PART III : REFERENCE SYSTEMS

THE REFERENCE SYSTEMS

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ABSTRACT. In order to discuss accurately the motions of the Earth in space, it is necessary to define rigorously two readily accessible reference systems. The conception and the realization of celestial absolute systems and terrestrial coordinate systems are discussed. It is suggested that these systems of reference ought to be defined with a minimum of theoretical or observational constraints. Examples of such ideal reference systems are given, together with some desirable properties for intermediate systems.

INTRODUCTION

"Toute l'astronomie repose sur l'invariabilité de l'axe de rotation de la Terre à la surface du sphéroide terrestre et sur l'uniformité de cette rotation".

If this statement by P.S. de Laplace (1825) still held, there would be no IAU symposium on "Time and the Earth's rotation". But since we know that it is not true, there arises the problem: since the description of the motion of a body is possible only with respect to something else, what shall we refer the motion of the Earth to?

As a matter of fact, we need two systems of reference: a terrestrial coordinate system that would represent the body "Earth" and to which observatories as well as the axis of rotation are referred, and an absolute external celestial system of reference in which the motion of the first system represents what we call the "Rotation of the Earth" - that is, the motion of the Earth around its centre of mass.

Many different definitions of the two systems are possible and some are reviewed in this presentation. They are not equivalent and may have different characters as far as their conceptual simplicity, their practical realization or their accessibility are concerned. Some may be practical for some kind of observations and completely unfit for others. Let us discuss these different points for the celestial system.

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1. CELESTIAL REFERENCE SYSTEMS

1.1. Dynamical systems

The main requirement for a celestial reference frame is that it should be inertial. By this, we mean that there exists no residual rotation of the system. Strictly speaking this requirement is a dynamical one and it implicity assumes the validity of Newtonian mechanics, corrected if necessary for relativistic effects. If we call (S_1) such an ideal absolute frame of reference, and if we assume that $\vec{\omega}$ is the rotation vector of another system (S) with respect to (S_1), then the absolute acceleration of a point P, $\vec{\Gamma}_1$, differs from its acceleration in the system (S) by:

$$\delta\vec{\Gamma} = \vec{\Gamma}' + 2\vec{\omega} \wedge \vec{V}' \qquad (1)$$

where $\vec{\Gamma}'$ and $\vec{V}'$ are the acceleration and the velocity of (S) relative to (S_1).

The quantity $\delta\vec{\Gamma}$ enters in the differential equations of the motion of P as referred to (S). The condition that (S) is an absolute system is:

$$\delta\vec{\Gamma} = 0 \qquad (2)$$

So the detailed analysis of the motion of a system of celestial bodies, like the Moon or the components of the solar system, may provide corrections that would make the chosen reference system absolute, by determining the parameters of equation (1). Hence, this provides an access to the absolute reference system.

The complete solution of the equations of motion may, however, contain terms having a structure similar to the solution of equation (1). For instance, if a single planet is taken as the material system, the mean motion n of the planet around the sum is linked to the semi-major axis a by Kepler's third law:

$$n^2 a^3 = km \qquad (3)$$

If there are errors Δa and Δkm in the assumed values of a and km, one has the following relation:

$$\frac{2\,\Delta n}{n} + \frac{3\,\Delta a}{a} - \frac{\Delta km}{km} = 0 \qquad (4)$$

and this implies that Δn and ω cannot be determined separately; the absolute system therefore cannot be derived from the measurement of n. In the strict case of the Newtonian two-body problem, the absolute system has to be defined by the condition that the pericenter is fixed in space. But in relativistic celestial mechanics this is no more true, and one must also know the exact value of the parameter of the Schwartzschild model in order to have access to the absolute frame. For the Moon, a similar difficulty arises since, in addition to the Newtonian accelerations due to the Earth, the Sun and the planets, and to the relativistic effects, the existence of a poorly modelled secular acceleration introduces new difficulties in determining $\vec{\omega}$.

In practice, more complex dynamical systems, including many bodies like the massive components of the solar system, are used. The complexity of the system appreciably decorrelates the equations in $\vec{\omega}$ from other unknowns, but there are proportionally many more parameters to be determined from the observations in order to solve the whole system of equations. So, finally, the actual determination of the reference frame is not necessarily improved.

