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Stream-orbit misalignment & a new algorithm for constraining the Galactic potential with streams

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Abstract. In general, a tidal stream is misaligned with the orbit of its progenitor. Here we present the formation of tidal streams in angle-action space to discuss the effect of this misalignment on orbit-fitting algorithms for constraining the Galactic potential. We close by presenting and testing an alternative algorithm which more fully accounts for the dynamics of streams by using the angle-action formalism.

Keywords. Galaxy: kinematics and dynamics, halo, structure – methods: numerical

1. Introduction

Structure in the Universe is formed hierarchically. The smallest structures are formed earliest, and these structures go on to combine and form ever larger structures. Observations of tidal streams give us the opportunity to see this process in action. Tidal streams are long filamentary structures formed by tidal forces from a host galaxy stripping stars from a smaller satellite cluster. In recent years it has been discovered that the Milky Way is rich with such structures (Belokurov *et al.* 2006). The structure of a stream reflects the underlying potential which formed it, so inferences about the host potential can be made from stream observations (McGlynn 1990, Johnston *et al.* 1996, Johnston *et al.* 1999). Importantly they probe the potential on a large scale where the dark-matter halo is expected to dominate. Multiple stream observations may be the best way to constrain the large-scale distribution of dark matter in our Galaxy.

Many methods for estimating the potential exist: corrected stream tracks (Johnston *et al.* 1999, Varghese *et al.* 2011), entropy minimisation (Peñarrubia *et al.* 2012) and N-body simulations (Law & Majewski 2010). However, perhaps the most attractive, due to its simplicity, is orbit fitting (Binney 2008). The stream track is assumed to delineate an orbit, such that if we observed for long enough we would see the stream members follow each other across the sky. This method has been employed most successfully by Koposov *et al.* (2010) for constraining the Galactic potential using the stream GD-1 (Grillmair & Dionatos 2006). However the validity of the assumption that a stream delineates an orbit has recently been questioned (Choi *et al.* 2007, Eyre & Binney 2011) and so must be tested.

The following work is taken from two papers. The first of these (Sanders & Binney 2013a) discusses the validity of fitting streams with orbits, whilst the second (Sanders & Binney 2013b) presents a new algorithm for constraining the Galactic potential using streams.

2. Stream formation in angle-action space

The assumption that a stream delineates an orbit is clearly fundamentally flawed. Stream formation requires the particles to have subtly different orbits. If all the stream members are at different phases on the same orbit the stream would not grow in time but instead stretch and compress as it moved along it's orbit. There is an intrinsic spread in the integrals of motion of stripped particles, which is governed by the properties of the cluster, primarily the mass. For a low-mass cluster it is tempting to say that the spread in the integrals of motion is small enough to assume all the particles are essentially moving on the same orbit. However, if we try to fit an orbit to the stream members this assumption will lead us to err. The difference in integrals of motion in the stream may be small, but over several orbital periods these small differences become important, as the particles drift slowly apart, and hence away from the assumed orbit track. The power cold streams have for constraining the Galactic potential is due to the wide range of orbital phases in the stream. We may be able to find many potentials which produce consistent integrals of motion for all stream particles, but some of these solutions may produce inconsistent phases of the stream.

The above discussion can be formalised neatly using angle-action coordinates (Tremaine 1999, Helmi & White 1999). The angles, $\boldsymbol{\theta}$, and the actions, $\boldsymbol{J}$, describe the dynamics of a free particle in the external Galactic potential in a pleasingly simple way – the actions are integrals of motion, whilst the angles increase linearly in time:

$$\boldsymbol{J} = \text{const.}; \;\; \boldsymbol{\theta} = \boldsymbol{\Omega} t + \boldsymbol{\theta}(0). \tag{2.1}$$

