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CLASSICAL TREATEMENT OF THE COMPTON COLLISION IN GENERAL RELATIVITY

A systematic development of the photon 4-momentum in the non-quantic perspective of General Relativity is very difficult to find in the scientific litterature. However, the photon 4-momentum is a fundamental concept if one wants to study relativistic transport theory or Compton collision formulae. For these reasons, a coherent formalism has been elaborated. It sysmetically allows one to formulate these problems the criterion of validity being the fact that in the case of the Minkowski metric, the general formulae give the classical results of the Special Relativity.

This paper based on previous work (1) which contains detailed calculations, deals with the classical treatment of the Compton collision in a non diagonal Riemannian metric under the hypothesis of conservation of the global 4-momentum. Following a deep analogy with the non-zero rest mass particle, one defines the 4-momentum and the apparent mass of the photon, M. Then one writes the generalized Doppler effect and defines a new scalar parameter ξ to describe the trajectory of the photon and one calculates the frequency and velocity of the emerging photon and electron in a Compton collision, the collision parameters being the frequency and the direction of the incident photon, the velocity of the incident electron and the direction of the emerging photon. We have :

(1) Pierre PAILLERE :"Le corpuscule photon en espace de Riemann"
Note CEA n° 2401 (March 1984)

E. R. Priest and V. Krishan (eds.), Basic Plasma Processes on the Sun, 95–96.

$q^\alpha = h\nu\Omega^\alpha/\sqrt{goo}$, $\Omega\alpha\Omega^\alpha = 0$, $\gamma ij\Omega^i\Omega^j = 1$, $\gamma ij = -gij + \gamma i\gamma j$, $\gamma i = goi/\sqrt{goo}$,

$H = cq_o = h\nu$, $a = \gamma i\Omega^i$, $a' = \gamma i\Omega^{i'}$, $M = h\nu(1-a)/c^2 goo$,

$q^\alpha = M_o c\, dx^\alpha/cd\xi$, with $M_o c = q_\alpha u_s^\alpha$, u_s^α : source velocity ,

and (R) : $d\lambda = \frac{cd\xi}{M_o} = \frac{cdt}{M}$ = relativistic invariant, so that :

$$\boxed{\begin{aligned} \frac{\nu'}{\nu} &= \frac{\sqrt{goo} + \gamma i\beta^i - \gamma ij\Omega^j\beta^i}{\sqrt{goo} + \gamma i\beta^i - \gamma ij\Omega^{j'}\beta^i + \Gamma\Lambda(1-\vec{\Omega}\vec{\Omega}')}, \quad \Gamma = \frac{h\nu}{moc^2\sqrt{goo}} \\ \frac{\beta^{i'}}{\sqrt{goo}} &= \frac{\beta^i + \Gamma\Lambda(\Omega^i - \nu'\Omega^{i'}/\nu)}{\sqrt{goo} + \Gamma\Lambda\,[1-a-(1-a')\nu'/\nu]}, \quad \Lambda = [(\sqrt{goo} + \gamma i\beta^i)^2 - \beta^2]^{\frac{1}{2}}. \end{aligned}}$$

In the Schwarzschild's case one has : $a = a' = 0$ and

$$\frac{\nu'}{\nu} = \frac{\sqrt{1-2m/r} - (\vec{\Omega}.\vec{\beta})}{\sqrt{1-2m/r} - (\vec{\Omega}.\vec{\beta}) + \Gamma\Lambda(1-\vec{\Omega}\vec{\Omega}')}, \quad \Lambda = [1-2m/r - \beta^2]^{\frac{1}{2}} \quad , \alpha = 1,465.10^5 \text{ C.G.S.}$$

while, with the help of the (R) relation we obtain by the geodesic equation $Dq_o = 0$, $h\nu$ = constant along the trajectory that we can write, with $u = 1/r$:

$$\frac{du}{dct} = -u^2(1-2mu)\;[1-(Ru)^2\;(1-2mu)]^{\frac{1}{2}}, \quad \frac{d\varphi}{Dct} = Ru^2(1-2mu),$$

whence the transit time Δt from Sun to Earth :

$$\frac{c\Delta t}{R} = \int_{R/DSE}^{1} dx/[x^2\cdot(1-\alpha x)][1-x^2(1-\alpha x)]^{\frac{1}{2}}; \quad (\alpha = 2m/R \quad , x = Ru)$$

(DSE = distance of Sun to Earth/R = Sun radius)

We find Δt = 498,99669 s while the quotient of DSE by c gives $\Delta t'$ = 499,006813 s !

Notice that for goo = 0 we get : $\beta = 0$, $\beta' = 0$, $\nu = 0$, $\nu' = 0$, $1 = 0$, $\frac{du}{dt} = 0$ and $\frac{d}{dt} = 0$ (R = 6,9598 10^{10} cm, DSE = 1,495985 10^{13} cm).

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