Magnetic Field of a Contracting Protostar

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IN a recent paper, Mestel and Spitzer[1] discussed the problem of star formation from cool matter in the presence of a magnetic field of energy density comparable with the thermal energy density. If the field were frozen into the gas, the magnetic pressure would put a lower limit of order $10^3 M_\odot$ to the mass that could be gravitationally bound, and this limit is unaltered by contraction of the cloud. However, in a lightly ionized gas, the field moves not with the gas as a whole, but with the plasma, and the motion of the plasma through the cloud is determined by a balance between mgnetic force and friction between neutral gas and plasma. A cloud containing sufficient dust can extinguish the galactic ionizing radiation; the plasma density decays quickly enough for the plasma and neutral gas to become uncoupled during the time of gravitational contraction. In this way high densities are built up without correspondingly high magnetic energy, and the cloud can break up into stars.

As a protostar contracts and the temperature rises, the plasma density rises to a level sufficient for freezing of the internal field to be restored. If the field within the star is initially uniform, it remains so under homologous contraction, and by flux conservation its strength increases as $1/R^2$ as the radius R decreases. However, the *external* field is tied to the cloud plasma, and so its shape and strength are determined by a balance between the magnetic force density $\mathbf{j}\times\mathbf{H}/c$ and the other forces acting on the plasma. The star's gravitational field acting on the surrounding gas causes mass motions which have been studied, under the heading "accretion," chiefly for their possible relevance to stellar evolution. It is the friction between the inflowing neutral gas and the plasma, which is retarded by the magnetic force, that provides the principal balancing nonmagnetic force. Even when the rate of accretion of matter by the star is negligible, the flow of the surrounding gas should not be ignored in magnetic problems, because of the strong coupling between plasma flow and field.

The star is supposed at rest in a nonrotating HI cloud, containing a sufficient proportion of metallic dust grains to keep the flow isothermal at about 100°K. As a first approximation the retarding effect of the magnetic force (acting indirectly through the plasma-hydrogen friction) on the inflow of the hydrogen is neglected, so that the density-velocity field of the hydrogen is that given by Bondi[2] in his study of spherically symmetric accretion. An initially straight cloud field $\mathbf{H}_\infty$ would be distorted by the inward flow of matter until a steady state is reached in which the plasma velocity perpendicular to the field vanishes. Beyond the radius $r_c \cong 10^{16}$ $(M/M_\odot)$ (M being the stellar mass) the flow of the hydrogen is subsonic, and the component across the field of the hydrogen-plasma friction is

$$\beta\rho^2 v_T(\mathbf{H}\times(\mathbf{v}\times\mathbf{H})/|\mathbf{H}|^2), \tag{1}$$

where β = coupling constant, ρ = density of hydrogen, v_T = thermal velocity of hydrogen, and $\mathbf{v}$ = bulk velocity of hydrogen. For distances $r<r_c$, the hydrogen flow is supersonic, and the transverse component of friction is approximately

$$\beta\rho^2|\mathbf{v}|(\mathbf{H}\times(\mathbf{v}\times\mathbf{H})/|\mathbf{H}|^2). \tag{2}$$

Hence the equation to the field is

$$\frac{(\nabla\times\mathbf{H})\times\mathbf{H}}{4\pi}=\frac{\beta\rho^2 V}{H^2}(\mathbf{v}\times\mathbf{H})\times\mathbf{H}, \tag{3}$$

where $V=v_T$, $|\mathbf{v}|$ respectively, in the two zones. As the field is purely poloidal (rotation being ignored), (3) is equivalent to

$$\nabla\times\mathbf{H}=\frac{4\pi\beta\rho^2 V}{H^2}(\mathbf{v}\times\mathbf{H}). \tag{4}$$

This equation holds everywhere except in a narrow region close to the equator (defined by the direction of $\mathbf{H}_\infty$). The plasma flowing down those lines of force not connected with the star rapidly builds up a "plasma zone" near the equator, its density being fixed by equating the gravitational inflow of plasma to the slow equatorial drift across the field toward the star. The field on the equator turns out to be small, so that this drift is not neglibible, although it is slow compared with normal accretion velocities. The equation to the magnetic field in the plasma zone is again given by equating the magnetic force to the largest nonmagnetic forces on the plasma, which in this zone are the gravitational pull of the star and the pressure gradient.

At any instant during the contraction of the star, the field in the surrounding gas is given by the solutions of (4) and the equation for the plasma zone, subject to the boundary conditions $\mathbf{H}\rightarrow\mathbf{H}_\infty$ as $r\rightarrow\infty$, and $\mathbf{H}$ continuous at the stellar surface with the frozen internal field. (To insure this the field lines within the star are not accurately straight, but bend in a narrow surface region, this to avoid horizontal discontinuity in the field, surface currents and infinite magnetic forces.)

