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Comparison of the Short Period Rigid Earth Nutation Series

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Abstract. We present a comparison of the diurnal and subdiurnal nutations of the three last theories of rigid Earth nutation: SMART97, RDAN97 and REN 2000. For a better interpretation of the observations, we characterize their contribution to the polar motion and we estimate the nonrigid effects.

1. Introduction

The short period nutation terms have been put forward by Kinoshita and Souchay (1990). They are due to the fact the Earth is not exactly an axial and homogeneous ellipsoid. They present variations up to 20 microarcseconds in the time domain. This is the level of accuracy of the Earth orientation parameters determined by modern geodetic technques. Hence astrometric determination of the Earth's orientation as well as the interpretation of the Earth's orientation parameters requires knowledge of the short period nutation terms.

Three available series for short period nutations, established by three independant teams in the frame of a rigid Earth model, are available. We aim at providing a comparison between these series, as well a complete description of the phenomenom for astrometric purposes.

2. Comparison of the available series for the short period nutations

Presently, there are three accurate available rigid Earth nutation series computed by different analytical methods:

- SMART97 *(Solution du Mouvement de l'Axe de Rotation de la Terre)*: the rigid Earth nutation series of Bretagnon *et al.* (1997, 1998) based on the Eulerian equations for the motion of the Earth in the space,
- RDAN97 *(Roosbeek Dehant Analytical Nutation)*: the rigid Earth nutation series of Roosbeek and Dehant (1998), computed using the torque approach,

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- REN 2000 *(Rigid Earth Nutation)*: the rigid Earth nutation series of Souchay and Kinoshita (1996, 1997) and Souchay *et al.* (1999), based on the Hamiltonian equations of a rotating body.

All the effects having an influence at the 0.1 microarsecond (μas) level such as Earth's triaxiality (C_{22} and S_{22} effects), J_3 and J_4 effects, non-zonal harmonics of third and fourth degree influences, planetary indirect and direct effects are taken into account in these three theories. The non-zonal harmonics induce the short-period nutations and can be divided into three categories (Souchay *et al.*, 1999):

- The quasi-diurnal terms, related to C_{31}, S_{31}, C_{41} and S_{41} harmonics,
- The semidiurnal terms, due to C_{22}, S_{22}, C_{32} and S_{32} harmonics,
- The third-diurnal terms, related to C_{33} and S_{33} harmonics.

The nutation terms are classically expressed as a nutation in longitude $\Delta\psi$ and a nutation in obliquity $\Delta\varepsilon$:

$$\Delta\psi = \psi_s \sin\theta(t) + \psi_c \cos\theta(t) \tag{1}$$

$$\Delta\varepsilon = \varepsilon_s \sin\theta(t) + \varepsilon_c \cos\theta(t) \tag{2}$$

where θ is the astronomical argument of the nutation given by a linear combination of the Delaunay arguments and the Greenwich mean sidereal time Φ:

$$\theta(t) = i_1 l_M + i_2 l_S + i_3 F + i_4 D + i_5 \Omega + i_6 \Phi$$

where l_M is the mean anomaly of the Moon, l_S the mean anomaly of the Sun, F the difference between the mean longitude of the Moon and the mean longitude of the node of the Moon, D the difference between the mean longitudes of the Moon and of the Sun, Ω the mean longitude of the node of the Moon. The argument $\theta(t)$ can be expressed as the sum of a secular term λt, where λ is the frequency of the nutation concerned and a quasi-constant term.

In Table 1 we present the main quasi-diurnal and subdiurnal terms of REN 2000 series according to the above representation (the complete series contain 252 terms in longitude and 190 in obliquity). In the same table, we report the term-to-term differences with SMART97 and RDAN 2000 models. The largest offsets are below 0.8 μas.

We produced also the time comparison of those series over one thousand years. Table 2 provides us with the maximum absolute values of the differences between each of the series.

3. Circular components and nonrigid effects

From a geophysical point of view, the best information is contained in the circular components of the nutation terms. The prograde and retrograde circular terms, expressed in the terrestrial frame have totally different frequencies. In turn

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Table 1. Main quasi-diurnal and semidiurnal nutations of the figure axis: REN-2000 values and differences with SMART97 and RDAN97 models. The unit is μas.

