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ON AN INTEGRAL FORMULATION FOR HEAT-DIFFUSION MOVING BOUNDARY PROBLEMS

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Moving boundary problems involving heat conduction or diffusion occur in many practical situations and are generally referred to as Stefan problems. In this thesis the classical one dimensional Stefan problems for planar and inward cylindrical and spherical phase change problems are considered. Also, three extensions of these problems are considered. These are a problem involving both a fast and slow chemical reaction, the melting or freezing of a binary mixture with two distinct fusion temperatures, and finally the melting or freezing of a pure material which is not initially at its fusion temperature. For each of these problems we develop an integral formulation, which is related to the weak or enthalpy formulation, and which leads to a new formal integral for the boundary motion. In many practical contexts a simple analytic approximation is more revealing than a numerical result, and the formal integral is exploited to obtain simple analytic bounds for the boundary motion. These bounds are particularly tight in the context of the simultaneous chemical reaction problem, where an equivalent accuracy could not be achieved by either series approximation or numerically without extensive computation.

The first three chapters of this thesis contain an introduction and literature survey for Stefan problems, and then deal with the classical

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single phase Stefan problems for slabs, cylinders and spheres. The integral formulation is obtained by two different methods, and an approximate iterative analytic technique arising from the integral formulation is discussed. Formal series solutions for the single phase Stefan problem are also derived from the integral formulation. Extensions to Stefan problems with time dependent boundary conditions, and radially or temperature dependent thermal properties are considered briefly, and the relationship of the integral formulation to the enthalpy formulation is described. Simple bounds for the moving boundary are obtained from the new formal integral for the boundary motion. These bounds are improved by showing the standard pseudo steady state temperature to be an upper bound on the actual temperature and by finding an upper bound for the speed of the moving boundary. The relationship of these bounds to the approximate boundary motions arising from large Stefan number perturbation expansions is discussed. The utility of the bounds is considered by comparison with exact and numerical solutions, obtained from a finite difference enthalpy method. Possible approaches for obtaining further refinements for the bounds are noted.

The final three chapters deal respectively with the three problems cited above. First, we consider a single phase moving boundary problem involving two simultaneous chemical reactions and find the appropriate integral formulation and bounds for the boundary motion. For two component mixtures, we obtain a formal integral relating the motion of the two moving boundaries, but in the absence of another independent relationship between the boundaries, it is not possible to bound the moving boundaries. From the enthalpy formulation, however, it is possible to obtain the generalization of the formal integral, relating the boundary motions for an n-component mixture. These integrals may be used as an independent check on the accuracy of a numerical scheme. In the final chapter a variety of genuine two phase problems are considered, and bounds for the boundary motion are deduced The utility of the bounds and procedures for refining them are discussed and the relationship to the enthalpy formulation is noted. Finally, results for Langford's heat functions, obtained from the integral formulation given in this thesis, are developed in the appendix.

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