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CP violation: K mesons

15.1 Introduction

Most of the symmetries in elementary-particle physics are continuous. A typical example is the symmetry of rotations around an axis, where the angle of rotation can assume any value between zero and 2π. In addition to continuous symmetries, there are also discrete symmetries, for which the possible states assume discrete values classified with the help of a few integers. For instance, snowflakes exhibit the discrete symmetry of rotations under 60° and crystals exhibit various types of discrete symmetries. In elementary-particle physics there are three discrete symmetries of basic importance: parity, charge conjugation and time-reversal.

Parity is the reflection of space coordinates and will be denoted by P. Under parity there are two states – the object and its space reflection. Parity is familiar from quantum mechanics, where the eigenstates of Hamiltonians are classified according to their properties under space reflection. For spherically symmetric potentials the wave functions are proportional to the spherical harmonics $Y_m^\ell(\theta, \varphi)$ whose parity is $(-1)^\ell$. For a long time it was assumed that the fundamental interactions respect P, but a critical review of experimental evidence led two theoreticians, T. D. Lee and C. N. Yang, to suggest that parity may be violated by the weak interactions. One year later, an experiment led by C. S. Wu brought the proof that the P symmetry is indeed violated by weak interactions.

The symmetry of charge conjugation, to be denoted by C, exchanges particles with antiparticles. One can imagine building an antiworld by replacing all particles by antiparticles. In the antiworld the three interactions gravity, the strong force, and electromagnetism are the same, but the weak interactions are different. For example in the antiworld muon-type antineutrinos are right-handed and produce μ^+ which are also right-handed. In comparison neutrinos are left-handed and always produce, in high-energy reactions, left-handed μ^-. In the weak interactions the C symmetry is broken. However, it was assumed, at that time, that the observed processes do

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respect the combined CP transformation, the one obtained by applying both C and P transformations.

There is a fundamental reason why CP symmetry plays a crucial role. It is intimately linked to the time-reserval transformation (T). This transformation consists of "looking" at an experiment running backward in time. Although, at the macroscopic level, one can distinguish the real sequence of events from the time-reversed one in terms of large-scale phenomena such as entropy or the expansion of the Universe, this is not a priori evident for microscopic interactions, i.e. it is not a priori evident that the amplitudes for reactions and for the time-reversed reactions are equal.

The analysis of CP violation is facilitated by an important theorem known as CPT theorem. It states that any local field theory based on special relativity and quantum mechanics is invariant under the combined action of C, P, and T. A consequence of the theorem is that CP symmetry implies T symmetry, because any CP violation should be compensated by T violation.

Until 1964 the decays and interactions of particles showed that the CP symmetry was conserved; this created the belief that microscopic phenomena also obey the T symmetry. In 1964 CP violation was observed in an experiment dedicated to the study of K^0 and $\bar{\mathrm{K}}^0$ mesons. Since then it has become an active topic of research, with CP violation having been observed so far in the K and the B mesons. In this chapter we study the properties of mesons under discrete symmetries, leaving the study of fermions for more specialized articles and books.

15.2 General properties

We describe now the properties that govern the decays of neutral pseudoscalar mesons, such as K^0, D^0, and B^0_d, when the interactions obey the CPT and CP symmetries. The results guide us to properties of these reactions that indicate breakdown of CP and/or CPT symmetries.

For simplicity of presentation we shall consider the K^0 as an example and describe properties of the K^0–$\bar{\mathrm{K}}^0$ system; however, the results are general and hold for the other mesons too. We adopt the phase convention

$$P|K^0\rangle = |K^0\rangle, \quad P|\bar{K}^0\rangle = |\bar{K}^0\rangle, \tag{15.1}$$

$$C|K^0\rangle = |\bar{K}^0\rangle, \tag{15.2}$$

$$T|K^0\rangle = \langle K^0|, \tag{15.3}$$

with similar relations being valid for the antiparticle $|\bar{K}^0\rangle$. Even though $|K^0\rangle$ is a pseudoscalar particle, we chose a convention that under parity $|K^0\rangle$ transforms into itself, since with this choice it is easier to keep track of the minus signs. The

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freedom to make this choice comes from the fact that parity transformation requires $P^2 = 1$; thus there is still freedom of the overall sign.

The decays of the mesons are mediated by the weak interactions, whose operators are Hermitian. Non-Hermitian terms appear in loop diagrams from energy denominators; however, Hermiticity still holds when we consider the dispersive and absorptive parts separately, as we discuss next. For instance, the semileptonic decay has the amplitude

$$a_\ell = \langle \pi^- \ell^+ \nu | H_\mathrm{W} | K^0 \rangle \tag{15.4}$$

with H_W the weak Lagrangian, which is Hermitian. Similarly, the contributions to the mass matrix, with i and j being K^0 or $\bar{K}^0$, have the general form

$$H_{ij} = m_\mathrm{K}\delta_{ij} + \langle i|H_\mathrm{W}|j\rangle + \sum_n \frac{\langle i|H_\mathrm{W}|n\rangle\langle n|H_\mathrm{W}|j\rangle}{M_\mathrm{K} - E_\mathrm{n} + \mathrm{i}\varepsilon}. \tag{15.5}$$

The first two terms appear for $i = j$. The last term originates from box diagrams and is present for $\Delta S = 2$ transitions with $j = \mathrm{K}^0$ and $i = \bar{\mathrm{K}}^0$. The last term is decomposed into two Hermitian matrices by decomposing the energy denominator into a principal part denoted by P and a δ-function term. The Hamiltonian decomposes as follows:

$$H_{ij} = M_{ij} - \frac{\mathrm{i}}{2}\Gamma_{ij}, \tag{15.6}$$

with a dispersive term

$$M_{ij} = m_\mathrm{K}\delta_{ij} + \langle i|H_\mathrm{W}|j\rangle + P\sum_n \frac{\langle i|H_\mathrm{W}|n\rangle\langle n|H_\mathrm{W}|j\rangle}{M_\mathrm{K} - E_\mathrm{n}} \tag{15.7}$$

and an absorptive term

$$\Gamma_{ij} = 2\pi \sum_n \langle i|H_\mathrm{W}|n\rangle\langle n|H_\mathrm{W}|j\rangle\delta(E_n - M_\mathrm{K}). \tag{15.8}$$

These terms satisfy the Hermiticity relations

$$M_{ij} = M_{ji}^* \quad \text{and} \quad \Gamma_{ij} = \Gamma_{ji}^*. \tag{15.9}$$

More relations follow from CP and CPT invariance. We present the conditions as two theorems.

Theorem 1 *For a Hamilton operator that is CPT-invariant the amplitudes for the decays of particles and antiparticles are the complex conjugates of each other.*

Proof Let us denote the amplitude for K^0 decay by

$$A_I = \langle X_I|H_\mathrm{W}|K^0\rangle. \tag{15.10}$$

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Then, using $(CPT)H_W(CPT)^{-1} = H_W$, we obtain

$$\begin{aligned} A_I &= \langle X_I|(CPT)^{-1}H_W(CPT)|K^0\rangle \\ &= \langle \bar{K}^0|H_W|\tilde{X}_I\rangle = \bar{A}_I^*, \end{aligned} \tag{15.11}$$

where $|\tilde{X}_I\rangle = CP|X_I\rangle$, i.e. the CP conjugate state and

$$\bar{A}_I = \langle \tilde{X}_I|H_W|\bar{K}^0\rangle. \tag{15.12}$$

By applying the theorem to the diagonal elements of the mass matrix H_{ij} and using Hermiticity of the dispersive and absorptive parts, we obtain

$$M_{11} = M_{22} \quad \text{and} \quad \Gamma_{11} = \Gamma_{22}. \tag{15.13}$$

This is the statement that CPT invariance demands the equality of masses and widths for particles and antiparticles. When we apply the theorem to off-diagonal elements, we obtain the relation

$$M_{12} = M_{21}^* \quad \text{and} \quad \Gamma_{12} = \Gamma_{21}^*, \tag{15.14}$$

which is not new, but the Hermiticity relations in Eqs. (15.9).

