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GEOMETRIC INVERSE PROBLEMS

This up-to-date treatment of recent developments in geometric inverse problems introduces graduate students and researchers to an exciting area of research. With an emphasis on the two-dimensional case, topics covered include geodesic X-ray transforms, boundary rigidity, tensor tomography, attenuated X-ray transforms, and the Calderón problem.

The presentation is self-contained and begins with the Radon transform and radial sound speeds as motivating examples. The required geometric background is developed in detail in the context of simple manifolds with boundary. An in-depth analysis of various geodesic X-ray transforms is carried out together with related uniqueness, stability, reconstruction, and range characterization results. Highlights include a proof of boundary rigidity for simple surfaces as well as scattering rigidity for connections. The concluding chapter discusses current open problems and related topics. The numerous exercises and examples make this book an excellent self-study resource or text for a one-semester course or seminar.

Gabriel P. Paternain is Professor of Mathematics at the Department of Pure Mathematics and Mathematical Statistics at the University of Cambridge and a Fellow of Trinity College. His research has covered an ample mathematical landscape, including Hamiltonian dynamics, symplectic geometry, and geometric inverse problems. He is the author of the monograph *Geodesic Flows* (1999), and was awarded the Pilkington Teaching Prize at Cambridge for his ability to explain analysis and geometry with a clarity that has won him the admiration and respect of his students.

Mikko Salo is Professor of Mathematics at the University of Jyväskylä, Finland. He has received several awards for his work on inverse problems in partial differential equations and geometry, including the Calderón prize, the Väisälä prize, an ERC Starting Grant, and an ERC Consolidator Grant.

Gunther Uhlmann is the Walker Family Endowed Professor at the University of Washington and the Si Yuan Professor at the Hong Kong University of Science and Technology. He has worked on microlocal analysis and a broad spectrum of inverse problems. He was awarded the AMS Bocher Prize, the Kleinman Prize from SIAM, the Solomon Lefschetz Medal from the Mathematical Council of the Americas, and the Birkhoff Prize, awarded jointly by SIAM and the AMS.

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Geometric Inverse Problems

With Emphasis on Two Dimensions

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