[1] L. Mestel and L. Spitzer, Jr., Monthly Notices Roy. Astron. Soc. **116**, 503 (1956).
[2] H. Bondi, Monthly Notices Roy. Astron. Soc. **112**, 195 (1952).

Then the slow contraction of the star leads to changes in the form of the external field. For consider a homologous contraction of the whole system, in which all linear dimensions are decreased by a factor L. This increases the left-hand side of (3) by $1/L^5$. In the subsonic zone $\rho \cong$ constant and $v \propto 1/r^2$; in the supersonic zone $\rho \propto 1/r^{3/2}$, $v \propto 1/r^{1/2}$. Hence the right-hand side of (3) increases by $1/L^2$ and $1/L^4$, respectively: under homologous contraction the magnetic forces would increase too rapidly to balance the friction, and so the external field lines must lag behind the contracting star and its field. Similar conclusions follow for the field in the plasma zone.

In spite of this, however, all the field lines initially linked with the star tend to remain so—there is no immediate breakage of any line into an infinite part, unconnected with the star, and a closed loop that passes through the star. Such a field would possess an X-type neutral point, external to the star; near this point the magnetic force would be unable to withstand the inward forces on the plasma, so that the field lines would be dragged into the star. Hence as the star contracts there arises near the equator a zone of low but nonvanishing field-strength, with $\nabla \times \mathbf{H}$ correspondingly large enough to balance the nonmagnetic forces. The lines of force on either side of the equator are approximately radial, and make sharp hairpin bends on the equator.

Long before the star reaches the main-sequence, the field far from the equator is approximately curl-free through most of the supersonic zone—the friction term in (4) is small compared with H/r. Further out this is no longer the case, although the irrotational field that satisfies the boundary conditions at infinity and at the star's surface does give a fair picture of the field at high latitudes for all radii. Near the equator, however, the curl-free approximation is hopelessly wrong, and nearly all the lines of force linked with the star pass through the region in which $\nabla \times \mathbf{H}$ is large before going off to infinity. The general shape of the field is given in Fig. 1.

Although the equatorial field is weak, the high curvature of the field lines yields a strong radial field on either side, exerting a strong transverse force toward the equator. This must be balanced by a steep pressure gradient: the situation is similar to the well-known "pinch effect."

As the start contracts the equatorial discharge (measured by $\nabla \times \mathbf{H}$) increases, and we may expect that the finite conductivity will ultimately allow a sufficiently rapid drift of plasma across the field for the gravitational pull to be balanced by inertial rather than magnetic force. At this stage a neutral point will form outside the star, but the field will still differ markedly from the curl-free field, both in its detailed structure and in the position of the neutral point.

The retarding effect on the neutral gas of the plasma-hydrogen friction is greatest in the equatorial plane,

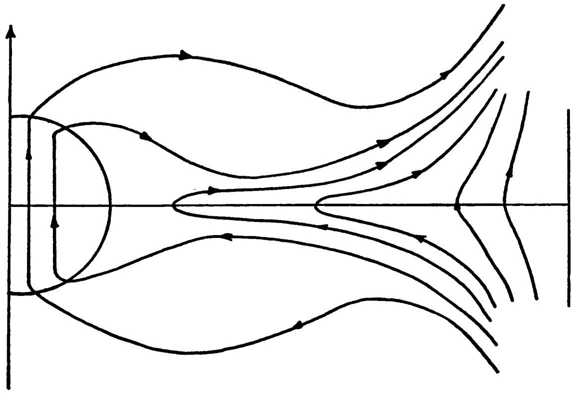

FIG. 1. Shape of the magnetic field near the contracting protostar.

and zero in the direction of $\mathbf{H}_\infty$. The high plasma density near the equator yields a large friction, which cuts down enormously the inflow of neutral matter through the plasma zone. Outside this zone an upper limit to the effect of the field on the accretion rate is given by assuming the friction at a given radius spherically symmetric, and equal to its value just outside the plasma zone. In the subsonic region the ratio of friction to gravitation is a constant which is small for the number density at infinity $N_\infty \ll 4\times10^3$. In the supersonic region the frction steadily increases until it just balances gravity; the neutral matter at low latitudes then flows in at approximately constant velocity, the accretion rate being maintained by an increase in density that just compensates the decrease in velocity below Bondi's value. Hence, for $N_\infty \ll 4\times10^3$, Bondi's ρ, v field is a good approximation in the subsonic zone, while in the supersonic zone (ρv) is the same as Bondi's value, except in the plasma zone. As ρ and v appear together as $(\rho v)^2$ in Eq. (4) for the supersonic zone, it follows that the magnetic field computed from (4) using Bondi's ρ, v field will be a good approximation except for clouds of very high density.

