DEPARTURE POINT, EARTH'S RATE OF ROTATION AND COORDINATE TRANSFORMATION IN QUASI-INERTIAL GEOCENTRIC EQUATORIAL COORDINATE SYSTEM (QIGECS)

H.J. Yan and E. Groten
Institute of Physical Geodesy, Technical University Darmstadt
Petersenstr.13, 6100 Darmstadt, FRG

__ABSTRACT__. The paper summarizes the discussion on the origin of right-ascension and puts forward new arguments in view of high-precision Geodesy and Astrometry. From the movement of the Celestial Departure Point, the classical right-ascension precession might be amended by an additional term $-0^s.000257$/century originating from the nutation-precession interaction movement. A similar term might also be introduced in the maintenance of a terrestrial reference system, while the concept of a Terrestrial Departure Point is considered. The definition of the Earth's rate of rotation in an inertial or quasi-inertial system is reviewed. A periodic erroneous term of maximum amplitude 2.65*mas* is pointed out in the conventional transfer relation between CRS and TRS, that can for its main part be compensated by introducing the periodic terms of Woolard's equation of the equinox.

1. INTRODUCTION

According to Aktinson's research the reference pole of the Earth altered from the previous conventional rotation pole to the Celestial Ephemeris Pole (CEP) both in theoretical computation of nutation and in practical routine. A new consideration of the origin of right-ascension of the equatorial coordinate system was first discussed by Guinot (1979), and its mathematical developments can be found in (Capitaine et al., 1987). As a matter of opinion, Aoki and others discovered that this "non-rotating point" was conceptually included in the construction of the Departure Point defined on the moving instantaneous true equator (Aoki and Kinoshita, 1983; Aoki, 1988). In the frame of the Proper Reference System (PRS) (Murray, 1983), as a new origin of right-ascension on the moving true equator the definition, mathematical representation and properties of the Departure Point in the Quasi-Inertial Geocentric Equatorial Coordinate System were investigated in detail by Yan and Groten (1992a) (abbreviated by YGa). A further discussion on the relationship between the Departure Point and Earth Rotation can be found in (Yan and Groten, 1992b) (abbreviated by YGb).

2. DEPARTURE POINT IN QIGECS AND NUTATIONAL PRECESSION IN RIGHT-ASCENSION

With respect to remote radio sources, Guinot's instantaneous celestial

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"non-rotating reference system" was defined as if, geometrically speaking, there were no components of rotation around the instantaneous axis. The "terrestrial non-rotating reference system" was similarly realized by the condition that there were no components of rotation around the instantaneous axis while it drifts on the surface of a two-dimensional Earth-fixed reference system.

After considering the ways of defining the celestial reference systems both in astronomy and in mathematics, Yan and Groten (YGa) pointed out that it was probably more reasonable to relate this new equatorial spherical coordinate system geometrically to the true or mean equatorial coordinate system at epoch T_0, and physically to PRS at date T.

Because of the effect of the parallel transport of PRS to the Solar System Barycentric Reference Frame (SSBRF) (Yan et al., 1990), we have to argue about the necessity and reasonableness in establishing a new geocentric celestial reference system to be geometrically fixed to the most remote bodies in universe. Yan and Groten (YGa) proposed a new definition of the origin of right-ascension: *the Celestial Departure Point (abbreviated by the Departure Point), which can be related to either the true or the mean equinox at epoch, will be the origin of right-ascension of the local quasi-inertial reference system on the moving true equator.* Physically speaking, with respect to such a Quasi-Inertial Geocentric Equatorial Coordinate System (abbreviated by QIGECS), the Earth rotates as if there were no torques around the selected instantaneous axis for a rigid Earth (Aoki, 1988). After employing the 1980 IAU Theory of Nutation, CEP becomes the unique choice as the reference pole of QIGECS. We may say that QIGECS is a kind of geocentric equatorial coordinate system which is related to PRS as close as possible.

Assuming the mean equator of epoch T_0 as the fundamental great circle the true right-ascension of the Departure Point of QIGECS can be expressed as (YGa):

$$\alpha_d = Z^* + \int_{T_0}^{T} \cos\theta^* \, d\zeta^* + \int_{T_0}^{T} \Omega \cos\epsilon' \, dT \ , \tag{1}$$

here, differed from the conventional usage, Z^* θ^* ζ^* are referred to the mean equator of epoch and the true equator of date:

$$Z^* = Z + \Delta Z^* \ , \quad \zeta^* = \zeta + \Delta\zeta^* \ , \quad \theta^* = \theta + \Delta\theta^* \ ; \tag{2}$$

and ϵ' is the true obliquity of date. The additional term Ω in Eq.(1) originates from the effect of parallel transport of PRS to SSBRF and is numerically equal to the value of geodesic precession: $\Omega \approx 1''92$/century. Similar to s, as introduced by Guinot (1979), the quantity s^* has here the form:

$$s^* = \int_{T_0}^{T} (1 - \cos\theta^*) \, d\zeta^* - \int_{T_0}^{T} \Omega \cos\epsilon' \, dT \ . \tag{3}$$