The time derivatives of the angles are $\boldsymbol{\Omega} = \partial H/\partial \boldsymbol{J}$, the frequencies of the Hamiltonian, H, t is the time, and $\boldsymbol{\theta}(0)$ is a constant. Let us consider a single star moving freely in the stream. The separation between the angle coordinates of the star and the cluster remnant, $\boldsymbol{\Delta\theta}$, obeys the equation

$$\boldsymbol{\Delta\theta} = \boldsymbol{\Delta\Omega} t + \boldsymbol{\Delta\theta}(0), \tag{2.2}$$

where $\boldsymbol{\Delta\Omega}$ is the separation in frequencies which is frozen-in after the particle has left the cluster, and the time t denotes the time since the particle was stripped from the cluster. $\boldsymbol{\Delta\theta}(0)$ is the initial separation in angles, which is insignificant compared to the term $\propto t$ when a stream has formed. Therefore, all stream stars obey the equation

$$\boldsymbol{\Delta\theta} \approx \boldsymbol{\Delta\Omega} t. \tag{2.3}$$

In Figure 1 we show five particles taken from an N-body stream simulation to illustrate this equation. We see that the angle separation for each of the particles oscillates until the particle is stripped. The angle difference then increases linearly in time. We note that the first particles to be stripped have the steepest gradients as the most energetic particles are the first to be stripped.

As a stream only spans a small range in actions we can express $\boldsymbol{\Delta\Omega}$ as a first-order Taylor expansion

$$\boldsymbol{\Delta\Omega} \approx \mathbf{D} \cdot \boldsymbol{\Delta J} \tag{2.4}$$

where we have introduced the Hessian matrix $\mathbf{D}$ and the difference in actions $\boldsymbol{\Delta J}$. Tremaine (1999) noted that for long thin streams, i.e. essentially 1D objects, to form the Hessian must be highly anisotropic. It is therefore dominated by a single large eigenvalue, λ_1, which has associated eigenvector, $\hat{\boldsymbol{e}}_1$. If the Hessian satisfies this condition, we have that

$$\frac{\boldsymbol{\Delta\theta}}{t} \approx \boldsymbol{\Delta\Omega} \propto \hat{\boldsymbol{e}}_1. \tag{2.5}$$

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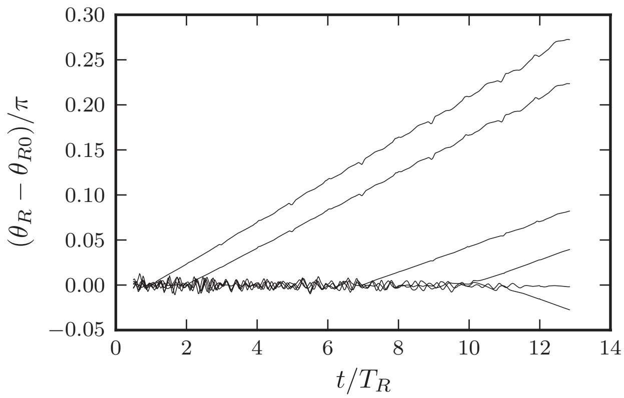

Figure 1. Difference between the radial angle of the progenitor and six particles taken from the N-body simulation presented in Section 4. The particles are stripped at pericentric passage (given by the units on the x-axis). Once stripped from the cluster the particles move freely in the external potential.

This means that the angle and frequency distributions of the particles will be confined to a straight line.

3. Orbit-fitting

With the above formalism we can return to the discussion of orbit-fitting. An orbit fit is valid if all the particles have the same frequency, *and* the angles increase at this frequency along the stream. The first of these conditions is entirely dependent on the mass of the cluster, with high-mass progenitors producing large spreads in frequencies. However, the second condition is mass-independent, and it depends only upon the potential and the progenitor actions. Therefore, for low-mass progenitors the second condition is more significant, so we will focus on it here. For an orbit-fit to be valid $\boldsymbol{\Delta\theta} \propto \hat{\boldsymbol{e}}_1$ must be aligned with the progenitor frequency $\boldsymbol{\Omega}_0$. In general the principal eigenvector of the Hessian will not be aligned with the progenitor frequency, and there will be a misalignment angle which we define as

$$\varphi = \arccos(\hat{\boldsymbol{\Omega}}_0 \cdot \hat{\boldsymbol{e}}_1). \tag{3.1}$$