It follows now that the mass matrix in the $|K^0\rangle$ and $|\bar{K}^0\rangle$ has the form

$$M - \frac{\mathrm{i}}{2}\Gamma = \begin{pmatrix} M_{11} - \frac{\mathrm{i}}{2}\Gamma_{11} & M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12} \\ M_{12}^* - \frac{\mathrm{i}}{2}\Gamma_{12}^* & M_{11} - \frac{\mathrm{i}}{2}\Gamma_{11} \end{pmatrix}. \tag{15.15}$$

As mentioned already, the form of the diagonal elements follows from conservation of the CPT symmetry. We can make them different, thus introducing by hand a violation of CPT invariance, and study the modifications in the lifetimes and other properties of the states.

The presence of the off-diagonal matrix elements implies the mixing of the states K^0 and $\bar{\mathrm{K}}^0$. The physical states are a mixture of them, obtained by diagonalizing the mass matrix, which will be presented in the next section. Additional restrictions, which we describe in the next theorem, are introduced by CP symmetry. □

Theorem 2 *For a Hamiltonian that is CP-invariant, the decay amplitudes for particles and antiparticles are relatively real.*

Proof

(i) As before we denote by A_I and $\bar{A}_I$ the decay amplitudes for particles and antiparticles, respectively.
(ii) CP invariance implies

$$H_W = (CP)^{-1}H_W(CP). \tag{15.16}$$

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(iii) Then the matrix elements are related,

$$\begin{aligned} A_I &= \langle X_I|(CP)^{-1}H_{\mathrm{W}}(CP)|K^0\rangle \\ &= \langle \tilde{X}_I|H_{\mathrm{W}}|\bar{K}^0\rangle = \bar{A}_I . \end{aligned} \tag{15.17}$$

We can apply the theorem in cases in which $|X_I\rangle$ is a specific final state or in the case $|X_i\rangle = |\bar{K}^0\rangle$ which refers to the mass matrix.

Let us consider the latter case first. Taking $|X_I\rangle = |\bar{K}^0\rangle$ and $|\tilde{X}_I\rangle = |K^0\rangle$, we obtain the relation

$$\langle \bar{K}^0|H_{\mathrm{W}}|K^0\rangle = \langle K^0|H_{\mathrm{W}}|\bar{K}^0\rangle . \tag{15.18}$$

In this and the following equations H_{W} can be the lowest-order Lagrangian or may include higher-order terms with possible contractions between fields. It follows now that, when we consider the dispersive and absorptive terms separately, they are relatively real. This is a stronger restriction than the Hermiticity requirement of Eq. (15.15), where they were complex conjugates of each other. We shall use these properties in the next section, where we will define the parameter ε.

For decays of particles the theorem says that the amplitudes for particles and antiparticles are relatively real. This indicates a strategy for detecting CP violation. It consists of measuring the phase difference of the two amplitudes relative to a third standard phase, such as the phase occuring in the time development of states, the phase in a Breit–Wigner propagator, or some other known phase. We shall describe several methods in the next sections. □

15.3 Time development of states

In the following sections of Chapter 15, we shall assume that CPT is a good symmetry of Nature and study cases in which the CP symmetry is broken. The fact that there are off-diagonal elements in Eq. (15.5) means that $|K^0\rangle$ and $|\bar{K}^0\rangle$ are not mass eigenstates but the physical states are mixed states. The physical states are obtained by diagonalizing the matrix in Eq. (15.15). Beyond the solution of the physical states we are interested in learning how the elements M_{12} and Γ_{12} are produced. In gauge theories they originate from box diagrams and lead, for the various mesons, to terms of different magnitudes, so that the physical properties of K^0, D^0, and B^0 mesons are very different. We describe first the time development of the states.

We are interested in defining a state that is a superposition of $|K^0\rangle$ and $|\bar{K}^0\rangle$ and has the time development

$$\psi_1(t) = (B_1|K^0\rangle + D_1|\bar{K}^0\rangle)e^{iE_1 t} , \tag{15.19}$$

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with B_1 and D_1 being constants. The time evolution is described by the Schrödinger equation

$$\mathrm{i}\,\frac{\mathrm{d}}{\mathrm{d}t}\psi_1(t) = \left(M - \mathrm{i}\frac{\Gamma}{2}\right)\psi_1(t), \tag{15.20}$$

whose stationary solutions are determined by the eigenvalue problem

$$\begin{pmatrix} H & H_{12} \\ H_{21} & H \end{pmatrix}\begin{pmatrix} B_1 \\ D_1 \end{pmatrix} = E_1\begin{pmatrix} B_1 \\ D_1 \end{pmatrix}, \tag{15.21}$$

with a similar equation for the second eigenvalue E_2.

The solutions have energies $E_{1,2} = H \pm \sqrt{H_{12}H_{21}}$ and the eigenfunctions

$$\psi_{1,2} = \begin{pmatrix} 1 \\ \pm q/p \end{pmatrix}\mathrm{e}^{-\mathrm{i}E_{1,2}t} \quad \text{with} \quad \frac{q}{p} = \left(\frac{H_{21}}{H_{12}}\right)^{\frac{1}{2}} = \mathrm{e}^{\mathrm{i}\xi}, \tag{15.22}$$

respectively. We have written the wave functions in terms of the parameter q/p and have not yet normalized them. The reason for this is that one frequently uses another parameter, ε, which is defined by

$$\frac{q}{p} = \frac{1-\varepsilon}{1+\varepsilon} = \frac{[M_{12}^* - (\mathrm{i}/2)\Gamma_{12}^*]^{1/2}}{[M_{12} - (\mathrm{i}/2)\Gamma_{12}]^{1/2}} = \mathrm{e}^{\mathrm{i}\xi}, \tag{15.23}$$

which will be used later on. The dynamics of each problem resides in the matrix elements of the Hamiltonian, which are calculated in terms of the box diagrams. We describe the results of these calculations in the next section. They provide us with values for M_{12} and Γ_{12}, which turn out to be complex functions indicating CP violation in the mass matrix.

Let us discuss the physical states. On substituting q/p into Eq. (15.19), we obtain at $t = 0$ two normalized states,

$$|K_\mathrm{S}\rangle = \frac{1}{\sqrt{2}(1+|\varepsilon|^2)^{1/2}}\left[(1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\bar{K}^0\rangle\right], \tag{15.24}$$

$$|K_\mathrm{L}\rangle = \frac{1}{\sqrt{2}(1+|\varepsilon|^2)^{1/2}}\left[(1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\bar{K}^0\rangle\right]. \tag{15.25}$$

Each state has its own time development given by $\mathrm{e}^{-\mathrm{i}E_{\mathrm{S,L}}t}$. The subscripts S and L indicate the short- and long-lived states. When $\varepsilon = 0$, $|K_\mathrm{S}\rangle$ and $|K_\mathrm{L}\rangle$ are even and odd eigenstates of the CP operator. For Re $\varepsilon \neq 0$ the states are no longer CP eigenstates, indicating that the symmetry is broken in the construction of the states.