The values of $\Delta\theta^*$, $\Delta\zeta^*$, ΔZ^* can be obtained from the solutions of equation:

$$NP = R_1(-\epsilon - \Delta\epsilon) R_3(-\Delta\psi) R_1(\epsilon) R_3(-Z) R_2(\theta) R_3(-\zeta) = R_3(-Z^*) R_2(\theta^*) R_3(-\zeta^*) \ , \tag{4}$$

here $\Delta\epsilon$ and $\Delta\psi$ are the nutations in obliquity and longitude, respectively.

The quantity s^* can be formally written as:

$$s^* = s' - \Omega cos\epsilon_0(T-T_0) \approx \frac{1}{2}\int_{T_0}^{T}\theta^2 d\zeta + (s'_n)_s + (s'_n)_r - \Omega cos\epsilon_0(T-T_0) \ ; \tag{5}$$

in which the quantity s' is used corresponding to the quantity s originally applied by Guinot (1979). On the right side of Eq.(5), the first term originates from precession-precession which appears as a part of precession in right ascension in the conventional precession theory; $(s'_n)_r$ includes periodic and resonant parts coming from nutation-precession and nutation-nutation terms; $(s'_n)_s$ is a linear function being introduced only by nutation-nutation term:

$$(s'_n)_s = \frac{sin\epsilon}{2}\sum_i \Delta\psi_i \Delta\epsilon_i \frac{2\pi k_i}{T_i}(T-T_0) \ , \tag{6}$$

here T_i, $\Delta\epsilon_i$ and $\Delta\psi_i$ are the period, components of nutation in obliquity and longitude, respectively; k_i has the symbol of angular frequency of i-th nutation component, which takes the value +1 or -1. The magnitude estimation of terms in Eq.(6) was listed in Table 1 in (YGa). In QIGECS, the corresponding coordinate transformation of a vector $\underline{r}$ is written as:

$$\underline{r}^* = R_3(\zeta^*-s^*)R_2(\theta^*)R_3(-\zeta^*)\underline{r}_0{}^* \ . \tag{7}$$

The relation between the true right-ascension $\alpha(T)$ in the instantaneous true equatorial reference system and the instantaneous right-ascension A(T) in QIGECS is written as:

$$A(T) = \alpha(T) - (Z^*+\zeta^*-s^*) \ . \tag{8}$$

The expression $(Z^*+\zeta^*-s^*)$ is the true right-ascension of the Departure Point $\alpha_d(T)$ in Eq.(1). After eliminating the effect of the geodesic precession and taking the average value of distance between the Departure Point and true equinox, the accumulated precession in right-ascension in QIGECS might be written as:

$$M^*(T) \equiv \langle Z^*+\zeta^*-s^*+\Omega cos\epsilon_0(T-T_0)\rangle \equiv \int_{T_0}^{T} m^*(T)dT = Z + \zeta - \frac{1}{2}\int_{T_0}^{T}\theta^2 d\zeta - (s'_n)_s$$

$$= 307^s496_{178}T' + 0^s093_{104}T'^2 - 6^s2\cdot 10^{-6}T'^3 \ , \tag{9}$$

in which $m^*(T')$ is defined as the rate of precession in right-ascension in QIGECS, symbol $\langle\rangle$ denotes the average value, T' is the number of Julian centuries of dynamical time elapsed since JD2451545.0. In comparison with the conventional definition of precession in right-ascension $M(T')$ (Aoki et al., 1982); $-(s'_n)_s$ is a nutational precession in right-ascension:

$$\Delta M(T') = M^*(T') - M(T') = -(s'_n)_s = -3.85(T'-T_0{}')mas/cy \ . \tag{10}$$

