BOUNDARY LAYER IN NONLINEAR DYNAMO

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Abstract. The solution of the boundary layer problem in a nonlinear galactic dynamo is described.

Key words: nonlinear dynamo – boundary layer

The generation of the large scale magnetic field in a thin disk galactic dynamo is investigated. The linear or so called kinematic theory describing the initial stages of the field generation is known. For the subsequent stages one needs to consider a nonlinear model of the disk dynamo. The use of asymptotic methods is a fruitful way for investigating this. We shall consider steady states of the large scale magnetic fields. Let us add nonlinearity taking the helicity function as

$$\alpha(z,B) = \alpha_0(z)\,[\,1 - g(z)\,B^2\,] \tag{1}$$

where $\alpha_0(z)$ is the helicity distribution of the linear model and $g(z)$ is a slowly changing function of z (see Kvasz et al. 1992). Following Vainstein and Ruzmaikin 1972 we derive the differential equation for the azimuthal component B of the magnetic field.

$$\frac{d^3}{dz^3}B - D\,\alpha(z,B)\,B = 0 \tag{2}$$

where D is the dynamo number with the opposite sign. The boundary conditions for (2) in the case of the disk of the half-thickness 1 surrounded by vacuum are:

$$B(1) = 0, \qquad \frac{d^2}{dz^2}B(1) = 0, \qquad \frac{d}{dz}B(0) = 0. \tag{3,4,5}$$

Assuming the dynamo number D to be large and using asymptotic methods we search the solution of the boundary problem (2-5) in the form:

$$B = [g(z)]^{-1/2} + \Phi_0(x) + D^{-\eta}\,\Phi_1(x) + \dots . \tag{6}$$

Here the first term is a regular solution, the other terms correspond to the boundary layer. Here $x = (z-1)\,D^{\kappa}$ is a new fast variable, η and κ are constants. The boundary layer is introduced to fulfill the boundary conditions (3-4) at the point $z = 1$. The only regular solution cannot fulfill them. The characteristic thickness of the boundary layer is $1\,/\,D^{\kappa}$. Substituting (6) into (2-5) gives us $\kappa = 1/3$ and $\eta = 2/3$. To calculate the boundary layer we introduce a new function $\Psi(x) = -[\,g(1)\,]^{1/2}\,\Phi_0(x)$ and a new variable $t = -[\,\alpha(1)\,]^{1/3}x$ (7). Thus, we obtain an initial value problem for the nonlinear differential equation:

F. Krause et al. (eds.), The Cosmic Dynamo, 355–356.

$$\frac{d^3}{dt^3}\Psi = \Psi^3 - 3\Psi^2 + 2\Psi\,, \tag{8}$$

$$\Psi(0) = 1, \quad \frac{d^2}{dt^2}\Psi(0) = 0, \quad \Psi(\infty) = 0\,. \tag{9,10,11}$$

The main difficulty here is an infinity in the boundary condition (11). We change it to $d\Psi\,/\,dt = p$ at the point $t = 0$ and solve further an initial value problem for the several values of the parameter p. Depending on the values of p we obtain 6 qualitatively different cases of behaviour of the solution (see figure below the text). The cases 1-3 can be completely explained by linearization of (8) near to the point $\Psi = 1$. They never fulfill the condition (11) and are thus not of interests. The cases 4-6 can be explained by linearization of (8) near to the point $\Psi = 0$. The linearized solution has the form

$$\Psi(t) = C_1\,e^{\lambda_1 t} + e^{\lambda_2 t}(C_2 \sin\omega t + C_3 \cos\omega t)\,,$$

where $\lambda_1 = 2^{1/3}$, $\lambda_2 = -2^{-2/3}$, $\omega = -2^{-2/3}\,3^{1/2}$, C_1, C_2 and C_3 are constants. The unique case fulfilling condition (11) is 5. The value of the parameter p_0 is $-0.684\,781\,644\,265\,300 \pm 3\,10^{-15}$. This high precision is very important. If a lower precision of e.g. 6 digits is used, only one oscillation can be seen and it is not possible to study the phenomenon qualitatively. The increase of precision extends the interval of the oscillations but nevertheless the numerical solution goes further to $+$ or $-\infty$. The frequency of oscillation and degree of relaxation of the numerical solution completely correspond to λ_2 and ω. Thus, one can numerically calculate the boundary layer of the nonlinear dynamo problem as exactly as is necessary.

References

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Figure
1. $p \geq 4 \cdot 10^{-4}$
2. $0 < p \leq 4 \cdot 10^{-4}$
3. $p = 0$
4. $p_0 < p < 0$
5. $p = p_0$
6. $p < p_0$