There are two special properties we wish to discuss. The states are not orthogonal to each other, but have an overlap

$$\langle K_\mathrm{S}|K_\mathrm{L}\rangle = \frac{2\,\mathrm{Re}\,\varepsilon}{1+|\varepsilon|^2}. \tag{15.26}$$

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This follows from the fact that the mass matrix, in general, is not Hermitian. The second property occurs when ε is purely imaginary. We can use the states $|K^0\rangle$ and $|\bar{K}^0\rangle$ or new states

$$|\tilde{K}^0\rangle = \mathrm{e}^{\mathrm{i}\alpha}|K^0\rangle \quad \text{and} \quad |\tilde{\bar{K}}^0\rangle = \mathrm{e}^{-\mathrm{i}\alpha}|\bar{K}^0\rangle \tag{15.27}$$

rotated by a constant phase α. A purely imaginary ε can be eliminated by the appropriate redefinition of the states in Eqs. (15.24) and (15.25). The real part of ε cannot be eliminated. A real part of ε_K has been established for the K^0-meson system. For B^0 mesons ε_B is, to a good approximation, purely imaginary and there is no CP violation in the construction of the physical states.

For the time development of the states we separate the eigenvalues into their respective dispersive and absorptive parts,

$$M_{S,L} - \frac{\mathrm{i}}{2}\Gamma_{S,L} = E_{S,L} = E_{1,2},$$

which define the mass and width differences

$$M_L - M_S - \frac{\mathrm{i}}{2}(\Gamma_L - \Gamma_S) = 2\sqrt{H_{12}H_{21}}. \tag{15.28}$$

A general state is a superposition of the physical states K_S and K_L, with constant coefficients $C_{1,2}$ describing how the state was created at time $t = 0$:

$$\psi(t) = C_1\mathrm{e}^{-\mathrm{i}[M_S-(\mathrm{i}/2)\Gamma_S]t}|K_S\rangle + C_2\mathrm{e}^{-\mathrm{i}[M_L-(\mathrm{i}/2)\Gamma_L]t}|K_L\rangle. \tag{15.29}$$

The decay of the state proceeds through strangeness-changing couplings, which requires that we rewrite them in terms of $|K^0\rangle$ and $|\bar{K}^0\rangle$. The time development of a state that at time $t = 0$ began as $|K^0\rangle$ is given by

$$\psi_1(t) = N\left[f_+(t)|K^0\rangle + \frac{q}{p}f_-(t)|\bar{K}^0\rangle\right], \tag{15.30}$$

with

$$f_\pm = \mathrm{e}^{-\mathrm{i}[M_S-(\mathrm{i}/2)\Gamma_S]t} \pm \mathrm{e}^{-\mathrm{i}[M_L-(\mathrm{i}/2)\Gamma_L]t}, \tag{15.31}$$

with N a normalization constant. Similarly, a state that starts at $t = 0$ as $|\bar{K}^0\rangle$ has the time development

$$\psi_2(t) = N'\left[f_-(t)|K^0\rangle + \frac{q}{p}f_+(t)|\bar{K}^0\rangle\right]. \tag{15.32}$$

These equations indicate that a state that started as a pure $|K^0\rangle$ will develop in time a $|\bar{K}^0\rangle$ component through the interference of the two terms. The fact that it involves an interference phenomenon makes possible the separation of the amplitudes, as well as determination of the factor q/p.

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Let us consider an experiment in which a state $|K^0\rangle$ was created. This state will evolve into a mixture of both K^0 and $\bar{K}^0$. In fact the probabilities of finding at time t the $|K^0\rangle$ and $|\bar{K}^0\rangle$ components are

$$|\langle K^0|K^0(t)\rangle|^2 = |f_+(t)|^2 = \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2\cos(\Delta M\, t)e^{-\Gamma t}\right]$$

and

$$|\langle \bar{K}^0|K^0(t)\rangle|^2 = \left|\frac{q}{p}f_-(t)\right|^2 = \frac{1}{4}\left|\frac{q}{p}\right|^2\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2\cos(\Delta M\, t)e^{-\Gamma t}\right],$$

with $\Gamma_1 = \Gamma_S$, $\Gamma_2 = \Gamma_L$, and $\Gamma = \frac{1}{2}(\Gamma_S + \Gamma_L)$. Similar formulas hold for a state that starts as $|\bar{K}^0(t)\rangle$. The detection of $|K^0\rangle$ or $|\bar{K}^0\rangle$ in the final state is carried out by observing their decay products. Thus the final formulas involve an additional amplitude, which introduces its own phase. The time development of the states allows the accurate determination of ΔM and of relative phases. In fact, this property is used heavily in the analysis of experiments.

15.3.1 Simplified formulas for K^0 *mesons*

Numerous experiments with K-meson beams were able to determine ΔM, $\Delta\Gamma$, and the parameter ε_k. The results suggest several approximations that simplify the equation considerably. For neutral K mesons the mass and width differences are comparable:

$$\Delta M_K = M_L - M_S = 3.52\times 10^{-15}\,\text{GeV},\quad \Delta\Gamma_K = \Gamma_L - \Gamma_S = -7.36\times 10^{-15}\,\text{GeV}. \tag{15.33}$$

Measurements in the decays of the particles determine ε_K to be small,

$$|\varepsilon_K| = (2.27 \pm 0.02)\times 10^{-3}, \tag{15.34}$$

with a phase of approximately 45°. At the end of this section we describe an experimental method that determines $|\varepsilon_K|$.

For small ε_K the exponent ξ which occurs in Eq. (15.23) is small and allows the following approximations:

$$H_{12} \approx \sqrt{H_{12}H_{21}}(1 - i\xi),$$
$$H_{21} \approx \sqrt{H_{12}H_{21}}(1 + i\xi).$$

From the definition of ε_K it follows that

$$\varepsilon_K = \frac{H_{12} - H_{21}}{H_{12} + H_{21} + 2\sqrt{H_{12}H_{21}}}$$

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and, using the above approximations,

$$\varepsilon_{\mathrm{K}} = \frac{\mathrm{i}\,\mathrm{Im}\,M_{12} + \frac{1}{2}\,\mathrm{Im}\,\Gamma_{12}}{2\sqrt{H_{12}H_{21}}} = \frac{\mathrm{i}\,\mathrm{Im}\,M_{12} + \frac{1}{2}\,\mathrm{Im}\,\Gamma_{12}}{\Delta M - (\mathrm{i}/2)\Delta\Gamma}. \tag{15.35}$$

The fact that the phase is 45° means that $\mathrm{Im}\,M_{12}$ and $\mathrm{Im}\,\Gamma_{12}$ are comparable. Furthermore, the magnitude of $|\varepsilon_{\mathrm{K}}|$ implies that the denominator is much larger, i.e.

$$\mathrm{Im}\,M_{12},\ \mathrm{Im}\,\Gamma_{12} \leq \mathrm{Re}\,M_{12} \quad \text{or} \quad \mathrm{Re}\,\Gamma_{12}, \tag{15.36}$$

giving the final result

$$H_{12} = \mathrm{Re}\,M_{12} - \frac{\mathrm{i}}{2}\,\mathrm{Re}\,\Gamma_{12} \tag{15.37}$$

and

$$\sqrt{H_{12}H_{21}} \approx \mathrm{Re}\,M_{12} - \frac{\mathrm{i}}{2}\,\mathrm{Re}\,\Gamma_{12}. \tag{15.38}$$

The simplified formulas of this section hold only for the K^0 system. For this case the mass difference is

$$\Delta M = 2\,\mathrm{Re}\,M_{12} \tag{15.39}$$

and the width difference is

$$\Delta\Gamma = 2\,\mathrm{Re}\,\Gamma_{12}. \tag{15.40}$$

We shall discuss the theoretical determination of these quantities in the next section. Before leaving the discussion of the K mesons, we discuss the measurement of $\mathrm{Re}\,\varepsilon_{\mathrm{K}}$ from semileptonic decays.

