CORRESPONDENCE

To the Editor of the *Mathematical Gazette*

DEAR SIR,

In his article "Two problems on Impulsive Motion" (page 95 of the Mathematical Gazette, May 1960) Mr O'Keeffe writes "the process of taking moments about a moving joint B (O'?) is valid only in cases where the second term of the equation

$$[\mathbf{H}(O')]_{t_0}^{t_1} + \int_{t_0}^{t_1} \mathbf{V}(O') \wedge \mathbf{L}\, dt = \mathbf{X}(O') \qquad (2)$$

vanishes; the most important of which is .." (In this equation $\mathbf{H}(O')$ is the angular momentum of the system about O', $\mathbf{V}(O')$ is the velocity of O', $\mathbf{L}$ is the linear momentum of the system and $\mathbf{X}(O')$ is the sum of the moments about O' of the applied impulses.) In fact the second term *always* vanishes, in the limit as $t_1 \to t_0$, since it is clear from the equation

$$[\mathbf{L}]_{t_0}^{t_1} = \mathbf{J},$$

where $\mathbf{J}$ is the sum of the applied impulses, that although $\mathbf{L}$ has jump-discontinuity at t_0, in the limit, nevertheless it is bounded in the interval (t_0, t_1). It follows, therefore, that the second term of equation (2) always vanishes and there is no restriction, when considering impulsive motion, on the validity of taking moments about a moving point.

Viewed in another way, impulsive motion is concerned with instantaneous change in the particle-velocities, and the equations of impulsive motion state the equivalence of two sets of localised vectors, the first set being the vectors which represent the change in momentum of the system, and the second set being the applied impulses. The motion, or otherwise, of a point about which moments are taken is clearly irrelevant.

I agree with Mr. O'Keeffe that many solutions claiming to use Bertrand's Theorem use Kelvin's Theorem, in fact. The equations expressing Kelvin's Theorem are, of course, precisely Lagrange's equations of impulsive motion for the coordinates corresponding to which there is no generalised component of impulse. Bertrand's Theorem states that if a system is subjected to given impulses, the kinetic energy generated is greater than it would have been if the system had been subjected to the same impulses and also workless constraints. So in order to use Bertrand's Theorem to solve a problem it is necessary to apply to the system variable constraints, depending, say, on parameters x_i; these constraints must be such that they are capable, by variation of the x_i, of allowing all possible motions of the unconstrained system. The problem of the system subject to these variable constraints and the applied impulses must then be solved, and the resulting kinetic energy T evaluated as a function of the x_i and the applied impulses. Maximisation of T with respect to the x_i will then yield a solution of the unconstrained problem.

As an example, consider the simple problem of a uniform rod AB, of mass m and length $2a$, at rest on a smooth horizontal table. If the rod receives a horizontal impulse J, perpendicular to AB, at A, then it is easily shown that the instanteous centre of the resulting motion is between A and B at the point C where $AC = 4a/3$, and that the kinetic energy generated is $2J^2/m$. In order to use (or illustrate) Bertrand's theorem we apply a variable constraint by smoothly pivoting, to the table, the point P of the rod at distance x from A. The resulting kinetic energy T is easily shown to be given by

$$T = 3J^2x^2/2m\,(4a^2 - 6ax + 3x^2).$$

Differentiation with respect to x gives

$$\frac{dT}{dx} = 3J^2\,ax(4a - 3x)/m\,(4a^2 - 6ax + 3x^2)^2,$$

showing that T has, of course, a minimum at $x = 0$ and a maximum value, of $2J^2/m$, when $x = 4a/3$, that is when P coincides with C, and the resulting motion is the same as in the unconstrained case.

The conclusion seems to be that Bertrand's Theorem is not of much assistance in the exact solution of problems, though it may be of use in finding a lower bound for the kinetic energy.

Yours etc., S. T. Cook

To the Editor of the *Mathematical Gazette*

SUBTRACTION AND DIVISION

Dear Sir,

In a discussion on Subtraction, I see that I am being quoted as an authority for some modern method which teachers of infants have found useful.

Let us be frank about the duties of a member of a committee. Is he to be obstructive about every detail outside his own experience? On a matter of sacred principle or deep conviction let him dig in his heels in passionate protest; but if we all do this about every detail of which we know or care but little, what is left but a mosquito-like swarm of minority-reports on trivialities?

Like most Victorians, I subtract by the outmoded method of the 19th century; but if A or B prefers something better suited to this enlightened age, let him have it: it is out of place for me to object. This is surely a case for easy tolerance.

But if you want something for me to gnash a tooth about, take those mouldy little figures that look like indices and aren't, baffling enough even when neatly printed on page 180† of the current issue, and utterly chaotic when smudged about by a heavy-fisted boy with a fat pen.

† $\dfrac{5^1 2\ 7^3 6}{4}$