These examples show that the definition of a system of reference implies the existence of a model of the physical system that is used to define it. I have described elsewhere some of the models associated with dynamical systems (Kovalevsky, 1975). Other examples of the complexity of the parametrization of the material system may be found in Mulholland (1977) for the lunar motion, in Duncombe et al. (1975) in the case of the solar system and in Anderle and Tanenbaum (1975) in the case of a system defined by the motion of artificial satellites.

These examples show clearly that a dynamical system of reference having accuracy in the range of 0".1 to 0".01 can only be defined through a complex model incorporating many parameters which must be determined simultaneously from the observations. For example, almost all the system of astronomical constants (see Müller and Jappel, 1977), and a set of six mean or instantaneous orbital elements for each planet and for the Moon, are necessary to define and give access to a reference system based on the planetary system; the complexity would need to be even greater to yield the desirable accuracy of 0".001.

1.2. Kinematic systems

Another approach to the definition and the realization of a celestial reference frame is to consider that point-marks of the reference systems are distant celestial bodies, stars or galaxies. In the Universe, light follows well defined geodetic lines, the apparent directions of which can be easily reduced to the actual directions by appropriate aberration corrections; but since celestial bodies are not fixed, one also needs some model of their motion with respect to what we believe is an absolute reference system. A model of the distribution of proper motions has hence to be adopted. And here lies a major difficulty. If we consider the case of stars, there is no means of completely separating a general rotation of the system of stars (galactic rotation) from the rotation of the reference frame. Therefore, in the construction of a representation of the reference system by a catalogue of star positions and proper motions (like the FK 4 or the future FK 5), it is necessary to use a dynamical definition of the reference frame. Classically, this is done by the introduction of the equator and the ecliptic as reference planes obtained through the observations of the position of the Sun and the planets, and a model of their respective motions through theories of the rotational behaviour of the Earth (precession, nutation), and the motion of the Earth and planets around the Sun (planetary precession, motion of the ecliptic). This leads to the very difficult and delicate task of fitting the star system to the assumed motions of these planets (Fricke, 1974 and 1975).

So, at present, there does not exist any purely geometric reference system. However, if one assumes that extragalactic point sources have very small relative proper motions and that there is no transverse component in the expansion of the Universe, then we obtain a static reference system. This is a particularly simple model and, conceptually, an ideal one.

1.3. Practical realizations

Among these various possible types of reference system, which are those that are the most advantageous for the study of the Earth rotation? The reply to this question depends upon the intrinsic properties of the system, but also upon its accessibility to the type of instrument that is to be used to measure the Earth rotation parameters.

1.3.1. Laser techniques on the Moon

Laser ranging, to the Moon or to satellites, provides a very promising technique for determining UT1 and polar motion. These are obtained by analysis of the residuals obtained at several stations when nominal rotation parameters are used in the process of global determination of all the quantities involved in the physical description of the motion of the satellite (see, for instance, Kolenkiewicz et al., 1977) or the Moon (Harris and Williams, 1977). The determination of the absolute reference system is a part of the global dynamical discussion of the motion of the body and is one of the limiting factors in the precision of the results.

In the case of the Moon, the main references are the ephemeris of the centre of mass of the Moon and the ephemeris of the rotation of the Moon about its centre of mass. The construction of both ephemerides is subject to errors of modelling and, in particular, to errors in the values of the higher moments of the lunar gravitation field, the knowledge of the free libration of the Moon (Calame, 1976) and the inadequate modelling of the transfer of angular momentum due to the tidal dissipation of the Earth. Most of these parameters are gradually improved as more and more lunar ranging observations are included in the solution defining the ephemeris. Gradually, then, these ephemerides define a better absolute reference system, though it is difficult to say how much the outcome is free from a residual rotation and possible terms of long period. The short periodic terms are certainly well represented, so this system is fit for short range Earth rotation determinations. The worst part of the model is probably due to the dissipation terms. However, even those can be determined from lunar ranging with reasonable accuracy, and residual accelerations of the system are of the order of magnitude of the error in the determination of the secular acceleration of the Moon, at present about 5" per century square.

This system is a self-defined system that can be used only for lunar laser ranging. It cannot be used by other techniques unless other observations link it to another reference system. This is done for a star defined system like the FK4 using meridian observations and occultations by the Moon. But this procedure degrades the initial precision of the dynamical system, since the supplementary observations are not as precise as lunar ranging and, at the best, the residual long term drift of the transferred system will not be better than that of the comparison.

In conclusion, this system is certainly very good for short and medium range in time (say 10 years). The residual rotation (or accelerations) seems to be a limitation that can be reduced only by comparison with a better known reference frame.

