

# SOME PROBLEMS FOR GALACTIC HYDROSTATIC EQUILIBRIA

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ABSTRACT. Although locally a static, stable equilibrium seems consistent with observations of the interstellar medium (ISM), the extrapolation towards other galactic regions is not straightforward. Basically, the variation with galactic radius of the gravitational acceleration cannot be reconciled with the approximate constancy of the HI scaleheight. Moreover, halo gas located in the outer galaxy is prone to thermal instability and subsequent collapse towards the galactic plane.

## 1. INTRODUCTION

Ever since the classical work of Parker (1966,1969) on stratified equilibria for the galactic gas, magnetic fields (MF) and cosmic ray (CR) particles, the local hydrostatic equilibrium has been the subject of numerous stability analyses, often involving unrealistic simplifications. Bloemen (1987, hereafter Paper I) applied a general stability criterion for stratified equilibria (Lachièze-Rey *et al.* 1980) to a model based on an observational description of the ISM, and concluded that a stable hydrostatic equilibrium is not precluded by observations.

We have investigated similar models for  $R = 5$  and  $R = 15$  kpc ( $R_{\odot} = 10$  kpc), which are representative radii for the "inner" and "outer" galaxy. Detailed results will be presented elsewhere; here only the (im-)possibility of the radial extrapolation is discussed.

## 2. METHOD AND INGREDIENTS

From the equilibrium equation

$$\frac{d}{dz}[P_{GAS}(z) + P_{MF}(z) + P_{CR}(z)] = -\rho(z)g(z), \quad (1)$$

the total hydrostatic pressure

$$P_{TOT}(z) = \int_z^{\infty} \rho(x)g(x) dx \quad (2)$$

is calculated. Here  $P_{MF} = B^2/8\pi$  is ascribed to a systematic MF ( $\perp z$ , the distance to the galactic plane);  $P_{CR}$  is mainly due to CR protons. (Generally, the net tension due to a random MF can be neglected (*e.g.* Parker 1969).) The total density,  $\rho(z)$ , is the sum of the observed gas density distributions. For  $R = 5$  kpc and  $R = 10$  kpc we adopt the distributions listed in Table I. Note that, in contrast to Paper I, an ionized medium is now included (Reynolds 1989), with the same distribution for all considered galactocentric radii.

At  $R = 15$  kpc, the average HI density profile with respect to the warped mid-plane was derived from the density atlas of Burton and de Lintell Hekkert (1986), and molecular hydrogen was neglected.

For the acceleration perpendicular to the galactic plane,  $g(z)$ , we adopt Oort's (1960) determination; this is a good average of recently proposed curves and is consistent with the massive halo distribution ( $\epsilon(R)$  below) derived by Bahcall *et al.* (1982). The radial extrapolation of  $g$  is based on a constant vertical (*e.g.* van der Kruit and Searle 1982; van der Kruit and Freeman 1984; Lewis and Freeman 1989) and exponential radial (*e.g.* de Vaucouleurs and Pence 1978) stellar density distribution for spiral galaxies. The result is summarized by

$$g(R, z) \approx [\sigma^2(R)/z_0] \{ \tanh(z/z_0) + \epsilon(R)(z/z_0) \} \quad (3)$$

with  $z_0 = 250$  pc,  $\sigma^2(R) = (15.4 \text{ kms}^{-1})^2 \exp(-(R - R_\odot)/0.44R_\odot)$  and  $\epsilon = 0.04, 0.07$  and  $0.14$  for  $R = 5, 10$  and  $15$  kpc respectively.

The equilibrium is stable if  $P_{GAS} \geq -g\rho^2[\gamma(d\rho/dz)]^{-1}$ , assuming a very simple large scale equation of state for the gas:  $P \propto \rho^\gamma$  (we take  $\gamma = 1$ ). Model dependent stabilizing effects (CR diffusion, turbulent MF) are not taken into account, so that the above criterion defines a conservative lower limit on the gas pressure.

TABLE 1.

Parameters of the vertical gas distribution, for  $R = 5$  and  $R = 10$  kpc. Gauss:  $n(z) = n(0) \exp\{-\frac{1}{2}(z/h)^2\}$ ; Exponent:  $n(z) = n(0) \exp\{-z/h\}$ .

Component	Distribution	$n(0)$ ( $\text{cm}^{-3}$ )	$h$ (pc)	$\sqrt{\langle V_z^2 \rangle}$ ( $\text{km s}^{-1}$ )
Cold HI	Gauss	0.3	135	6-7
Warm HI	Gauss	0.07	135	9-11
	Exponent	0.1	400	9-15
H <sub>2</sub>	Gauss	1.6*-0.6**	60*-70**	5-6
HII	Exponent	0.025	1500	~ 20

\*( $R = 5$  kpc) \*\* ( $R = 10$  kpc)

### 3. OBSERVATIONAL CONSTRAINTS

#### 3.1 Mid Plane Pressures

The various pressure contributions of the ISM near the galactic plane are listed in Table II.

The gas pressure is the sum of the turbulent and thermal contributions of all gas phases. Note that the velocity dispersion probably increases with latitude (*e.g.* Kulkarni and Fich 1985); therefore the last column of Table I gives a *range* of dispersion values encountered in literature.