3. THE COORDINATES OF STATIONS IN THE EARTH-FIXED REFERENCE SYSTEM

From Guinot's "terrestrial non-rotating reference system" or the Instantaneous Quasi-Terrestrial Reference System (IQTRS) as denoted by Yan and Groten (YGa), a Terrestrial Departure Point on the instantaneous terrestrial equator should have similar definition and properties to the Departure Point. Under the accuracy requirement of 0.01*mas* per century and taking the movements of the Terrestrial Departure Point into account, the wobble matrix W(T) might be amended as:

$$W(T) = R_2(-x_p)R_1(-y_p)R_3((s_T)_s) \quad . \tag{11}$$

The quantity $(s_T)_s$ should only include the <u>intrinsic part</u> in quantity s_T:

$$(s_T)_s = \frac{1}{2}\int_{T_0}^{T} (\dot{x}_p y_p' - x_p' \dot{y}_p)\,dT \quad , \tag{12}$$

wherein quantities x_p' and y_p' are measured from the barycentres of the polar motion's track (the mean rotation axis in IERS Terrestrial Reference Frame), and not from the reference pole of the Terrestrial Reference System (the IERS Reference Pole). Therefore, the quantity $(s_T)_s$ in Eq.(12) is also independent of the choice of the reference pole of TRS, of which the main part is nearly linear and has the relation with the amplitudes of the periodic polar motion:

$$(s_T)_s = \tfrac{1}{2}\sum_i x_{pi} y_{pi} \omega_{pi} (T-T_0) \quad , \tag{13}$$

here x_{pi} , y_{pi} are the amplitudes of i-th periodic polar motion components in x- and y-direction, respectively, and are taken here as average values in the period from T_0 to T; ω_{pi} is the corresponding angular frequency.

It might be better, in our opinion, to take the term $(s_T)_s$ as a secular drift of the Zero-Longitude of IQTRS to be added to the coordinates of stations. The station's coordinates of date in IQTRS is then written as:

$$\underline{R}^*(T) = [\underline{I}+\Delta\underline{L}_0(T)]\times[\underline{R}^*(T_0)+\Delta\underline{R}(T)] \quad , \tag{14}$$

in which $\underline{R}^*(T_0)$ means the coordinates of reference epoch, $\Delta\underline{R}(T)$ represents the ordinate correction parts originating from the local and global crustal movements and deformations caused by astronomical and geophysical factors such as the tidal action, plate motion etc.; $\underline{I}$ is an unit vector and:

$$\Delta\underline{L}_0(T) = -(s_T)_s\underline{k} \tag{15}$$

is the Zero-Longitude correction of IQTRS; $\underline{k}$ is an unit vector pointing towards the direction of the IERS Reference Pole.

4. RELATION BETWEEN [TRS] AND [CRS]

The classical transformation matrix from [CRS] to [TRS] has a form:

$$[TRS] = R_2(-x_p)R_1(-y_p)R_3(GAST)NP(T)[CRS] \quad ; \tag{16}$$

in which GAST is Greenwich Apparent Sidereal Time at date T, NP(T) means the normal precession-nutation matrix.

In Eq.(16) the nutation and precession are considered separately and there is a small inaccuracy in the precession in right-ascension (Aoki, 1988). The transformation formula from QIGECS to IQTRS is rewritten as:

$$[TRS]^{*} = W'R_3((s_T)_s)R_3(\phi_A)R_3(\zeta^{*}-s^{*})R_2(\theta^{*})R_3(-\zeta^{*})[CRS]^{*} \;; \tag{17}$$

here Eq.(14) is employed and the wobble matrix W' implies the common form:

$$W' = R_2(-x_p)R_1(-y_p) \;;$$

ϕ_A is the Earth rotation angle with respect to the Departure Point of QIGECS and numerically equals

$$\phi_A = GMST + (Z+\zeta-\tfrac{1}{2}\int\theta^2 d\zeta) - [(s'_n)_s-\Omega cos\epsilon_0(T-T_0)] \;. \tag{18}$$

The conventional relationship between GMST and GAST is:

$$GAST = GMST + \delta\alpha_n = GMST + \Delta\psi cos(\epsilon+\Delta\epsilon) \;, \tag{19}$$

Except for the systematic discrepancy caused by the selection of the reference point of the Earth's rate of rotation that has been discussed in (YGb), the remaining difference between Eqs.(16) and (17) can be written as

$$\delta R_3 = R_3(\Delta Z^{*}+\Delta\zeta^{*}-(s'_n)_r-\delta\alpha_n) \;. \tag{20}$$

This inaccuracy of the conventional coordinate transformation from [CRS] to [TRS] has been incorporated in the present observation series of UT1 time or in the determination of the nutation constants. By solving Eq.(4) we have:

$$\Delta Z^{*}+\Delta\zeta^{*} = (cos\epsilon\Delta\psi-sin\epsilon\Delta\psi\Delta\epsilon)-\theta\frac{\Delta\epsilon}{2}+\frac{sin\epsilon\Delta\psi\Delta\epsilon}{2} \;. \tag{21}$$

and:

$$\delta R_3 \approx R_3(-\int_{T_0}^{T}[\Delta\epsilon d\theta-sin\epsilon\Delta\psi d(\Delta\epsilon)]-(s'_n)_s) \;. \tag{22}$$

The relation between GAST and GMST seems to be rewritten as:

$$GAST = GMST+\Delta\psi cos(\epsilon+\Delta\epsilon)+0''00265 sinL+0''00006 sin2L \;, \tag{23}$$

here L means the longitude of the mean ascending node of the lunar orbit on the ecliptic measured from the mean equinox of date.

