usefulness of the book for beginners. More specialized readers may find it quite useful.

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The Confluent Hypergeometric Function. BY HERBERT BUCHHOLZ. Springer-Verlag, New York (1969). 238 pp.

The book under review is the English translation of the original German edition which appeared in 1952. Since that time the text has become recognized as the standard treatise in the field. One of the more noteworthy features of the text is the emphasis placed on the physical applications of confluent hypergeometric functions to problems in mathematical physics, in particular the wave equation in parabolic coordinates. The author's viewpoint is basically that of Whittaker and hence aside from a few basic definitions and relationships for Kummer's function the text is devoted to the study of Whittaker functions.

Chapter 1 is concerned with the differential equations satisfied by the Whittaker and related functions, with applications to the wave equation. In Chapter 2 integral representations are obtained and used in Chapter 3 to derive asymptotic expansions. Chapter 4 is devoted to the derivation of a wide variety of series and integrals involving various special cases of the Whittaker functions. In Chapter 5 series of polynomials related to the Whittaker functions (such as those of Laguerre, Hermite, Charlier, etc.) are examined. In Chapter 6 integrals with respect to various parameters are discussed and applications are made to problems in wave propagation. Chapter 7 is concerned with the zeros of the Whittaker functions and to eigenvalue problems arising in physical applications. Included is a discussion of the Green's function for the reduced wave equation in a region bounded by confocal paraboloids of revolution. The book concludes with a summary of special cases of the Whittaker functions, which include cylinder functions, the incomplete gamma function, parabolic cylinder functions, the error and Fresnel integrals, Hermite and Laguerre polynomials, and many others.

The English translation of this important text is more than welcome and the reviewer feels it should occupy a prominent place on the bookshelf of every applied mathematician.

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Lie Theory and Special Functions. BY WILLARD MILLER, JR. Academic Press, New York (1968). xv+338 pp.

On the beginner, the theory of special functions makes the impression of a chaos of formulae which are not connected with each other by some general ideas. A systematic approach to this theory begins with the papers of E. Cartan in which he investigates the zonal spherical functions of representations of Lie groups. For the group $SO(n)$ these functions are expressed by the Gegenbauer-polynomials, for the motion groups of the n-dimensional euclidean space by the Bessel functions, etc. By means of special functions one can also express the matrix elements of the irreducible representations of Lie groups in a suitably chosen basis (usually chosen in such a way that to the elements of a certain subgroup correspond block-diagonal matrices). Such an approach clarifies the common origin of a great number of relations between the functions. For instance the addition theorems arise from the relation $T(g_1g_2)=T(g_1)T(g_2)$, and if the element g_2 is "infinitely near to the unit element" they become recurrence relations. Differential equations arise from Laplace operators on groups and homogeneous manifolds, and orthogonality relations from the orthogonality of the matrix elements of the irreducible unitary representations. These connections are presented in the reviewer's book "Special Functions and the Theory of Representations of Lie Groups."

In the book under review, the author W. Miller has chosen another group-theoretical approach to the theory of special functions. He constructs different realizations of representations of Lie algebras in the form of analytic differential operators in function spaces, analytic in a certain neighbourhood of some point. Further he selects an analytic basis in this space and with respect to this basis he represents the operators of the representation. In this way arise the following problems.

(1) Classify "all" the representations of a Lie algebra,

(2) Classify "all" the realizations of a representation of a Lie algebra in the form of generalized Lie derivatives,

(3) Select a suitable analytic basis in the space of a representation and find the matrix elements of the representation with respect to this basis.

In Miller's book these problems are solved for the Lie algebra $G(a, b)$ with four generators g^+, g^-, g^3, E and the commutator relations

$$[g^+, g^-] = 2a^2g^3 - bE$$

$$[g^3, g^+] = g^+, \qquad [g^3, g^-] = -g^-$$

$$[g^+, E] = [g^-, E] = [g^3, E] = 0.$$

For $a \neq 0$ this algebra is isomorphic to the algebra sl (2) $\oplus$ (E) and for $a=0$, $b \neq 0$ and $a=0$, $b=0$ to the algebra which are obtained from sl (2) $\oplus$ (E) by contraction.

The representations of the algebra $G(a, b)$ are realized in spaces of functions of one and two variables. Under certain additional conditions concerning the form of the operators all operator representations and the corresponding bases are established. The bases are expressed by means of hypergeometric functions (Laguerre and Hermite polynomials, functions of the parabolic cyclindre, etc.). In Chapters 3–5 the realizations of the algebras $G(1, 0)$, $G(0, 1)$, $G(0, 0)$ are discussed in detail together with the corresponding relations for special functions. Thus from the formula $T(g)e_k = \sum_m t_{km}(g)e_m$ are obtained the generating functions for various classes of special functions. A great number of relations arise from the decomposition of Kronecker products of irreducible representations. These and certain other results are due to the author of this book and appear for the first time in the monograph literature. The author also has established an interesting connection between the realizations of a Lie algebra and the Infeld-Helly method of factorization for the solution of differential equations.

There is no doubt that Miller's book will be of great interest and usefulness for the specialists in group representation theory and in special functions. It will also be useful for physicists since the Lie algebras dealt with in this book often occur in physical applications. It is to be regretted that certain aspects connected with the problems of the work, in particular the connections between representation theory and integral transforms, could not be taken into consideration.

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(Translated by H. Schwerdtfeger, McGill University)

Boundary Value Problems of Mathematical Physics and Related Aspects of Function Theory, Part I. Edited by V. P. Il'in. (Translation of Volume 5 of Stekov Institute, Seminars in Mathematics). Consultant's Bureau, New York (1969). vii + 96 pp.

This book is a collection of ten papers of well known authors in the field of Boundary Value Problems of Mathematical Physics. The papers are on both boundary value problems for equations of mathematical physics and on related questions of function theory and functional analysis. The first group of papers is concerned with a priori estimates and they investigate the solvability of boundary value problems for linear and quasilinear elliptic and parabolic equations. The second group of papers deals with classes of functions arising in the theory of partial differential equations. Most of the papers contain proofs of the result therein stated. One should express a pity that such important information is available only after two years of being published in Russian, and that the translation is not always originally correct, e.g. "compact carrier" instead of "compact support" (see p. 96) which proves that it has not been done by a professional mathematician.

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