Let us consider a beam that consists of $\mathrm{K_L}$ mesons. This beam is created in accelerators by producing intense beams of K^0 or $\bar{\mathrm{K}}^0$ mesons and setting up an experiment far away from the production region, where the $\mathrm{K_S}$ particles have already decayed. Next we consider the decays

$$\mathrm{K_L} \to \pi^-\ell^+\nu \tag{15.41}$$

and

$$\mathrm{K_L} \to \pi^+\ell^-\bar{\nu}, \tag{15.42}$$

distinguished by the charges of the pions and leptons. We denote the decay amplitudes as

$$a_\ell = \langle \pi^-\ell^+\nu | H_{\mathrm{W}} | K^0 \rangle, \tag{15.43}$$

$$\bar{a}_\ell = \langle \pi^+\ell^-\bar{\nu} | H_{\mathrm{W}} | \bar{K}^0 \rangle. \tag{15.44}$$

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Assuming CP invariance for the amplitudes, Theorem 2 says that the two amplitudes are equal,

$$a_\ell = \bar{a}_\ell, \tag{15.45}$$

which, together with the definition of K_L, determines the asymmetry

$$\delta = \frac{\Gamma(K_L \to \pi^-\ell^+\nu) - \Gamma(K_L \to \pi^+\ell^-\bar{\nu})}{\Gamma(K_L \to \pi^-\ell^+\nu) + \Gamma(K_L \to \pi^+\ell^-\bar{\nu})} = \frac{2\,\mathrm{Re}\,\varepsilon}{1+|\varepsilon|^2}. \tag{15.46}$$

The experimental value for the asymmetry is

$$\delta = (3.27 \pm 0.12) \times 10^{-3}, \tag{15.47}$$

which is consistent with the magnitude and phase given earlier in this section. The separation into magnitude and phase is obtained by comparing the semileptonic decay with the $K_L \to \pi\pi$ decays, which we shall describe in a following section.

15.4 The K^0–$\bar{K}^0$ transition amplitude

The theoretical calculations of the matrix elements M_{12} and Γ_{12} are far from being understood. For these as well as other matrix elements, there have been developed several methods that provide acceptable values and even make successful predictions. For K^0 mesons there are short- and long-distance contributions, with the dominance of the short-distance contributions being harder to justify, because the strong coupling constant $\alpha_S(q^2)$ is large and the quarks are confined into hadrons. For the B^0 mesons, on the other hand, the dominance of the top quark in intermediate states makes short-distance dominance more reliable.

The diagonal elements of the mass matrix are created by the strong interactions. The off-diagonal term M_{12} involves a $\Delta S = 2$ transition and receives contributions from the box diagrams, as described in Section 14.7. The method consists of calculating an effective $\Delta S = 2$ Hamiltonian in the free-quark model and then taking its matrix element between the K^0 and $\bar{K}^0$ states. The $\Delta S = 2$ Hamiltonian generated by this method was described in Section 14.7:

$$H_W^{\Delta S=2} = -\frac{G^2}{16\pi^2} M_W^2 Q_{\Delta S=2}\left[\lambda_c^2 E(x_c) + 2\lambda_c\lambda_t E(x_c,\, x_t) + \lambda_t^2 E(x_t)\right], \tag{15.48}$$

with the various terms defined as follows. The variable $x_i = m_i^2/M_W^2$ and the couplings of the quarks at the various vertices produce the factors

$$\lambda_i = V_{id}^* V_{is}. \tag{15.49}$$

Their numerical values are determined by the CKM-matrix elements with λ_c being of $O(\lambda)$ and λ_t of $O(\lambda^5)$ in the Wolfenstein parametrization. The functions $E(x_c)$

and $E(x_c,\ x_t)$ are obtained from the integration over the loop. We wrote down $E(x_i)$ in Eq. (14.52) and the second function is given as

$$E(x_i,\ x_j) = -x_i x_j \left[\frac{1}{x_i - x_j} \left(\frac{1}{4} + \frac{3}{2}\frac{1}{(1-x_i)} - \frac{3}{4}\frac{1}{(1-x_i)^2}\ln x_i \right) + (i \leftrightarrow j) \right.$$
$$\left. - \frac{3}{4}\frac{1}{(1-x_i)(1-x_j)} \right]. \tag{15.50}$$

For

$$m_c^2 \ll M_W^2, \quad E(x_c) \to -x_c$$

and for

$$m_t^2 \gg M_W^2, \quad E(x_t) \to -\frac{1}{4}x_t + \frac{3}{2}\ln x_t,$$

which indicates that the various terms in (15.50) are comparable.

Finally, there is the operator

$$Q_{\Delta S=2} = \bar{d}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma^\mu(1-\gamma_5)s,$$

which represents the external lines of the box diagram. The matrix element $X_K = \langle K^0|Q_{\Delta S=2}|\bar{K}^0\rangle$ contains the long-distance contribution of this calculation. A good deal of effort has been invested in its calculation. In various situations so far, we have encountered the calculation of two-quark operators (currents) between hadronic states, for which there are reliable numerical estimates – sometimes extracted from experimental data. Estimates of matrix elements for four-quark operators are less reliable and are still a subject of research. A simple estimate of such matrix elements is given in Eq. (16.5), which can be taken over for the K mesons by making the replacements $F_D \to F_K$ and $M_D \to M_K$.

The absorptive part Γ_{12} is in principle also calculable in terms of the box diagrams by setting the intermediate states on the mass shell, i.e. replacing the propagators by δ-functions. For K mesons the physical intermediate states are u quarks, making the absorptive part a long-distance effect. This term is calculated by low-energy methods with the intermediate states being 2π, 3π, ... mesons. The calculation carries a large uncertainty because the amplitudes and their relative phases are not known.

The situation is different for heavy mesons, in particular the B^0–$\bar{B}^0$ system, in which there are many intermediate states with multiparticle final states dominating the decay. For heavy mesons the sum over intermediate states will be replaced by the quarks and will be calculated as the absorptive part of the diagram. This is known as the quark–hadron duality, whereby hadronic matrix elements are replaced by

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the corresponding quark diagrams. Finally, the matrix elements are taken between hadrons, for which approximations are again necessary.

We close this section by deriving two approximate formulas describing the mass and width differences of neutral B mesons. The general formulas for width and mass differences are

$$\Delta M = 2\,\mathrm{Re}\left[\left(M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12}\right)\left(M_{12}^{*} - \frac{\mathrm{i}}{2}\Gamma_{12}^{*}\right)\right]^{\frac{1}{2}}, \tag{15.51}$$

$$\Delta \Gamma = -4\,\mathrm{Im}\left[\left(M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12}\right)\left(M_{12}^{*} - \frac{\mathrm{i}}{2}\Gamma_{12}^{*}\right)\right]^{\frac{1}{2}}. \tag{15.52}$$

For these equations, we need assume only that the CPT symmetry is exact. For K^0 mesons the data imply the approximations which were described in Section 15.3.1. For the B mesons the situation is different. Estimates of Γ_{12} and M_{12} using the effective Hamiltonians of this section lead to the estimate (Paschos and Türke, 1989)

$$\Gamma_{12} \sim 0.1 M_{12}$$

and with almost the same phase; consequently for B mesons

$$\Delta M_{\mathrm{B}} = 2|M_{12}| \quad \text{and} \quad \Delta\Gamma_{\mathrm{B}} = 2|\Gamma_{12}| \tag{15.53}$$

to a good approximation. The differences in the masses and widths of the K^0 and B^0 mesons indicate that each system must be treated separately. The qualitative differences are understood in terms of the quark substructure which enters the box diagrams.

