GRAVITATIONAL, INERTIAL AND TOROIDAL OSCILLATIONS OF THE OUTER CORE AND THEIR RELATED FREE WOBBLES

Po-Yu Shen
Gravity and Geodynamics Division, Earth Physics Branch
Department of Energy, Mines and Resources
Ottawa, Ontario K1A 0Y3, Canada

According to the respective restoring forces of self-gravitation, Coriolis force, and inertial coupling at the boundaries, the free oscillations of a contained fluid can be classified into gravitational, inertial, and toroidal oscillations. For the outer core of the Earth, however, due to the interplay of rotation, elasticity, and self-gravitation, the gravitational undertones and inertial oscillations are not distinguishable. Both have eigenfunctions consisting of spheroidal and toroidal parts of about equal amplitude, and exist in alternating allowed and forbidden zones depending on the gravitational stability of the outer core. The forbidden zones for a stable core correspond to the allowed zones for an unstable core, while for a neutrally stratified core there appear to be no forbidden zones. The eigenfunction of a toroidal mode consists essentially of a primary toroidal field and a secondary spheroidal component of the order of ellipticity, coupled at the outer core-mantle or outer core-inner core boundary. Therefore, in general, toroidal core modes appear in doublets $S^m_{n+1}T^m_n$ and $S^m_{n-1}T^m_n$ with degenerate frequency equal to $2m/n(n+1)$ times the angular frequency of rotation of the Earth. The ellipticities of the outer core boundaries are responsible for the removal of the degeneracy. It is shown that the primary toroidal and secondary spheroidal fields constitute the "generalized Poincare motion" which, for the fundamental mode $S^1_2T^1_1$, reduces to the "simple motion" defined by Poincare in 1910. Numerical solutions have been obtained for all three types of free core modes. While those for gravitational undertones and inertial oscillations are obtained by arbitrarily truncating the hydrodynamic equations, it is shown that the eigensolutions for toroidal modes are correct to first order in ellipticity due to the particular geometry of the outer core of the Earth.

The $\underline{T}^1_1$ toroidal displacement field represents a rotation relative to the frame of reference. Therefore for each free oscillation of the outer core where $\underline{T}^1_1$ is a component of the eigendisplacement, there is

E. P. Fedorov, M. L. Smith and P. L. Bender (eds.), Nutation and the Earth's Rotation, 185–186.

a corresponding free wobble of the Earth. In particular, the $S_2^1T_1^1$ fundamental toroidal mode gives rise to the "nearly diurnal wobble" and manifests itself through the resonance effects on diurnal earth tides and astronomical nutations. Our results agree with those obtained by Molodensky in 1961.