In Figure 2 we plot the magnitude of φ for a realistic Galactic potential from McMillan (2011). This potential consists of a disc, bulge and spherical NFW halo. We see that far out in the halo (high J_z) the misalignment angle is a few degrees, whereas for more disc-like orbits (low J_z) the misalignment angle increases to tens of degrees. Clearly the misalignment is non-zero but it is difficult to assess the impact of the magnitude of the misalignment on estimates of potential parameters when orbit-fitting. We introduce a family of two-parameter potentials which vary in their halo flattening, Q, and the ratio k of the dark to visible matter force ratio on the Sun. We estimate the errors we will make when orbit-fitting by finding the parameters (k, Q) which causes the angle distribution of a stream to be aligned with the progenitor frequency. The results of this experiment for the halo flattening, Q, are shown for known streams in Figure 3. We have estimated the actions of the stream progenitors from information in the literature. We see that for some known streams (specifically GD-1, Anticenter and Aquarius) the error we make in the halo flattening by orbit-fitting is of order one.

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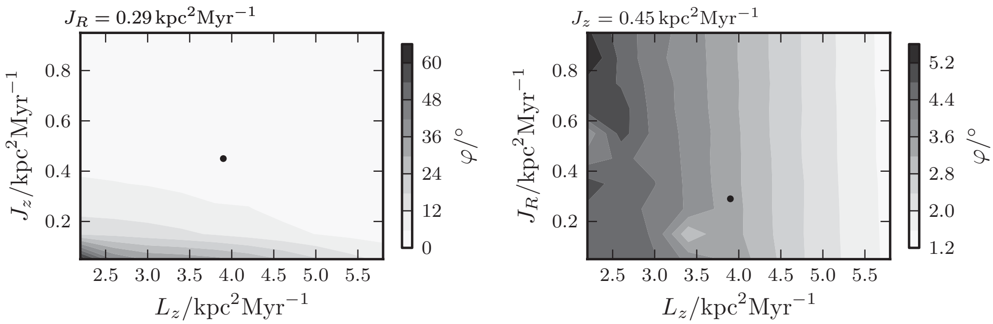

Figure 2. Misalignment angle for two action-space planes in a realistic Galactic potential from McMillan (2011). The black dot shows the approximate action coordinates of GD-1 which the simulation in Section 4 was chosen to emulate.

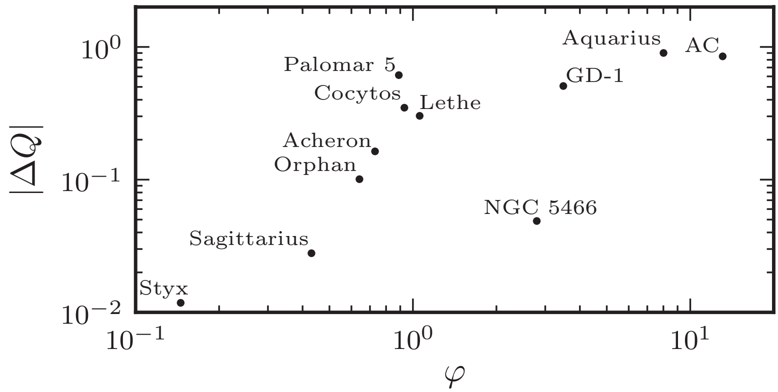

Figure 3. Expected error in the halo flattening using orbit-fitting algorithms to analyse known streams.