1.3.2. Laser techniques on artificial satellites

In the case of satellites analogous difficulties arise, but in a much shorter timescale. For instance, the longitude origin is essentially arbitrary and many forces which cannot be accurately modelled act on satellites to give effects that exceed the acceptable limits in a few weeks or, at the best, a few months (Anderle et al., 1975). Imperfections in the models used for calculation of solar and terrestrial radiation pressure and for the representation of the Earth's gravity field produce estimated uncertainties of a few centimeters per week even for LAGEOS, the satellite that has been most nearly optimised in these respects. The laser observations therefore do not provide full orientation in inertial space and the accumulated effects may reach 0"01 per year, which is worse than the present knowledge of the precession.

Another drawback of this system is that it is specific to the laser observations of a given satellite and, unlike that based on observations of the Moon, it cannot be tied to other systems. So a dynamical system defined by artificial satellites is accurate only over a short period of time. It can be useful for the measurement of transient and short-period phenomena, but it is not fit for middle-term and long-term studies.

1.3.3. Stellar systems

These are well known and will not be analysed here (see Fricke 1974). The best examples are the FK 4 and the FK 5 systems, the latter being now under construction (Fricke 1975). As we have said, they are essentially dynamical and are based on dynamical models of the Earth and of the solar system. The dynamical discussion of the residual rotation in longitude (Laubscher, 1976) showed a residual rotation in the system that is determined to 0".1 per century. There is also a residual drift of 0".3 per century in the obliquity that exists in all planetary observations. It has been suggested (Duncombe and Van Flandern, 1976) that this could be an artificial effect of an incomplete reduction to a fundamental star reference system. This is not the place to discuss this assumption, but whatever is the reason, it is a good illustration of the limitations in the construction of a classical system of reference.

One can describe a stellar reference frame by the following characteristics:

- It is easily accessible through observations of bright stars ($m < 10$), planets or the Moon;

- Its long term precision is limited to about ± 0".1 per century;

- It has regional errors (± 0".05 in position and ± 0".003 per year in proper motions for the FK 4 and, it is hoped, about half these values in the FK 5). These errors may introduce biases that are very difficult to analyse in the motions referred to the system; the problems are well known to all users of such catalogues;

- Its extension to higher magnitudes (12 or more) is possible only through photographic observations that significantly alter the accuracy of the realization.

1.3.4. Extra-galactic radio system

Such a system would be defined by the fixed positions of a few extra-galactic radio-sources whose positions are determined by very long base interferometry. The relative accuracy of these positions represents the accuracy of the system. And since they are fixed sources, there is no rotation or drift of the system. Actually this relies on the assumptions that the sources have no transverse velocities and are physically well defined, and are not, for instance, transient patches of synchrotron radiation. This requirement reduces the already small number of accessible sources. However, it is not too optimistic to consider that there might be over 20 reasonably bright sources having these qualities that could be the basis of a

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reference system. A list of such sources was proposed by Elsmore and Ryle (1976). The choice of the spherical coordinate system in which these coordinates are expressed is completely irrelevant. It could be related in some way to right ascension and declination, but could equally well be completely disconnected.

The adopted final coordinates of the sources may be obtained by some kind of weighted compensation of the angular distance between sources, as obtainable from VLBI observations. The final outcome is a base B formed of three orthogonal unit vectors ($\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$). The direction of each source S_i is given by a unit vector $\vec{S}_i$ expressed in the base B.

Any other object that can be observed with respect to at least two of the basic sources of S_i S_j, has a direction defined by a unit vector $\vec{V}$ such that ($\vec{V}$, $\vec{S}_i$) and ($\vec{V}$, $\vec{S}_j$) are equal to the observed values. So, the densification of the catalogue representing the system is possible using only relative observations.

The possibility of using such a system was demonstrated theoretically by Walter (1974) and practically by Elsmore (1976) who made first - epoch observations for the determination of the constant of precession. He used 5 km base radio-interferometric observations of ten extra-galactic radio-sources that played the role of the markers of the absolute system.

However, the densification of this catalogue will be very difficult, since there exist only very few radio stars and well defined points like extra-galactic radio-sources. But the situation might be completely different if an astrometric satellite is launched.

1.3.5. The astrometric satellite

Such a satellite has been imagined by Lacroute and presented several times (Bacchus and Lacroute, 1974; ESRO, 1975) and recently studied by ESA as a phase A project (ESA, 1978). It will permit, if it is successful, to measure with an accuracy of 0".002 the positions of 100 000 stars and, in a single mission, their proper motions to 0".002 per year. A second mission 10 years later would improve the proper motions by another factor of 5.