15.5 CP violation in amplitudes

Besides the phase introduced in the mass matrix the decay amplitudes have their own phases. Theorem 1 states that the amplitudes for particle and antiparticle decays are the complex conjugates of each other. This is a consequence of CPT. CP symmetry goes one step further and requires the amplitudes to be real relative to each other. Consequently, evidence for the breakdown of CP symmetry requires the measurement of phases.

In quantum mechanics the overall phase of a sum of amplitudes can always be removed, but relative phases among amplitudes are measurable observables. For this reason all measurements must include at least two phases and the experiments measure one phase relative to the other.

Let us denote the final state by $\langle f|$ and in addition select the final state to be a CP eigenstate with eigenvalue unity. Examples of such decays are

$$\mathrm{K}^0 \to \pi^+\pi^-, \quad \pi^0\pi^0. \tag{15.54}$$

There are two decay amplitudes specified by the isospin of the two pions being 0 or 2. Searches for direct CP violation measure the relative phase of the two amplitudes and try to establish whether it is the same in K^0 and $\bar{\mathrm{K}}^0$ decays. We denote the amplitude

$$\begin{aligned} \langle (2\pi)_I | H_{\mathrm{W}} | K^0 \rangle &= A_I \mathrm{e}^{\mathrm{i}\delta_I}, \\ \langle (2\pi)_I | H_{\mathrm{W}} | \bar{K}^0 \rangle &= \bar{A}_I \mathrm{e}^{\mathrm{i}\delta_I} \quad \text{with} \quad I = 0 \text{ or } 2. \end{aligned} \tag{15.55}$$

The phases δ_I are created by final-state interactions of the two pions, which is a strong-interaction effect independent of the initial state but a function of the isospin I. Beyond the strong phase there is also a phase of weak origin, which changes sign as we go from particles to antiparticles. Consequently, we can write the amplitudes as follows.

$$A_I = |A_I| \mathrm{e}^{\mathrm{i}\theta_I}, \tag{15.56}$$

$$\bar{A}_I = |A_I| \mathrm{e}^{-\mathrm{i}\theta_I}, \tag{15.57}$$

where θ_I is now a phase of weak origin.

Experiments starting with a $|K^0\rangle$ or a $|\bar{K}^0\rangle$ beam also observed the mixing phenomenon, described in Section 15.3, in the decays to $\pi^+\pi^-$ and $\pi^0\pi^0$. At a distance corresponding to six to seven lifetimes of the $\mathrm{K_S}$ mesons the two amplitudes interfere and show a difference. In this way one can separate the ratios

$$\eta_{+-} = \frac{A(\mathrm{K_L} \to \pi^+\pi^-)}{A(\mathrm{K_S} \to \pi^+\pi^-)} \tag{15.58}$$

and

$$\eta_{00} = \frac{A(\mathrm{K_L} \to \pi^0\pi^0)}{A(\mathrm{K_S} \to \pi^0\pi^0)}. \tag{15.59}$$

If CP is a good symmetry (the CP quantum number is conserved), then these ratios vanish. The experiments found these ratios to be different from zero. It is customary to make an isospin analysis of the amplitudes and write them as

$$A(\mathrm{K}^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}} A_0 - \frac{2}{\sqrt{3}} A_2 \tag{15.60}$$

and

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 + \frac{1}{\sqrt{3}}A_2 \tag{15.61}$$

and rewrite the ratios in terms of isospin amplitudes.

Straightforward substitution of the amplitudes gives

$$\eta_{+-} = \frac{(\sqrt{2}A_0 + A_2 e^{i\Delta}) - e^{i\xi}(\sqrt{2}A_0^* + A_2^* e^{i\Delta})}{(\sqrt{2}A_0 + A_2 e^{i\Delta}) + e^{i\xi}(\sqrt{2}A_0^* + A_2^* e^{i\Delta})}, \tag{15.62}$$

with $\Delta = \delta_2 - \delta_0$. It is mentioned here that the ratio is phase-convention independent. A popular phase convention was introduced by Wu and Yang, which selects the A_0 amplitude to be real and then the answers will depend on the phase of the A_2 amplitude denoted by θ_2. We adopt this convention; then, by substituting $e^{i\xi}$ in terms of ε and collecting terms together, we obtain

$$\eta_{+-} = \frac{\varepsilon\left[1 + \frac{1}{\sqrt{2}}\left|\frac{A_2}{A_0}\right|\cos\theta_2\, e^{i\Delta} + \left(\frac{i}{\sqrt{2}}\right)\left|\frac{A_2}{A_0}\right|\sin\theta_2\, e^{i\Delta}\right]}{\left[1 + \frac{1}{\sqrt{2}}\left|\frac{A_2}{A_0}\right|\cos\theta_2\, e^{i\Delta} + \left(\frac{i\varepsilon}{\sqrt{2}}\right)\left|\frac{A_2}{A_0}\right|\sin\theta_2\, e^{i\Delta}\right]}. \tag{15.63}$$

The expression simplifies if we neglect the second term in the denominator, since $\varepsilon|A_2/A_0|\sin\theta_2 \ll 1$. In this case

$$\eta_{+-} = \varepsilon + \frac{\varepsilon'}{1 + \omega/\sqrt{2}} \tag{15.64}$$

with

$$\varepsilon' = \left(\frac{i}{\sqrt{2}}\right)\left|\frac{A_2}{A_0}\right|\sin\theta_2\, e^{i\Delta}$$

and

$$\omega = \left|\frac{A_2}{A_0}\right|\cos\theta_2\, e^{i\Delta}.$$

On repeating the analysis for η_{00} with the same approximations, we obtain

$$\eta_{00} = \varepsilon - \frac{2\varepsilon'}{1 - \sqrt{2}\,\omega}. \tag{15.65}$$

In many models both A_0 and A_2 are complex and rephasing of the amplitudes is necessary in order to bring them into accord with the Wu–Yang phase convention.

In summary, in addition to the CP parameter discussed in Section 15.3.1, there is the parameter ε'. The parameter ε arose from phases in the mass matrix and ε' from relative phases in the decay amplitudes. The former is referred to as indirect

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CP violation and the latter as direct CP violation. It is also customary to define the ratio ε'/ε because the phase $\pi/2 - \Delta \approx 45^\circ$ of ε' is approximately equal to the phase of ε and cancels out in the ratio.

Going back to a general phase convention whereby both A_0 and A_2 are complex, we should replace θ_2 by $\theta_2 - \theta_0$ and the definition of ε'/ε becomes

$$\begin{aligned}\frac{\varepsilon'}{\varepsilon} &= \frac{1}{\sqrt{2}|\varepsilon|}\left|\frac{A_2}{A_0}\right|(\sin\theta_2 - \sin\theta_0)\\ &= -\frac{\omega}{\sqrt{2}\,\varepsilon}\frac{1}{\mathrm{Re}\,A_0}\left(\mathrm{Im}\,A_0 - \frac{1}{\omega}\,\mathrm{Im}\,A_2\right),\end{aligned} \tag{15.66}$$

with

$$\omega = \left|\frac{A_2}{A_0}\right| \approx \frac{\mathrm{Re}\,A_2}{\mathrm{Re}\,A_0} \simeq \frac{1}{22.2}.$$

The remaining problem is the calculation of the imaginary parts of the amplitudes in Eq. (15.66) because the real parts are much larger, with their numerical values known from experiments.