5. EARTH'S RATE OF ROTATION IN QUASI-INERTIAL SYSTEMS

The present definition of Universal Time UT1 is based on a fictitious mean Sun on the mean equator of date (Aoki et al., 1982):

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$$GMST1(0^hUT1) = 24110^s548_{41} + 8640184^s812_{866}T'_U + 0^s093_{104}T'^{2}_U - 6^s_2 \cdot 10^{-6}T'^{3}_U \quad , \tag{24}$$

here: $T'_U = d'_U/36525$, and d'_U represents the number in days of Universal Time elapsed since JD2451545.0UT1, taking on values of ±0.5, ±1.5····.

Eq.(24) originates from the consideration that we should relate Universal Time to the Earth rotation and should keep the convenience of using the mean length of solar day as the time unit in practice and scientific research. This conventional way of definition of Universal Time UT1 had been criticized sometimes (Capitaine et al., 1987).

It has been shown that if the same precession theory is employed, the selection of the reference point of the Earth's rate of rotation does not intrinsically change the accuracy of the definition of Universal Time and does introduce only a formal systematic difference in the expression of Universal Time. According to the different considerations of the Departure Point, the different relationships between Universal Time UT1 and the corresponding Earth Rotation Time (ERT) are investigated in detail in (YGb).

It might be considered to relate ERT to the rotation angular velocity referred to the Departure Point or simply related to ϕ_A in Eq.(18). With respect to the mean equinox at epoch J2000.0, the relationship between Universal Time UT1 and the Greenwich Earth Rotation Time (GERT) is generally written as:

$$GERT = 67310^s548_{41} + (876600^h + \beta)T'_U \quad , \tag{25}$$

or:

$$T_{UT1} = \lambda(T_{GERT} - 67310^s548_{41}) + 12^h \quad ; \tag{26}$$

here T_{GERT} represents a continuous time number of GERT which is defined as:

$$T_{GERT}(J2000.0) = 67310^s548_{41} \quad , \tag{27}$$

T_{UT1} is also a continuous time number of UT1 which satisfies the definition

$$T_{UT1}(2000 \text{ January } 1,\ 0^hUT1) = 0 \quad , \tag{28}$$

and in QIGECS the coefficients β and λ have the values:

$$\beta = 8639877^s199_{938} \quad , \qquad \lambda = 0.99726966327_{417} \quad . \tag{29}$$

6. CONCLUSION

In (YGa, YGb) we have analyzed the properties and the applications of the Departure Point and have compared the physical definitions and mathematical relationship between the Departure Point and the conventionally used equinox. In our opinion, at present, the equinox will keep its dominant role as the origin of the fundamental reference system. But it is surely significant that the discussion of the Departure Point has brought us some new and fresh understanding in Astrometry and Geodesy.

From QIGECS, the conventional precession in right-ascension should include an additional nutational right-ascension precession originating from the nutation-nutation terms of the position of the Departure Point. This term would further cause a new consideration of the the Earth's rate of rotation in a quasi-inertial system. The parallel transformation in the

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general theory of Relativity also causes a difference in the definition of the Departure Point in QIGECS.

Similar to the Departure Point of QIGECS, the Terrestrial Departure Point might introduce a secular drift of the Zero-Longitude of the IQTRS, of which the calculation formula is reconsidered in detail in this paper.

Aoki (1988) noticed the problem that the conventional transformation relationship between [CRS] and [TRS] should be amended by introducing the Departure Point. In this paper, the errors of the conventional formulae have been reestimated in the range of the square terms of nutation. These terms can essentially be compared with Woolard's equation of the equinox (Woolard, 1953).

In both, conventional equatorial coordinate system and QIGECS, we could get exactly the same results if the same nutation theory were employed. The adoption of the Departure Point does not completely free us from the errors in precession-nutation parameters. Because the Departure Point has only an one-dimensional physical restraint, it cannot directly be observed kinematically and improved dynamically. As a supplementary method, the Departure Point could be used in astronomical practice, but it may not ultimately replace the equinox.

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