PHASE-TYPE APPROXIMATIONS TO FINITE-TIME RUIN PROBABILITIES IN THE SPARRE-ANDERSEN AND STATIONARY RENEWAL RISK MODELS

BY

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Abstract

The present paper extends the "Erlangization" idea introduced by Asmussen, Avram, and Usabel (2002) to the Sparre-Andersen and stationary renewal risk models. Erlangization yields an asymptotically-exact method for calculating finite time ruin probabilities with phase-type claim amounts. The method is based on finding the probability of ruin prior to a phase-type random horizon, independent of the risk process. When the horizon follows an Erlang-l distribution, the method provides a sequence of approximations that converges to the true finite-time ruin probability as l increases. Furthermore, the random horizon is easier to work with, so that very accurate probabilities of ruin are obtained with comparatively little computational effort. An additional section determines the phase-type form of the deficit at ruin in both models. Our work exploits the relationship to fluid queues to provide effective computational algorithms for the determination of these quantities, as demonstrated by the numerical examples.

Keywords

Sparre-Andersen model, ladder height, maximal aggregate loss, deficit at ruin, phase-type distribution.

1. Introduction

The present paper is concerned with the determination of the probability of ruin in finite time in the Sparre-Andersen risk model and the stationary renewal risk model (also known as the stationary Sparre-Andersen model).

Traditionally, the exact determination of finite time ruin probabilities, in both the classical and the Sparre-Andersen risk models, has required the solution of rather complicated integro-differential equations. Explicit formulae for the probability of ruin exist for a limited number of cases, such as the classical model with exponential claim sizes. Even then, the form of the solution entails the evaluation of Bessel functions (see, for instance, Drekic & Willmot (2003)).

than the regular values for $l = 7$ when $T = 10$, which is atypical. Furthermore, the extrapolated value for $l = 6$ when $T = 50$ actually exceeds the ultimate ruin probability very slightly. Nonetheless, we would submit that the accuracy of the results for $l = 7$ is already quite sufficient for any decision making purpose.

The numbers for this second example were run completely using the more time-consuming Mathematica code, yet still each value for $l = 7$ ran in about 30 seconds.

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