15.6 The effective Hamiltonian

K-meson decays involve low-energy interactions mediated by the exchanges of hadrons and at least one W boson. It is customary to appeal to the quark–hadron duality and replace the hadrons by quarks and gluons. The weak interaction is a short-distance phenomenon that is represented by the couplings of the W to quarks. This is not the only part of the interaction, because there are strong interactions produced by the exchanges of gluons. A complete calculation must include both of them. Thus a method has been developed in field theory for this purpose. The method consists of summing the leading logarithmic contributions of the diagrams. The final result is an effective field theory with the W and the heavy quarks eliminated or, as one says, “they have been integrated out.”

Even if we start with one weak operator at momenta comparable to M_W, the exchange of gluons introduces more operators coming from loop diagrams, like penguin and box diagrams. The effective Hamiltonian has the form

$$H_{\mathrm{eff}} = \sum_{a,b} C_{ab}(M_\mathrm{W},\ \mu)Q_{ab}(\mu), \tag{15.67}$$

with $Q_{ab}(\mu) = \bar{q}(x)\Gamma_a q(x)\bar{q}(x)\Gamma_b q(x)$ with Γ_a and Γ_b being matrices in Dirac space. The constants $C_{ab}(M_\mathrm{W}, \mu)$ are the coefficient functions (Wilson coefficients) obtained from the renormalization of the operators. They depend on a high energy,

M_W, a low energy scale, μ, and the quarks in the intermediate states. Their general form is

$$C_{ij}(M_W,\mu) \sim \ln\left(\frac{M_W}{\mu}\right)^{\frac{\gamma}{b}}, \tag{15.68}$$

which is obtained by integrating renormalization equations of quantum chromodynamics (QCD). The exponent γ is known as the anomalous dimension and b arises from the running of the coupling constant. The calculation of the coefficients and their accuracy is a theoretical topic of active research whose study is beyond the scope of this book (Buchalla *etal*., 1996).

In order to give a general impression of the results, we present here the effective Hamiltonian for K-meson decays. As mentioned already, it depends only on the light quarks and contains eight operators:

$$\mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}\sum_{i=1}^{8}\left[C_i^{u}(\mu)\lambda_u + C_i^{c}(\mu)\lambda_c + C_i^{t}(\mu)\lambda_t\right]Q_i \quad \text{for} \quad \mu < m_c, \tag{15.69}$$

where $\lambda_q = V_{qd}^{*}V_{qs}$ are again the couplings from the CKM matrix, the unitarity of which implies

$$\lambda_u + \lambda_c + \lambda_t = 0 \tag{15.70}$$

and makes possible the elimination of one of them. Once we decide to eliminate λ_u, the coefficient functions will appear as differences $C_i^c - C_i^u$ and $C_i^t - C_i^u$. The substitution makes the coefficient functions less sensitive to the up quark. The operators which appear are defined as follows:

$$\begin{aligned}
Q_1 &= 4\bar{s}_L\gamma^{\mu}d_L\bar{u}_L\gamma_{\mu}u_L, & Q_2 &= 4\bar{s}_L\gamma^{\mu}u_L\,\bar{u}_L\gamma_{\mu}d_L,\\
Q_3 &= 4\sum_q \bar{s}_L\gamma^{\mu}d_L\,\bar{q}_L\gamma_{\mu}q_L, & Q_4 &= 4\sum_q \bar{s}_L\gamma^{\mu}q_L\,\bar{q}_L\gamma_{\mu}d_L,\\
Q_5 &= 4\sum_q \bar{s}_L\gamma^{\mu}d_L\,\bar{q}_R\gamma_{\mu}q_R, & Q_6 &= -8\sum_q \bar{s}_L q_R\,\bar{q}_R d_L,\\
Q_7 &= 4\sum_q \frac{3}{2}e_q\bar{s}_L\gamma^{\mu}d_L\,\bar{q}_R\gamma_{\mu}q_R, & Q_8 &= -8\sum_q \frac{3}{2}e_q\bar{s}_L q_R\,\bar{q}_R d_L.
\end{aligned} \tag{15.71}$$

Operator Q_2 is the original charged-current operator and Q_1 is generated from box-type diagrams, where in addition to the W a gluon is being exchanged (Fig. 15.1).

The penguin diagrams in Fig. 15.2 generate $Q_3, \ldots, Q_6$. Finally, penguin diagrams with the exchange of photons generate Q_7 and Q_8 (electroweak penguins). Since the penguin diagrams are important, we present several steps of the calculation in Section 15.7, where it is also explained how the penguin diagrams generate the various operators.

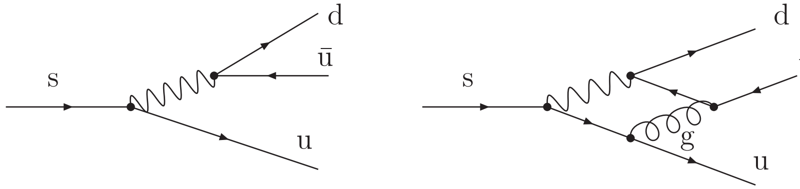

Figure 15.1. Tree and box diagrams.

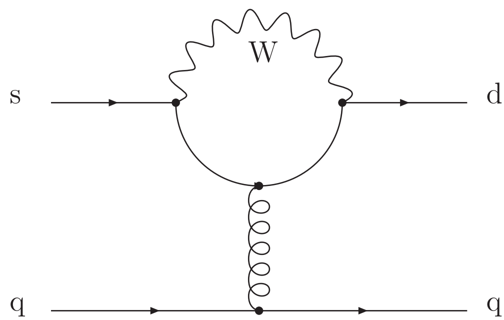

Figure 15.2. A penguin diagram.

Finally, one may also show that not all the operators are independent, since they satisfy the relation

$$-Q_1 + Q_2 + Q_3 = Q_4.$$

The coefficient functions $C_i(M_{\mathrm{W}}, \mu)$ originate from the short-distance interaction of QCD and have been calculated at the one-loop, as well as the two-loop, level. The hadronic matrix elements

$$\langle Q_i(\mu)\rangle_I = \langle \pi\pi, I|Q_i(\mu)|K^0\rangle \tag{15.72}$$

originate from long-distance interactions, since they involve low energies and momenta. They represent the low-energy limit of QCD and must be calculated by low-energy methods. They have been the subject of various calculations, which we shall mention briefly. Within the framework described in this section we can outline the calculation of ε'/ε.