The star catalogue that will be obtained will be free of all systematic regional distortion, because stars separated by about 70° are directly connected one to another in such an intermingled manner, that no regional shift may occur. However, the whole system may have a residual rotation.

Three new techniques that are now being developed should provide links between objects of magnitude 16-19 and stars within about $0^\circ.5$ of them to an accuracy of 0".001; these are optical interferometry, now being tested with small telescopes by Labeyrie in CERGA, Connes' techniques using photographic masks for small fields, and the large space telescope. So we may expect that, ten years from now, we shall be able to tie the basic extragalactic sources defining the VLBI absolute system to stars observed by space astrometry techniques. The stellar catalogue of the astrometric satellite will also include some radio stars observable by VLBI and it is in this form that the VLBI system may become fully accessible.

2. TERRESTRIAL REFERENCE SYSTEMS

The terrestrial reference systems play two roles:

- Their first objective is to represent unambigously the position of points on the Earth.

- Their second objective is to represent the Earth as a whole, so that their rotational motion with respect to the celestial system represents the angular motion of the Earth.

The problem is that these two objectives are not fully compatible and therefore different reference systems have been used in each case.

2.1. Geodetic data

The objective of geodetic data is to give a unique computational procedure for defining the coordinates of a point on the Earth. The definition is very complicated and involves the introduction of a reference ellipsoid defined by its size, flattening, and an initial point of triangulation. Provisions are made that this ellipsoid has axes directed to the CIO and the Greenwich mean astronomical meridian.

However, the necessary reduction to the Geoid, the complex problem of the deflection of the vertical that permits linkage between astronomical and geodetic coordinates, and numerous other corrections that must be applied to the observed quantities, require the use of models of the local variations in the density of the crust. These drawbacks are well known to geodesists and have been reviewed recently by Mueller (1975) who clearly shows that the existing geodetic data are not fit to serve as acceptable terrestrial reference frames.

However there exists a conceptually much simpler solution: this is the world-wide geometric datum. It would consist, if the Earth could be considered as a rigid body, of a constellation of first order geodetic points with given observed rectangular coordinates in a single system, preferably geocentric. This is possible already for a few stations in the world that could have their relative positions determined by several techniques like satellite laser, lunar laser and VLBI. The coordinates of these stations would be treated like those of the radio-sources in the celestial VLBI system. Such systems already exist in principle as the world geodetic network or as parts of dynamical geodetic models (SAO Standard Earth, GEM, GRIM, etc...). They may be greatly improved by using lunar laser ranging and VLBI and having a geophysically tested model of tidal displacements.

As these stations are on the real non-rigid Earth subject to plate motions, one will have to associate a "proper motion" with each point and treat the whole system of points as a catalogue of stars with proper motions but with, also, an undeterminable general rotation of the whole system. As a matter of fact, only relative motions of the plates can be determined, and there is no geophysical phenomenon that could "nail" the system. It would probably be best to consider one plate as fixed and consider the motions of all the others with respect to it.

In conclusion, one would obtain, as on the celestial sphere, a base B' of three perpendicular unit vectors ($\vec{e}_1{}'$ $\vec{e}_2{}'$ $\vec{e}_3{}'$), and the position of a point B_i would be given as a unit vector $\vec{P}_i$ expressed in the base B', and a distance r_i to the origin.

For the determination of the coordinates of other points in this system, various space or Earth-based techniques can be used. It is also to be noted that, in this case, the directions of $\vec{e}_1{}'$ $\vec{e}_2{}'$ $\vec{e}_3{}'$ are arbitrary and should not be defined otherwise than by the coordinates of the reference points. Any other definition would introduce other phenomena (as, for instance, the position at some instant of the axis of rotation) and would complicate the realization as well as the definition of the system.

2.2. Other terrestrial systems

Many systems that are used, or can be imagined, are not linked to practically accessible points, but to dynamically or kinematically defined axes. Examples are:

- The principal axis of inertia; the inertial tensor is diagonal in this system.

- The axis of figure, about which the moment of inertia is a maximum.

- The mean axis, defined in such a way that the total angular momentum of motion relative to the system is zero.

- The instantaneous rotation axis of the Earth.

