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BOOK REVIEWS

ASCHBACHER, M., *Finite group theory* (Cambridge studies in advanced mathematics 10, Cambridge University Press, 1986), ix + 274 pp., £22.50.

One of the biggest achievements of recent years in mathematics has to be classification of the finite simple groups. It brings to a successful conclusion what has been the major project of finite group theory for the whole of this century. So it is of interest to get some insight into this monumental work, without having to spend a great deal of time picking up the background. This book is part of the attempt to bring the Classification Theorem within the reach of common mortals. It does its job well, but there is still a long, long way to go before any one other than the experts can really claim to understand fully what is going on.

D. Gorenstein has credited the author of this book, M. Aschbacher, with bringing the successful conclusion of the Classification Theorem forward by several years. This makes him a most suitable person to write this book as long as he can get rid of the jargon of the inner circle. He has done so, and thus presents a good introduction to the whole area of finite group theory mentioned above. In the words of the author, it "is intended to serve both as a text and as a basic reference on finite groups". It fulfils the second object better than the first, as it is rather too condensed to be a good textbook. The numerous exercises at the end of each of the sixteen chapters improve its quality as a textbook, and lead to results which otherwise would not be covered. But to me, it is as a basic up-to-date reference for finite simple group theory that this book succeeds.

The major theme for most of the book is that is that of a representation, which is quite a general idea in this context. A representation of a group G in a category $\mathscr{C}$ is a homomorphism of G into the automorphism group of some object of $\mathscr{C}$. In practice there are three kinds of representation which are considered to any extent: permutation representations on sets, representations of groups on groups, and, most widely used, representations on vector spaces, the so-called linear representations. All these representations are considered in this book as well as the geometry associated with many of the groups considered. There is much else as well.

Starting with preliminaries, including sections on categories, graphs and geometries, and abstract representations, there follows a chapter each on permutation representations, representations of groups on groups and linear representations. More information is given on permutation groups, then there is a chapter on extensions, which deals with 1-cohomology and coprime action. A very important topic is that of spaces with forms as it gives rise to the classical groups, the major source of finite simple groups. Another important class is that of p-groups which comes next. The topics covered in the second half of the book are: change of field of a linear representation, presentation of groups, the generalized Fitting subgroup, linear representations of finite groups, transfer and fusion, the geometry of groups of Lie type, signalizer functors and finally a chapter on finite simple groups giving a list of the minimal simple groups and an outline of the Classification Theorem. Suggestions for the contents of courses based on this book are given, as well as advice on what can be safely omitted. Each chapter starts with an overview of the contents and ends with remarks placing the work in context and exercises. It is slightly odd that advice on which parts of the chapter can be left out on a first reading comes at the end in the remarks.

This is a very useful book for anyone who is interested in the whole field of finite group theory. It is a good basic reference for the aspiring specialist, and gives a good idea of the results and techniques for the interested outsider with a sound basic knowledge of group theory. As a textbook it is too condensed to be used on its own without help from a lecturer. But it is wellwritten and helps the reader to gain confidence in dealing with the whole subject of finite simple groups. Most libraries should have a copy and many algebraists will find it sufficiently useful and instructive to want to have their own copy. The standard of printing is high and the book is well-produced.

J. D. P. MELDRUM

CONWAY, J. B., *A course in functional analysis* (Graduate Texts in Mathematics 96, Springer-Verlag, 1985), xiv+404 pp. DM 118.

For many years, the only general account of the basics of functional analysis could be found in the magnificent treatise *Linear Operators* by N. Dunford and J. T. Schwartz and, as a result, this work served both as the definitive source for the established researcher and as a text for beginning graduate students to cut their teeth on. Although "Dunford and Schwartz", as it is always affectionately referred to, remains the best reference—I recall being told of the apocryphal mathematician who needed three copies: one for the office, one for home and one for the car—at the same time, a number of less ambitious texts aimed more directly at the beginner have appeared in recent years. Conway's book is one of the latest of these and is to be recommended. As with many such books, it grew out of a year-long course given to graduate students over a number of years. The contents probably represent the union of the topics covered in these courses as they evolved and changed and so, by suitable selection, a number of different courses could be based on the book. Such courses would differ in detailed content, but all would have the same underlying theme, namely linear analysis.

When approaching the fundamentals of any subject, an author must choose whether to proceed from the particular to the general or vice versa. In the present text, the former approach is adopted. This inevitably leads to a certain amount of repetition, but this will probably be welcomed by the student reader and, as the author observed in the introduction, it is the way mathematics usually develops. The book starts with an account of Hilbert spaces and the basic associated operator theory (adjoints, projections, compact operators and so on) and then turns to Banach spaces (Hahn–Banach theorem, duality, the closed graph theorem and other applications of Baire's theorem etc.). A limited amount of locally convex space theory is included, primarily so that weak topologies can be discussed, and some basic Banach algebra theory is covered, to be applied in the approach to spectral theory. The remainder of the book is somewhat more specialized, reflecting the author's particular interests in Hilbert space operator theory. There is a good account of the basic properties of C^*-algebras and this is put to work in the analysis of normal operators. The book ends with chapters on unbounded operators and Fredholm theory.

Many applications and examples are included, together with references to further developments, so there is much to whet the appetite of the interested reader. There are also plenty of exercises at the end of each section. Some of these might be criticized as being a trifle on the dull side, though a few routine problems are probably helpful for the beginner, and there are a fair number of interesting ones to offset them. All in all, this is an excellent book which will prove invaluable to any graduate student wanting a groundwork in the principles of the subject.

T. A. GILLESPIE

GARLING, D. J. H., *A course in Galois theory* (Cambridge University Press, 1986), pp. 167, cloth £22.50, paper £8.95.

As a course of study for undergraduates, Galois theory certainly has a lot going for it. Its clear purpose—finding the conditions for solubility of a polynomial equation using the usual arithmet-