Among the amplitudes which enter the calculation, Re A_0 and Re A_2 are taken from experimental data (Devlin and Dickey, 1979):

$$\mathrm{Re}\, A_0 = 0.338 \times 10^{-6}\,\mathrm{GeV} \quad \text{and} \quad \mathrm{Re}\, A_2 = 0.015 \times 10^{-6}\,\mathrm{GeV}.$$

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They are much larger than their imaginary parts. The amplitudes Im A_0 and Im A_2 are calculated in terms of the effective Hamiltonian

$$\mathrm{Im}\, A_I = \frac{G}{\sqrt{2}} \sum_{i=1}^{8} \left[\left(C_i^{\mathrm{c}} - C_i^{\mathrm{u}} \right) \mathrm{Im}\, \lambda_{\mathrm{c}} + \left(C_i^{\mathrm{t}} - C_i^{\mathrm{u}} \right) \mathrm{Im}\, \lambda_{\mathrm{t}} \right] \langle Q_i \rangle_I . \tag{15.73}$$

With a phase convention of the CKM matrix whereby V_{us} and V_{ud} are real, the unitarity of the CKM matrix provides one additional relation,

$$\mathrm{Im}\, \lambda_{\mathrm{t}} = -\mathrm{Im}\, \lambda_{\mathrm{c}} = \lambda^5 A \eta . \tag{15.74}$$

Substitution of this relation eliminates the u quarks in intermediate states, since the Wilson coefficients for top and charm quarks are subtracted from each other. This leads to the result

$$\frac{\varepsilon'}{\varepsilon} = \frac{G_{\mathrm{F}}}{\sqrt{2}} \frac{\omega}{|\varepsilon|} \frac{1}{\mathrm{Re}\, A_0} \mathrm{Im}\, \lambda_{\mathrm{t}} \sum y_i^{\mathrm{t}}(\mu) \left[\langle Q_i(\mu) \rangle_0 - \frac{1}{\omega} \langle Q_i(\mu) \rangle_2 \right], \tag{15.75}$$

with $y_i^{\mathrm{t}}(\mu) = C_i^{\mathrm{t}}(\mu) - C_i^{\mathrm{c}}(\mu)$ and $\omega = \mathrm{Re}\, A_2 / \mathrm{Re}\, A_0$. The superscripts denote contributions from top and charm quarks in the intermediate states. The unitarity of the CKM matrix helps by eliminating the Wilson coefficients of the u quarks and making the QCD contribution sensitive to the energy scales between m_{c} and m_{t}, where the short-distance expansion is acceptable. The Wilson coefficients are available and have been tabulated (Buchalla *et al.*, 1996).

The hadronic matrix elements have been the subject of numerous calculations. From the early estimates it was evident that $\langle Q_6 \rangle_0$ plays an important role. The matrix element is generated by the penguin diagrams and, since it involves pseudoscalar densities, it is enhanced. In chiral perturbation theory it is expressed in terms of coupling constants divided by the mass of the strange quark. It was also calculated by vacuum saturation or the tree contribution of the chiral perturbation theory. It was noted that the lowest-order contribution must be supplemented by chiral loops (Bardeen *et al.*, 1987, 1998). The final results indicate that $\langle Q_6 \rangle_0$ is important, especially because it is further enhanced by contributions from chiral loops.

An additional complication arises from the matrix element $\langle Q_8 \rangle_2$, whereby in the penguin diagrams the gluon is replaced by a photon. It was argued that the electroweak term can be very important because it is multiplied by a large factor, $1/\omega$. Calculations in chiral perturbation theory indicate that its contribution is moderate and it is further reduced by loops. A good approximation consists of taking the dominance $\langle Q_6 \rangle_0$ and $\langle Q_8 \rangle_2$. As an illustrative example we give typical values for the Wilson coefficients,

$$y_6 = -0.110 \qquad \text{and} \qquad y_8 = 1.15 \times 10^{-3}, \tag{15.76}$$

and the matrix elements,

$$\langle Q_6\rangle_0 = -3.4\,\text{GeV}^3 \qquad \text{and} \qquad \langle Q_8\rangle_2 = 0.46\,\text{GeV}^3, \tag{15.77}$$

obtained in chiral perturbation theory for $m_s = 150$ MeV. The CKM term is precisely known to be

$$\text{Im}\,\lambda_t = (1.35 \pm 0.35) \times 10^{-3}. \tag{15.78}$$

On collecting all terms together in Eq. (15.75), we obtain the value

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_K = 16.9 \times 10^{-4}, \tag{15.79}$$

which is consistent with the experimental values. The contribution of the electroweak penguin is less than 20%. Many quantities entering the calculation carry uncertainties and the final range for the ratio is larger, in the range $(10-20) \times 10^{-4}$ (Hambye *et al.*, 2000). Calculations in the chiral quark model give a similar range. These values are consistent with the newest experimental values,

$$\left(\frac{\varepsilon'}{\varepsilon}\right)_K = \begin{cases} (14.7 \pm 2.2) \times 10^{-4} & \text{NA(48)}\ \ \text{(Fanti } et\ al.\text{, 1999),} \qquad (15.80) \\ (22.7 \pm 2.8) \times 10^{-4} & \text{KTeV}\ \ \text{(Alavi-Harati } et\ al.\text{, 1999).} \qquad (15.81) \end{cases}$$

At this time it seems that the experiments are more precise than the theory. This is the outcome of four large experiments that invested great efforts in measuring precisely decays and interference phenomena in K-meson decays.

It would be an omission not to mention a good deal of work done on lattice gauge theories, which tries to determine the matrix elements. Unfortunately, their results are not stable enough yet. They give a wide range of values for the matrix elements and the CP parameter.

This is an introduction to the calculations of the CP parameter intended for students who may use it as a guide to the published articles. The bottom line is that theoretical analyses in the standard model are consistent with experimental measurements. The CKM paradigm gives a consistent – albeit not very accurate – picture for the K-meson decays and it remains to find out whether it continues being successful for mesons containing heavy quarks.

15.7 Calculation of a penguin diagram

In K-meson and B-meson decays an important contribution comes from the penguin diagram. We have mentioned already that in Eq. (15.71) the penguin diagram with gluonic corrections produces four operators. It is worthwhile to give several steps of the calculation, which demonstrate how the various operators are generated. This

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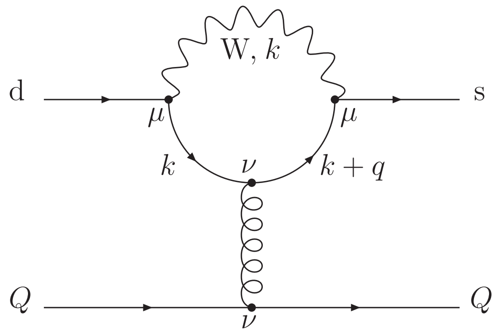

Figure 15.3. Momentum assignments for the penguin diagram.

section contains long algebraic manipulations and is presented here for those who are theoretically inclined.

The notation for the penguin diagram is introduced in Fig. 15.3. The external momenta are those of the strange and down, quarks, which correspond to typical momenta within light mesons and are small relative to the mass of the W boson. For this reason external momenta are kept in the spinors but will be neglected within the four-dimensional integral. A peculiarity of the penguin diagram is the presence of the gluon propagator with momentum q, which is kept throughout the calculation. Following standard rules, the matrix element is written in the form

$$\mathcal{M}_{\mathrm{P}} = \frac{g_{\mathrm{w}}^2}{8}\frac{g_{\mathrm{s}}^2}{q^2}\int \bar{s}\gamma_\mu\gamma_- \frac{\not{k}+\not{q}+m_i}{(k+q)^2-m_i^2}\gamma_\nu \frac{\lambda^\alpha}{2}\frac{\not{k}+m_i}{k^2-m_i^2}\gamma^\mu\gamma_- d\,\frac{1}{k^2-M_{\mathrm{W}}^2}\frac{\mathrm{d}^4k}{(2\pi)^4}$$
$$\times\, \bar{Q}\gamma^\nu\frac{\lambda^\alpha}{2}Q\cdot V_{i\mathrm{s}}^* V_{i\mathrm{d}}. \tag{15.82}$$

The quarks in the loop and their masses are denoted by the subscript i and m_i, respectively. The index for the intermediate quarks occurs also in the CKM matrix elements $V_{i\mathrm{s}}$ and $V_{i\mathrm{d}}$. The rest of the notation is standard, with g_{w} and g_{s} the weak and strong coupling constants, respectively, $\gamma_- = (1-\gamma_5)$, and λ^α the color matrices.