None of these axes is both directly observable and fixed. So they have to be defined through a parametrized model of the gravitational field of the Earth, or need to be followed by continuous observations, and in the latter case one would again ask the question, with respect to what?

And, with the improvement of observations, these systems would tend to be modified with time. Furthermore, the reference to some initial position at time t_o of such or such axis would imply a continuous chain of measurements that will have to control the reduction of later observations to the reference system. This also is not advisable.

2.3. Ideal and practical systems

The most conceptually simple "ideal" reference systems that could be constructed in the foreseeable future are:

- A celestial reference system defined by VLBI and densified by the astrometric satellite.
- A terrestrial reference frame defined by a constellation of stations whose motions are referred to a given "origin" tectonic plate.

The rotation of the Earth would therefore be described by the rotation vector R(t,t') that superposes the base B'(t') on the base B'(t) as expressed in the system of reference defined by B.

$$B'(t) = R(t,t') \times B'(t')$$

The instantaneous rotation of the Earth at time t is the time derivative R'(t) of R(t,t') when $t' \to t$ (figure 1)

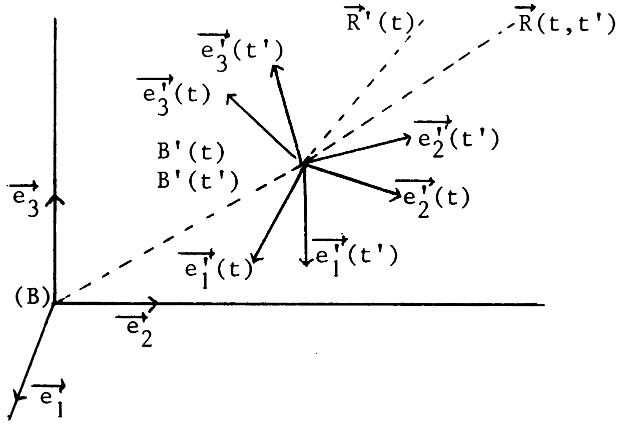

Figure 1. Bases and unit vectors of systems to be used for the rotation of the Earth

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The direction of the rotation vector corresponding to R'(t) as measured in the base B' gives the motion of the pole and, in the base B, gives the direction of the instantaneous celestial pole.

The actual definitions of both bases B and B' may be completely arbitrary and probably ought to be so. For historical reasons they might be set close to the existing systems, but they certainly should not be set to coincide, not even for a given moment, since this would introduce new parameters to be more or less well determined and a model that would be more or less good. These factors would introduce an unnecessary inaccuracy in the realization of the systems. For the same reasons, no mention should be made of the equinox, which is defined by physical systems that are particularly complicated and difficult to model.

However, for practical reasons, several celestial systems may have to be used; for instance, a dynamical one for satellite work. It is possible to link these to the basic ones by measuring the "rotation of the Earth" by different methods referred to each of the systems. Let us call the bases of these systems B_1 and B_2. The rotation should be the same in both systems; if it is not then this defines the difference between B_1 and B_2 and, therefore, permits linkage of B_1 and B_2.

In some cases it may also be useful to introduce intermediate systems; for instance, a system in which the origin remains close to the equinox and the main circle is close to the equator of the Earth. Such an intermediate system should be simply and unambiguously related to the basic system by formulae that do not depend upon current observations or models. The transformation formulae should be given in full in a closed form as a conventional definition, and not as a consequence of a model that would need to be changed as knowledge of the underlying physics improved.

The same precautions should also be applied to any transformation of the basic geodetic system on the Earth. In particular, I would favour the total disappearance of the astronomical system of coordinates on the Earth and consider that each observatory is defined by its geodetic position in the terrestrial system and has a deflection of the vertical that is part of the instrument if the local vertical is used as instrumental reference to the observations. This would have the advantage of permitting periodic or secular variations.

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DISCUSSION

W.G. Melbourne: With regard to the inertial celestial system and to the use of very accurate optical astrometric positions of extragalactic radio sources, it should be noted that the centroids of these objects at microwave and visual wavelengths may differ by the order of 0.01 arsec, at least for those accessible with present VLBI technology.

P. Brosche: One should also use optical positions of galaxies, since quasars may have large apparent velocities and may show structure at the desired level of accuracy.

J. Kovalevsky: Yes, I agree. However, one should still have radio sources for the definition of the absolute system.

J.D. Mulholland: The estimate that photographic positions are limited to 0.1 arcsec is probably wrong, in that there will be a certain amount of astrometry on the Space Telescope.

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