The results of Figure 3 have not accounted for the masses of the progenitors. We note that some streams have very massive progenitors, particularly the Sagittarius stream. For these streams the spread in frequencies will be more important than the misalignment in angle-space. In the case of Sagittarius, Figure 3 suggests that the misalignment is small enough for an orbit fit to be valid. However, an orbit fit on the entire Sagittarius stream will be flawed due to the spread in the frequencies. Still Figure 3 is of interest as it tells us that sections of the Sagittarius stream with small spreads in frequencies (for instance, the leading or trailing tail) are well fitted by orbits.

4. A new algorithm

For many streams orbit-fitting is inappropriate. Therefore, to successfully constrain the Galactic potential using streams we must develop superior methods which avoid the assumption that a stream delineates an orbit. We have already mentioned several of these methods. However, the angle-action formalism suggests an alternative. From the discussion we found that the angles and frequencies of all the stream members lie in the same straight line distribution with gradient determined by the principal eigenvector

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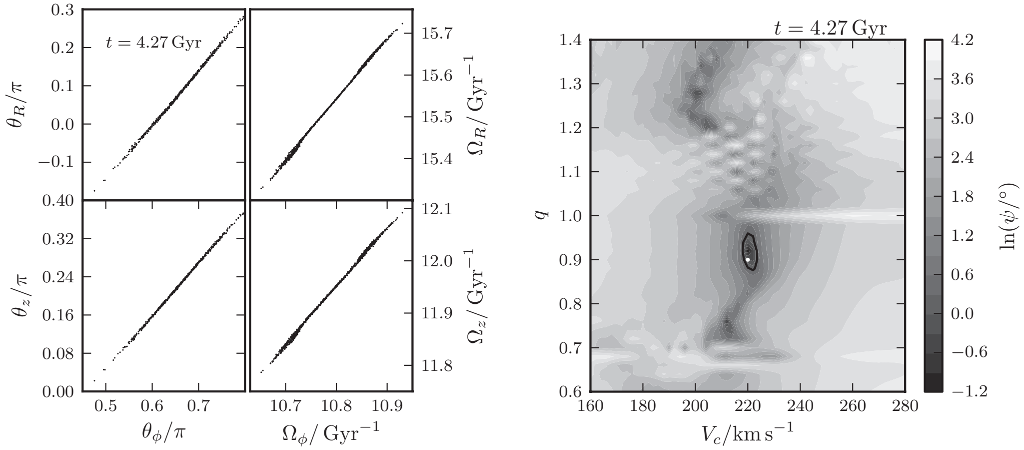

Figure 4. Results of the algorithm presented in Section 4. The left panel shows the angle- and frequency-space distributions of the stream in the correct potential, and the right panel shows the surface of the angle between the angle- and frequency-space distributions. The minimum is an estimate of the true potential parameters, which are shown by a white dot.

of the Hessian, $\hat{\boldsymbol{e}}_1$. In the correct potential the angle distribution will be aligned with the frequency distribution. Therefore, to find the correct potential we maximise $\cos\psi = \hat{\boldsymbol{\Delta\theta}} \cdot \hat{\boldsymbol{\Delta\Omega}}$. In practice we find $\hat{\boldsymbol{\Delta\theta}}$ and $\hat{\boldsymbol{\Delta\Omega}}$ by calculating the angles and frequencies of the stream members in each potential, and then performing a linear regression. This method has the nice properties that we are correctly accounting for the dynamics of the stream without knowledge of the properties of the progenitor, and without knowledge of the time since each particle was stripped. These properties do not affect the gradient of the angle and frequency distributions.

We test this method by applying it to a low-mass stream simulation. Using the N-body code GYRFALCON (Dehnen 2004) we evolve a $2 \times 10^4 M_\odot$ mass cluster on a GD-1-like orbit for ~ 10 periods in a two-parameter logarithmic potential given by

$$\Phi(R, z) = \frac{V_c^2}{2} \log\left(R^2 + \frac{z^2}{q^2}\right). \tag{4.1}$$

$V_c = 220\,\mathrm{km\,s^{-1}}$ is the circular speed and $q = 0.9$ is the flattening. We observe 500 particles from the resultant stream at pericentre. In Figure 4 we show the surface of ψ as a function of the two potential parameters. We see that the minimum (corresponding to maximum alignment of the angle and frequency distributions) lies very close to the true potential parameters.

