

The promise of Bayesian analysis for prominence seismology

Iñigo Arregui, Andrés Asensio Ramos, and Antonio J. Díaz

Instituto de Astrofísica de Canarias, E-38205 La Laguna, Tenerife, Spain
Departamento de Astrofísica, Universidad de La Laguna, E-38205 La Laguna, Tenerife, Spain
email: iarregui@iac.es

Abstract. We propose and use Bayesian techniques for the determination of physical parameters in solar prominence plasmas, combining observational and theoretical properties of waves and oscillations. The Bayesian approach also enables to perform model comparison to assess how plausible alternative physical models/mechanisms are in view of data.

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Prominence seismology aims to determine physical parameters in prominence plasmas by a combination of observed and theoretical properties of waves and oscillations (Joarder *et al.* 1997). The technique has been successful in the determination of a number of parameters using prominence fine structure oscillations (Arregui *et al.* 2012). Yet, solving the inversion problem is not an easy task, because observational information is always incomplete and uncertain. As a consequence, extracting information from model parameters by comparison of their predictions with observed data has to be carried out in a probabilistic framework. The Bayesian formalism is the only fully correct way we have to obtain information about physical parameters from observations (inference) and to compare the performance of alternative models to explain observed data (model comparison) (see e.g., Gregory 2005; von Toussaint 2011).

Two Bayesian data analysis tools are here used:

- the marginal posterior, $p(\theta_i|d) = \int p(\theta|d)d\theta_1 \dots d\theta_{i-1}d\theta_{i+1} \dots d\theta_N$, provides us with the most probable values of a given parameter, θ_i , compatible with observed data d , in the form of a conditional probability distribution, $p(\theta_i|d)$.
- the marginal likelihood, $p(d|M) = \int p(d, \theta|M)d\theta = \int p(d|\theta, M)p(\theta|M)d\theta$, provides us with the probability of the observed data d , given that the model M is true. It tells us how well the observed data are predicted by model M , with parameter set θ .

Determination of field strength and transverse inhomogeneity. We used observations of period (P) and damping time (τ_d) of transverse thread oscillations to obtain information on magnetic field strength (B_0) and transverse density inhomogeneity length scale (l/R). Figure 1 shows the marginal posteriors for these two parameters for given observed oscillation data. The posteriors provide a well constrained fully consistent solution to the inverse problem, with correct propagation of uncertainty.

Discrimination of damping mechanisms. We used our Bayesian model comparison tool to quantify the ability of different mechanisms to explain the damping of transverse thread oscillations in prominences. We considered three proposed mechanisms and computed their evidence in view of data (the damping ratio τ_d/P). The mechanisms were: (1) Alfvénic resonance; (2) slow resonance; and (3) Cowling's diffusion (see Soler 2010). The computation of the marginal likelihood for each model – the probability of the data given that the model is true – enables us to assess how well each one reproduces the observed damping ratio (Figure 2). Alfvénic resonance (model M1) is able to properly

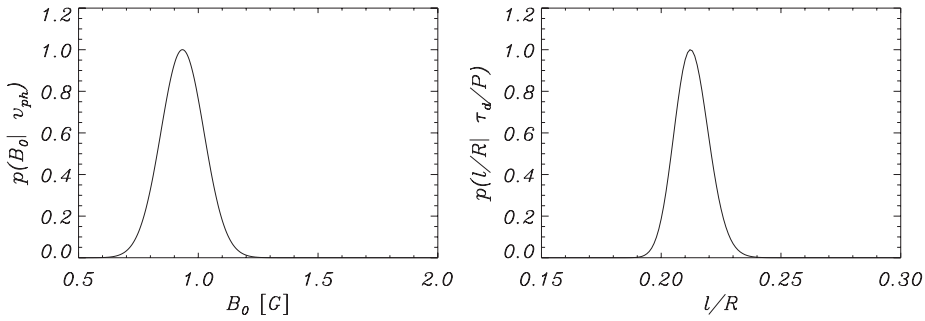


Figure 1. Posterior probability distributions for field strength (B_0) and transverse density length-scale (l/R , in units of the thread radius) for a thread oscillation with $P = 3$ min, $\tau_d = 9$ min, and phase speed $v_{ph} = 16$ km s $^{-1}$. Uncertainty of 10% in data has been considered.

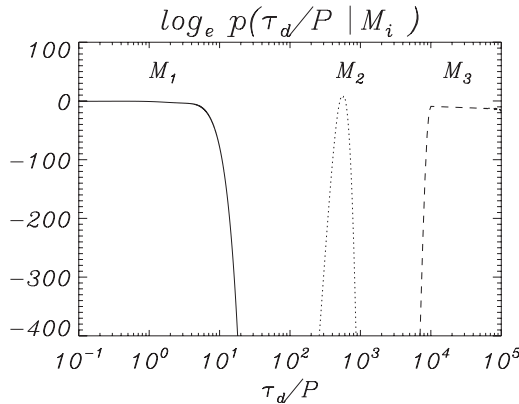


Figure 2. Graphical representation of the validity of each considered damping mechanism as a function of the observable damping ratio, in the form of marginal likelihoods in natural logarithm.

explain low damping ratios, such as the observed ones. Cowling's diffusion (M2) has the ability to reproduce damping ratios of the order of 10^3 . Model M3 (slow resonance) can only account for damping ratios above 10^4 , even for plasma- β values near unity.

These and additional examples (omitted for brevity) show that Bayesian analysis techniques exhibit great promise for prominence seismology.

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