The radial unfolding of the *COS-B* and *SAS-2*  $\gamma$ -ray data, indicates that the scalelength of both CR electrons and protons amounts to  $\simeq 15$  kpc (Bloemen 1989). Together with the 4 kpc radial scalelength of the synchrotron emissivity (Beuermann *et al.* 1985), this implies a scalelength of  $\simeq 5$  kpc for  $B^2$ . The radial gradient of  $P_{MF+CR}$  thus derived is based on energy independent scalelengths for the CR particles. For energy dependent CR distributions the gradient will generally be smaller.

#### 3.2 Radio Continuum Data

The radio continuum intensity towards the galactic poles can be reconciled with a stable equilibrium if a gaseous halo is included. For an exponential halo distribution the constraint  $(n_{halo}(z=0)/0.01 \text{ cm}^{-3})(H_{halo}/1 \text{ kpc})^2 > \Gamma$  was derived in Paper I. When an ionized medium is included, we find  $\Gamma \simeq 8$ , ( $\sim$  half the value found by Bloemen). In short, the models of Paper I required the halo thermal pressure for support of the colder gas components, thus allowing a long tail in the MF and CR distribution (albeit constrained by the stability criterion).

### 4. RESULTS AND DISCUSSION

We find that, within the uncertainty of our knowledge of the ISM parameters and their radial variation, a locally consistent hydrostatic equilibrium cannot be extrapolated to other  $R$ . Table II learns that, near  $z = 0$ , a discrepancy of  $\sim$  a factor 2 exists at  $R = 5$  kpc between hydrostatic and seemingly available pressure, even when the CR and MF pressures are maximized. More conservative values of  $P_{MF+CR}$  imply a discrepancy larger than a factor 3! The problem cannot be solved by altering the local parameters under the constraint of local equilibrium. If the "missing" pressure at small  $R$  is contributed by a halo gas, the minimum halo temperature must be  $4.5 \times 10^6$  K, implying an overpressure  $>$  a factor 20 with respect to the other ISM gas components. Moreover, the halo cooling rate versus possible supernovae energy input (*e.g.* Heiles 1987) cannot be rendered consistent if the halo mid-plane density and scaleheight are weak functions of galactocentric radius. Particularly, halo gas located at large  $R$  will cool significantly in  $\sim 10^6$  yr,

TABLE 2.  
Pressures at  $z = 0$ , in  $10^{-12}$  dyne  $\text{cm}^{-2}$  (corrected for He).

$R$ (kpc)	5	10	15
$P_{TOT}^*$	15.5	4.8	2.4
$P_{MF}$	2.7–5.2	1.0–1.9	0.4–0.7
$P_{CR}$	0.7–1.4	0.5–1.0	0.4–0.7
$P_{GAS}$	1.8–2.7	1.3–1.9	0.9–1.9

\* (from equation (2), excluding halo gas)

even if all available SNR power is dissipated in the halo; the estimated ISM pressure at  $R = 15$  kpc is however consistent with the hydrostatic pressure.

The hydrostatic equilibrium condition thus seems to be violated on a galactic scale, if the gravitating material has a constant scaleheight. Only if the gas velocity dispersion is strongly dependent on  $R$ , simple hydrostatics can be maintained. This result does not depend on stability considerations. The stability criterion mainly prescribes the halo density and temperature profile (which depends also strongly on the gas velocity dispersions). We want to stress, that the found discrepancies may be partly rooted in the naive application of equation (1), which ignores the details of gas-phase interactions in the ISM.

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#### REFERENCES

- Beuermann, K., Kanbach, G., and Berkhuyzen, E.M. 1985, *Astr. Ap.* **153**, 17  
 Bloemen, J.B.G.M. 1987, *Ap. J.* **322**, 694 (Paper I)  
 Bloemen, J.B.G.M. 1989, *Ann. Rev. Astr. Ap.* **27**, 469  
 Burton, W.B., and te Lintel Hekkert, P. 1986, *Astr. Ap. Suppl.* **65**, 427  
 Heiles, C. 1987, *Ap. J.* **315**, 555  
 van der Kruit, P.C., and Freeman, K.C. 1984, *Ap. J.* **278**, 81  
 van der Kruit, P.C., and Searle, L. 1982, *Astr. Ap.* **110**, 61  
 Kulkarni, S.R., and Fich, M. 1985, *Ap. J.* **289**, 792  
 Lachièze-Rey, M., Asséo, E., Cesarsky, C.J., and Pellat, R. 1980, *Ap. J.* **238**, 175  
 Lewis, J.R., and Freeman, K.C. 1989, *Astron. J.* **97**, 139  
 Oort, J.H. 1960, *Bull. Astr. Inst. Netherlands* **15**, 45  
 Parker, E.N. 1966, *Ap. J.* **145**, 811  
 Parker, E.N. 1969, *Space Sci. Rev.* **9**, 651  
 Reynolds, R.J. 1989, *Ap. J. (Letters)* **339**, L29  
 de Vaucouleurs, G. and Pence, W.D. 1978, *Astron. J.* **83**, 1163