5. Closing remarks

We have demonstrated that observed streams do not delineate orbits in a realistic Galactic potential, and assuming they do can lead to large errors when constraining properties of the dark matter distribution of the Galaxy. As an alternative to orbit-fitting we have presented a new algorithm for constraining the Galactic potential using tidal streams which exploits the correlations in angle-action space. We have demonstrated that the algorithm works by inspecting an N-body simulation. The next step is to apply the algorithm to a real data-set, such as the 6D GD-1 data from Koposov *et al.* (2010).

References

Belokurov, V., Zucker, D. B., Evans, N. W., *et al.* 2006, *ApJ*(Letters), 642, L137
Binney, J. 2008, *MNRAS*, 386, L47
Choi, J.-H., Weinberg, M. D., & Katz, N. 2007, *MNRAS*, 381, 987
Dehnen, W., 2004, *ApJ*(Letters), 536, L39
Eyre, A., & Binney, J. 2011, *MNRAS*, 413, 1852
Grillmair, C. J., & Dionatos, O. 2006, *ApJ*(Letters), 643, L17
Helmi, A. & White, S. D. M. 1999, *MNRAS*, 307, 495
Jin, S. & Lynden-Bell, D. 2008, *MNRAS*, 383, 1686
Johnston, K. V., Hernquist, L., & Bolte, M. 1996, *ApJ*, 465, 278
Johnston, K. V., Zhao, H., Spergel, D. N., & Hernquist, L. 1999, *ApJ*(Letters), 512, L109
Koposov, S. E., Rix, H.-W., & Hogg, D. W. 2010, *ApJ*, 712, 260
Law, D. R. & Majewski, S. R. 2010, *ApJ*, 714, 229
McGlynn, T. A. 1990, *ApJ*, 348, 515
McMillan, P. J. 2011, *MNRAS*, 414, 2446
Peñarrubia, J., Koposov, S. E., & Walker, M. G. 2012, *ApJ*, 760, 2
Sanders, J. L. & Binney, J. 2013, *MNRAS*, in press
Sanders, J. L. & Binney, J. 2013, *MNRAS*, in press
Tremaine, S. 1999, *MNRAS*, 307, 877
Varghese, A., Ibata, R., & Lewis, G. F. 2011, *MNRAS*, 417, 198

Discussion

Vasily Belokurov: Is the misalignment between the stream and the progenitor's orbit mainly due to the flattening of the potential and is this misalignment different from the offset that exists between the stream and the progenitor's orbit that is due to the stream's debris having distinct integrals of motion?

Jason Sanders: I have been investigating the misalignment in angle-space between the stream and progenitor orbit. This effect is mass-independent and depends only on the potential. It seems that a more flattened potential increases the misalignment between stream and orbit (see Figure 2), whilst spherical potentials have small, but still significant, misalignments.

There are two related effects which cause a stream to be misaligned with the progenitor orbit. First, like you say, the stream's debris has a spread in actions. Second, this action spread produces a corresponding spread in the angle variables, which is misaligned with the progenitor orbit. As the spread in angles increases in time, this second effect is significant even for low-mass progenitors when the first effect is insignificant. The source of both these effects is the finite spread in the actions of the debris.

Hans-Walter Rix: The parameters you have found from analysing the GD-1 stream using your new technique are very similar to those found by Koposov *et al.* (2010) using orbit-fitting. If the assumption that the GD-1 stream delineates an orbit is so wrong, why is this?

Jason Sanders: I have only analysed mock data chosen to be similar to GD-1. These mock data have been generated using the logarithmic potential parameters taken from Koposov *et al.* (2010). I intend to analyse the real GD-1 dataset and then perhaps we will see the difference between the two methods.