

Appendice A - Théorèmes de de Rham: Isomorphisme entre la cohomologie singulière et la cohomologie des formes sur une variété compacte.

Appendice B - Le cup-produit: L'isomorphisme précédent est un isomorphisme d'anneau.

Appendice C - Théorème de Hodge: L'isomorphisme entre la cohomologie et l'espace des formes harmoniques n'a été démontré au chap. Il qu'en admettant la propriété de l'espace des formes harmoniques d'être de dimension finie. C'est cette propriété qui est démontrée ici, par des méthodes d'espace de Hilbert.

Appendice D - Partition de l'unité: Existence d'une partition différentiable de l'unité sur une variété différentiable dénombrable à l'infini.

Les applications de certaines techniques ne sont pas toujours bien choisies: l'auteur aurait pu signaler que la nullité de l'invariant d'Euler-Poincaré d'un groupe de Lie compact est une conséquence immédiate de l'existence d'un champ de vecteurs invariant à gauche partout différent de 0; la démonstration qu'il en donne, valable seulement dans le cas semisimple, ne peut servir qu'à illustrer la théorie des formes harmoniques. Regrettons aussi que certaines méthodes élégantes de calcul ne figurent que dans les exercices (qui, en plus, font ainsi double emploi).

Certaines initiatives "pédagogiques", par contre, nous semblent particulièrement réussies. Par exemple:

- l'introduction des formes harmoniques et de l'opérateur $*$, avec le cas des surfaces de Riemann (p. 64-68),
- l'introduction de la cohomologie (de Čech) à valeurs dans un faisceau par l'exemple du cas 1-dimensionnel (p. 270-271).

Rassemblant enfin un grand nombre de résultats, ce livre facilitera grandement le travail des chercheurs.

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Combinatorial Mathematics, by H. J. Ryser. John Wiley and Sons, Inc., New York, 1963. ix + 141 pages. \$4.00.

This is a most welcome addition to the series of Carus mathematical monographs published by the Mathematical Association of America. In spite of the rapidly increasing utilization of the area of combinatorial mathematics in an impressive variety of applications, this is only the second book on the subject to appear in recent times. The first was, of course, the treatise by John Riordan "An introduction to combinatorial

analysis", New York, Wiley, 1958. The difference in emphasis between these two books may be described approximately by saying that the present monograph is mainly concerned with existence questions while Riordan's work emphasized enumeration problems. Not surprisingly, Ryser has paid particular attention to the development of those parts of combinatorics to which he has made substantial contributions.

The monograph is concise, clear, and remarkably free of misprints! The Table of Contents is as follows:

Chapter 1. Fundamentals of combinatorial mathematics

Chapter 2. The principle of inclusion and exclusion

Chapter 3. Recurrence relations

Chapter 4. A theorem of Ramsey

Chapter 5. Systems of distinct representatives

Chapter 6. Matrices of zeros and ones

Chapter 7. Orthogonal Latin squares

Chapter 8. Combinatorial designs

Chapter 9. Perfect difference sets.

In the first chapter, combinatorial mathematics is described rather than defined. Several historical illustrations are provided and it is properly asserted that there are now non-empty intersections of combinatorics with each of the mathematical domains: finite projective planes, abstract algebra, topology, foundations, graph theory, game theory, and linear programming. The chapter also includes standard observations on permutations and combinations without ever using the latter word.

The second chapter gives a generalization of the observation that the number of elements in the union of two sets is the sum of the number of elements in the first set and the second set less the number in their intersection. This generalization has the classical sieve formula as a corollary. Applications are made to number theory, to derangements (permutations with no fixed elements) and to permanents.

Recurrence relations are illustrated in the third chapter using binomial coefficients, the number of regions in a plane obtained from n lines in general position, the Fibonacci numbers, derangements a la Euler, and ménage numbers, the latter being the number of permutations of n objects such that no object is either fixed or carried forward into the next object in cyclic order.

The fourth chapter presents Ramsey's existence theorem, and applications thereof to convex polygons and binary matrices (with entries 0 and 1).

Chapter Five relates the now classical theorem of P. Hall (giving a criterion for the existence of a system of distinct representatives) with Latin rectangles and the term rank of a binary matrix, i. e., the maximum number of 1's in a matrix with no two 1's on the same row or column. Other presentations in book form of this theorem of Hall appear in

L. R. Ford, Jr., and D. R. Fulkerson, Flows in networks, Princeton University Press, 1962, pp. 67-75.

D. Konig, Theorie der endlichen and unendlichen Graphen, New York, Chelsea, 1950, pp. 234-235.

O. Ore, Theory of graphs, Amer. Math. Soc. Colloq. Publis., 38, 1962.

Chapter 6 presents a criterion for the existence of a binary matrix with prescribed row and column sum vectors, a result discovered independently by the author, D. R. Fulkerson and D. Gale. Applications are made to problems involving the term rank of a matrix.

An introduction to the very recent and already celebrated result of the "Euler Spoilers", Bose, Shrikhande and Parker, is given in Chapter 7, "Theorem 2.1. Let $n \equiv 2 \pmod{4}$ and let $n > 6$. Then there exists a pair of orthogonal Latin squares of order n ." The chapter also demonstrates the interconnection between finite projective planes and orthogonal Latin squares.

The eighth chapter introduces the reader to balanced incomplete block designs, a subject of considerable interest in contemporary statistics, especially important in the design of experiments.

The final chapter relates perfect difference sets of residue classes of integers with certain kinds of balanced incomplete block designs.

It was a pleasure for the reviewer to read this excellent little monograph. Most if not all of the material in the last three chapters has probably never appeared in book form previously, and this is also true of a considerable amount of the material in earlier chapters as well. Unfortunately, there are no exercises in this monograph so that it is not practicable to use it by itself as a textbook for an introductory course on combinatorics. But perhaps in conjunction with Riordan's book cited above (which abounds in exercises), an effective text for such a course would be available. Such courses are recently being offered for the first time at several leading universities.

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