Once the star has reached the main sequence, further evolution of the field (in the absence of a dynamo mechanism within the star) is determined by Joule decay. Currents flow in the narrow surface region in which the field lines bend so as to link up smoothly with the external field; the dissipation of energy by the Ohmic field leads to a gradual leakage of lines of force out of the star. In the ultimate steady state the internal field will be curl-free and hence uniform, its value being given by that solution of the external field equations which reduces to $\mathbf{H}_\infty$ at infinity, and is parallel to $\mathbf{H}_\infty$ at the star's surface.

In this paper, isothermal conditions are assumed for the external matter. If the star is surrounded by an HII zone (due either to stellar ultraviolet radiation or to compressional and frictional heating of the inflowing gas) a new treatment is required, including a hydromagnetic shock front between the two zones, and a new equation to the field in the HII zone.

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A magnetic field is often appealed to as a means of reducing the angular momentum of a contracting system. A field with lines of force that are part of the local galactic field will transfer angular momentum to the cloud as a whole, presumably rather efficiently; by contrast, a broken-off field will not send out hydromagnetic waves far from the star, and so will be less efficient. For this reason it is important to determine the structure of the external field, and to decide how far the star must contract before its field lines begin to break off from the galactic field. Further work should take into account rotation in the star and cloud; we may anticipate that a strong centrifugal field will accelerate the detachment of the field.

A fuller account of this work will appear in the *Monthly Notices of the Royal Astronomical Society*.

DISCUSSION

G. C. McVittie, *University of Illinois, Urbana, Illinois:* I would appreciate a clarification of the problem treated, first with respect to the geometry of the magnetic field, and second as to how gravitational and hydromagnetic equations have been combined. Is the gravitational accretion problem solved first and then the magnetohydrodynamic equations used merely to calculate the corresponding permissible H?

L. Mestel, *Cambridge, England:* The magnetic field has symmetry about its direction at infinity. As a first-order approximation, to keep the problem manageable without the help of an electronic computer, I neglect the effect of a magnetic field on accretion velocities and use Bondi's computations on the inflow of neutral matter. I indicated that the magnetic field, computed on this assumption, is a good approximation. The equation of hydromagnetics is used by asserting that, except in the plasma zone, there is no relative drift of plasma and field. In a steady state, the magnetic force acting on the plasma is balanced by the transverse component of the friction; the friction results from the inflow of *neutral* matter across the field, but there is no flow of ionized matter across the field, except in the plasma zone.

A. Schlüter, *Max Planck Institut für Physik, Göttingen, Germany:* Do I understand correctly that you assume the equation of continuity to hold for the plasma by itself, so as to arrive at the high density of the plasma near the equatorial plane, rather than assuming, for instance, that the degree of ionization is kept constant by the processes of ionization and recombination?

L. Mestel: I am assuming an ordinary HI region where at large distances the plasma density is 10^{-4} the total density. This is approximately maintained as the material flows in everywhere, except near the equator, simply because the plasma is forced to follow the lines of force. Those lines of force which are not linked with the star lead to a plasma zone along the equator. The thickness of the zone is given by balancing the gravitational pull of the star against the partial pressure of the plasma. The density of the plasma is determined by the condition that the flow toward the equator just balances the much slower drift toward the star across the lines of force at the equator.

A. R. Kantrowitz, *Avco Research Laboratories, Everett, Massachusetts:* I wonder if the charge exchange mechanism is not perhaps more important as the mechanism of interaction between plasma and neutral gas than elastic collision, which is what I assumed you have been mostly taking into account. This may be modified by the fact that many of the ions might be sodium or something like this, but still a good many of them would be hydrogen, for instance.

A. Schlüter: Mestel's assumptions are essentially the same as in our paper presented by Biermann, namely, that the friction between the plasma and the neutral gas is simply proportional to the relative velocity of the two components as long as the diffusion velocity is subsonic. This will be a good approximation whatever the mechanism of friction is in detail—whether it is charge-exchange or there are elastic collisions controlling the exchange of momentum. On the other hand, in a typical HI region, most of the ions are indeed of sodium or carbon or something like this, and so the charge exchange is not too large.

M. P. Savedoff, *Department of Astronomy, University of Rochester, New York:* I take it that this is a mechanism which concentrates the un-ionized heavy components of the interstellar gas into the star and leaves carbon or silicon atoms, which are generally ionized, outside the star in the plasma zone. Is that correct?

L. Mestel: I think not, in that once this fairly high density has been built up, equilibrium is maintained by a slow drift across the lines of force toward the star. You will get an initial separation, but the mass in the plasma zone is small compared with the amount of plasma available in the cloud.