(L)$). Let $\mu(x)=\log\gamma(x)$ (using the principal value). Then $\mu(x)$ is a derivation on $\mathcal{LC}(L)$ [3; p. 313]. If p is a polynomial in one variable then $\lambda\mapsto p(\lambda);\mathcal{L}(\mathcal{LC}(L))\to\mathcal{L}(\mathcal{LC}(L))$ is point-norm continuous on bounded sets and so $x\mapsto\mu(x)$ is point-norm continuous. We have $\log\alpha(x)=\delta(x)$ where $\delta(x)(a)=i\pi(aA(x)-A(x)a)/8$. Also $\|\delta(x)-\mu(x)\| =\|\log\alpha(x)-\log\gamma(x)\|\leq\sum_{n>0}n^{-1}\|(\alpha(x)-\text{id }\mathcal{LC})^n-(\gamma(x)-\text{id }\mathcal{LC})^n\|\leq\sum_{n>0}(\frac{4}{5})^{n-1}\|\alpha(x)-\gamma(x)\|\leq 5\frac{1}{70}<\pi/40$. For each $x\in[0,1]$ define $B(x)\in\mathcal{L}(L)$ by $B(x)c\xi_1=(\delta(x)-\mu(x))(ce_{11})\xi_1(c\in\mathcal{LC}(L))$. Then as in [5; Theorem 3.1], $(\delta(x)-\mu(x))c=cB(x)-B(x)c$, $\|B(x)\|\leq\pi/40$ and $\langle B(x)\xi_1,\xi_1\rangle=0$. Put $A_\infty(x)=$

$A(x)-8B(x)/\pi i$. Then $\|A_\infty(x)-A(x)\|\le\frac{1}{5}$ and $\mu(x)a=i\pi(aA_\infty(x)-A_\infty(x)a)/8$. For $\eta,\zeta\in L$ let $\eta\otimes\zeta$ be the rank one operator $\xi\mapsto\langle\xi,\zeta\rangle\eta$ and put $e_{ij}=\xi_i\otimes\xi_j$. We have

$$8e_{ii}\mu(x)(e_{ii})=i\pi\xi_i\otimes(A_\infty^*(x)\xi_i-(A_\infty(x)_{ii})^-\xi_i)$$
$$8\mu(x)(e_{ii})e_{ii}=i\pi(A_\infty(x)_{ii}\xi_i-A_\infty(x)\xi_i)\otimes\xi_i$$
$$8e_{ii}\mu(x)(e_{ij})e_{jj}=i\pi(A_\infty(x)_{jj}-A_\infty(x)_{ii})e_{ij}$$
$$A(x)_{11}=A_\infty(x)_{11}$$

Since the left of the first three equations is a norm continuous function of x and $A(x)_{11}$ is continuous we see that all the $A_\infty(x)_{ii}$ are continuous and $x\mapsto A_\infty(x)$ is strong * continuous. This contradicts the properties of A in Lemma 2.

By identifying a diagonal matrix with its diagonal sequence we can consider $c_0\subseteq\mathscr{L}\mathscr{C}(L)$.

COROLLARY 4. *Let* $\mathfrak{C}_0=\alpha j(c_0)$ *and* $\varepsilon'<(1000^{-1})$. *Then there is no* * *homomorphism* $\beta_0:\mathfrak{C}_0\to\mathfrak{A}$ *with* $\|c-\beta_0(c)\|\le1000^{-1}\|c\|$ $(c\in\mathfrak{C}_0)$.

Proof. We shall use the method of [2; Theorem 6.4] to extend β_0 to $\mathfrak{C}$. Consider $\mathfrak{D}$ as an algebra of operators on the Hilbert space $K=\ell_L^2[0,1]$. Then there is a unitary operator W on K with $\|I-W\|<999^{-1}$ [1; Theorem 5.4] and $\beta_0(c)=W^*cW$ $(c\in\mathfrak{C}_0)$. Put $\mathfrak{C}_1=W^*\mathfrak{C}W$. Then $\mathfrak{C}_1\overset{\varepsilon''}{\subseteq}\mathfrak{A}$ where $\varepsilon''=3(999)^{-1}$ and $\beta_0(\mathfrak{C}_0)\subseteq\mathfrak{C}_1\cap\mathfrak{A}$. Put $p_{ij}=W^*\alpha j(E_{ij})W$. For each $n\in\mathbb{Z}^+$ let $f'_n\in\mathfrak{A}$ with $\|p_{1n}-f'_n\|\le332^{-1}$ and put $f_n=p_{11}f'_np_{nn}$ so $\|P_{1n}-f_n\|<332^{-1}$. Thus $\|f_nf_n^*-p_{11}\|\le\|f_n\|\,\|p_{1n}-f_n\|+\|p_{1n}-f_n\|<165^{-1}$ and so $(f_nf_n^*)^{-1/2}$ exists in the algebra $p_{11}\mathfrak{A}p_{11}$ and we have $\|p_{11}-(f_nf_n^*)^{-1/2}\|<(1-165^{-1})^{-1/2}-1<328^{-1}$ so that $g_n=(f_nf_n^*)^{-1/2}f_n$ has $\|g_n-f_n\|\le328^{-1}(1+332^{-1})<327^{-1}$. Also $g_n\in p_{11}\mathfrak{A}p_{nn}$ and $g_ng_n^*=p_{11}$ so $g_n^*g_n$ is a projection in $p_{nn}\mathfrak{A}p_{nn}$ with $\|g_n^*g_n-p_{nn}\|<\|g_n\|\,\|g_n-p_{1n}\|+\|g_n-p_{1n}\|<4.327^{-1}$ and so $g_n^*g_n=p_{nn}$. Put $V=\sum_n p_{n1}g_n$, the series converging because for each n the nth term is a unitary operator on $p_{nn}K$. As $\sum_n j(e_{nn})$ converges weakly to I on K we see $\sum p_{nn}$ converges weakly to I and so V is unitary and $\|I-V\|=\sup_n\|p_{nn}-p_{n1}g_n\|=\sup_n\|p_{1n}-g_n\|<2.327^{-1}$. Now put $\beta(c)=V^*W^*cWV$ $(c\in\mathfrak{C})$. Then $\beta\alpha(j(e_{ij}))=V^*p_{ij}V=g_i^*p_{11}g_j\in\mathscr{A}$, so that

$$\beta(\mathfrak{C})\subseteq\mathfrak{A}$$

and

$$\|c-\beta(c)\|\le2\,\|I-WV\|\,\|c\|\le2(\|I-W\|+\|I-V\|)\,\|c\|\le70^{-1}\|c\|\ (c\in\mathfrak{C}).$$

Although K is not separable the subalgebra of $\mathfrak{D}$ generated by $\mathfrak{A}$ and $\alpha\mathfrak{A}$ is and so could be represented on a separable Hilbert space. The algebras $\mathfrak{A}$ and

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$\alpha(\mathfrak{A})$ do not have units but we could adjoin the identity on K to $\mathfrak{A}$, $\alpha(\mathfrak{A})$ and $\mathfrak{C}$ and the identity on L to $\mathscr{L}\mathscr{C}(L)$ and the proofs would apply. The algebra obtained by adjoining a unit to $\mathfrak{A}$ is postliminal but does not have continuous trace.

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