We follow several of the steps for the calculation of loops described in Section 14.7.2. We rewrite the matrix element as

$$\mathcal{M}_{\mathrm{P}} = \frac{g_{\mathrm{w}}^2}{8}\frac{g_{\mathrm{s}}^2}{q^2}\bar{s}\gamma_\mu\gamma_-\gamma_\alpha\gamma_\nu\left(\frac{\lambda^\alpha}{2}\right)\gamma_\beta\gamma^\mu\gamma_- d\,\bar{Q}\left(\frac{\lambda^\alpha}{2}\right)Q I^{\alpha\beta}(m_i,q), \tag{15.83}$$

with

$$I_{\alpha\beta}(m_i,\, m_i, q) = \int \frac{\mathrm{d}^4k}{(2\pi)^2}\frac{(k+q)_\alpha k_\beta}{\left[(k+q)^2-m_i^2\right]\left(k^2-m_i^2\right)\left(k^2-M_{\mathrm{W}}^2\right)}V_{i\mathrm{s}}^* V_{i\mathrm{d}}. \tag{15.84}$$

In this way we have separated the spinor structure from the four-dimensional integral. We ignored fermion masses in the numerator, since they are small relative

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to the integration momenta, and set $\bar{s}\not{q}d = 0$ because we consider masses for external quarks that are small. There is a logarithmic divergence independent of the quark masses. It is multiplied by $\sum_i V_{is}^* V_{is} = 0$ and vanishes. Similarly, in the limit in which the gluon momentum transfer q^2 is much larger than all internal quark masses, it is easy to check that the penguin diagram vanishes. However, for problems involving $q^2 \ll m_c^2$ or m_t^2 the cancellation is not complete.

The remaining two integrals appear with the factors $k_\alpha k_\beta$ or $q_\alpha k_\beta$ in the numerator. Using the method of Feynman parameters and the integrations described in Problem 4, the dominant contribution of the integral for $m_c,\ m_t \ll M_W$ is

$$I_{\alpha\beta}(m_i,\ q) = -\frac{q_\alpha q_\beta}{M_W^2}\frac{1}{16\pi^2 i}\ln\left(\frac{M_W^2}{m_i^2}\right)\left(\frac{1}{3}-\frac{1}{2}\right)V_{is}^* V_{id}, \tag{15.85}$$

with the $\frac{1}{3}$ coming from the $k_\alpha k_\beta$ term and the $\frac{1}{2}$ from the $q_\alpha k_\beta$ term. Finally, we simplify the spin structure by using known identities:

$$q^\alpha q^\beta \bar{s}\gamma_\beta\gamma_\nu\gamma_\alpha\gamma_- d = \bar{s}(2q_\nu - \gamma_\nu\not{q})\not{q}\gamma_- d = -q^2\bar{s}\gamma_\nu\gamma_- d. \tag{15.86}$$

The q^2 factor cancels out the gluon propagator in Eq. (15.82).

On collecting terms together, we arrive at the final result

$$\mathcal{M}_P = -\frac{G}{\sqrt{2}}\frac{\alpha_s}{12\pi}\ln\left(\frac{M_W^2}{m_i^2}\right)\bar{s}_L\gamma_\nu\lambda^\alpha d_L\bar{Q}\gamma^\nu\lambda^a Q(V_{is}^* V_{id}). \tag{15.87}$$

We note that the coupling $\bar{s}_L \ldots d_L$ contains left-handed quarks, in contrast to the gluon coupling $\bar{Q}\gamma_\nu Q$ being a vector. By decomposing the latter into left- and right-handed couplings, we generate two distinct operators. Finally, using an identity for the product of color matrices,

$$\sum_a \lambda_{ij}^a\lambda_{kl}^a = 2\left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right), \tag{15.88}$$

we double the number of operators. In the end, the penguin diagram generates four operators, Q_3, Q_4, Q_5, and Q_6, which were absent at the tree level.

There are two ways to treat the penguin diagram. One of them considers its contribution as a short-distance operator creating a four-fermion interaction among the quarks. This would be the case when the top quark dominates a process. The exchange of additional gluons may still be soft and some sort of summation is again necessary. A final step is the estimation of the four-quark operator between hadronic states.

The alternative method considers the four operators generated by the penguin diagrams as basic operators and sums up higher-order QCD corrections. This is achieved by considering gluonic corrections to each of the operators $Q_1, \ldots, Q_6$, which renormalizes and in addition mixes them up; that is, gluonic corrections to

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one operator generate several of the others. The problem to be solved is one of coupled differential equations. The initial conditions are defined at high momenta when only Q_2 has an initial value and all other operators are zero. Following this method (Peskin and Schroeder, 1995; Buchalla *et al.*, 1996), one arrives at the effective Hamiltonian similar to that in Eq. (15.69). The theory is effective because the additional corrections are proportional to higher powers of $\alpha_s(p^2)$, which for large momenta become very small.

Problems for Chapter 15

1. Introduce in Eq. (15.62) the Wu–Yang phase convention, then substitute for $e^{i\xi}$ in terms of ε and derive Eqs. (15.63) and (15.64).
2. (i) The normalizations that occur in Eqs. (15.30) and (15.32) describe how many K^0 or $\bar{K}^0$ mesons, respectively, are present at time $t = 0$. Argue that for normalized wave functions $|K^0\rangle$ and $|\bar{K}^0\rangle$ they should be $N = N' = \frac{1}{2}$.
   (ii) Consider the time development of the state $|K^0\rangle$ and the decays to $\pi^+\pi^-$. Describe the interference term and find an argument justifying the large interference at a distance corresponding to six to seven lifetimes of the K_S meson.
3. Show that the operators Q_1, Q_2, Q_3, and Q_4 satisfy the relation
$$-Q_1 + Q_2 + Q_3 = Q_4.$$
4. The calculation of the integral in Eq. (15.84) contains in the numerator two terms: one with $k_\alpha k_\beta$ and the other with $q_\alpha k_\beta$. Write each of the integrals in terms of Feynman parameters. The four-dimensional integrations are of the form
$$\int \frac{d^4k}{(2\pi)^4} \frac{\{k_\alpha, k_\alpha k_\beta\}}{(k^2 + 2k \cdot p - \Delta)^3}$$
$$= \frac{1}{32\pi^2 i}\left[\frac{\{-p_\alpha;\ p_\alpha p_\beta\}}{\Delta + p^2 + i\varepsilon} + \left\{0;\ \frac{1}{2}g_{\alpha\beta}\ln(\Delta + p^2 + i\varepsilon) + A_0\right\}\right].$$
   The function Δ contains masses of the quarks, the mass M_W, q^2, and Feynman parameters. The constant A_0 is cut-off-dependent but independent of quark masses; it disappears when we sum over the quarks in the loop. The remaining two integrations are elementary. Arrange the integrations in an appropriate way to extract the leading $\ln(M_W/m_i)$ term and obtain Eq. (15.85).

   Comment We described the integrals in the limit $m_i \ll M_W$. For $m_t > M_W$ you can again study the elementary integrals and obtain a modified logarithmic term.

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