	Argument						Period	Longitude ($\Delta\psi$)		Obliquity ($\Delta\varepsilon$)	
	Φ	l_M	l_S	F	D	Ω	(days)	sin	cos	sin	cos
	1	0	0	-1	0	0	1.03521	-5.30	-.65	-.27	2.17
RDAN97-REN2000								-.03	.01	.02	-.02
SMART97-REN2000								-.03	-.00	.00	-.01
	1	0	0	-1	0	-1	1.03505	-35.40	-4.35	-1.59	12.91
RDAN97-REN2000								.58	.07	-.04	.39
SMART97-REN2000								.58	.08	-.05	.46
	1	1	0	-1	0	-1	.99758	-19.94	-2.45	-.97	7.91
RDAN97-REN2000								.03	.02	.00	-.02
SMART97-REN2000								.09	-.04	-.02	-.03
	1	-1	0	1	0	1	.99696	24.14	2.97	1.16	-9.47
RDAN97-REN2000								-.21	-.03	.01	-.05
SMART97-REN2000								-.22	.03	.03	-.05
	1	-1	0	1	0	0	.99682	4.03	.50	.20	-1.60
RDAN97-REN2000								-.01	-.01	.00	.00
SMART97-REN2000								-.01	.01	.00	.00
	1	1	0	1	-2	1	.99216	-7.06	-.87	-.35	2.81
RDAN97-REN2000								-.04	-.01	-.00	.01
SMART97-REN2000								-.02	-.01	-.00	.01
	1	0	0	1	0	1	.96215	-38.23	-4.70	-1.86	15.11
RDAN97-REN2000								.09	.01	-.07	.02
SMART97-REN2000								.10	.00	-.01	.02
	1	0	0	1	0	0	.96201	-6.04	-.74	-.29	2.35
RDAN97-REN2000								.03	.12	.05	-.01
SMART97-REN2000								-.00	-.00	-.00	.00
	2	-1	0	-2	0	-2	.52743	-5.10	2.93	1.18	2.06
RDAN97-REN2000								-.09	.05	-.00	.00
SMART97-REN2000								-.05	.03	-.01	-.01
	2	0	0	-2	0	-1	.51756	-4.71	2.70	1.08	1.88
RDAN97-REN2000								-.11	.07	.02	.04
SMART97-REN2000								-.07	.04	.01	.02
	2	0	0	-2	0	-2	.51753	-25.53	14.65	5.68	9.89
RDAN97-REN2000								-.00	-.00	.15	.26
SMART97-REN2000								.17	-.10	.11	.20
	2	0	0	-2	2	-2	.50000	-10.44	6.00	2.37	4.12
RDAN97-REN2000								-.23	.13	.07	.12
SMART97-REN2000								-.19	.10	.06	.10
	2	0	0	0	0	0	.49863	31.25	-17.94	-7.12	-12.40
RDAN97-REN2000								.81	-.47	-.21	-.35
SMART97-REN2000								.60	-.34	-.16	-.27
	2	0	0	0	0	-1	.49860	4.28	-2.46	-.95	-1.66
RDAN97-REN2000								.06	-.04	-.04	-.06
SMART97-REN2000								.04	-.02	-.03	-.06

Table 2. Comparison in time domain between SMART97, RDAN97 and REN2000 short period nutations. The unit is μas.

	$\Delta\psi \sin\varepsilon$	$\Delta\varepsilon$
SMART97 – REN2000	1.74	3.52
RDAN97 – REN2000	2.40	6.80
RDAN97 – SMART97	1.40	6.80

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they constitute different phenomenona with respect to the Earth. Thus we can expect, that they are perturbed in different ways by the geophysical properties of the Earth.

Miscellenous ways for expressing the circular components of the nutations exist. A given nutation term, in the mean equatorial frame, can be expressed by the complex coordinate

$$P = \Delta\psi \sin\varepsilon_0 + i\,\Delta\varepsilon. \tag{3}$$

According to equations (1) and (2) and by using Euler decomposition of cosine and sine terms, we get the following expression in circular components:

$$P = i(a^+ e^{i\theta(t)} + a^- e^{-i\theta(t)}) \tag{4}$$

with:

$$a^+ = a_r^+ + i\,a_i^+ = \frac{1}{2}(-\Delta\psi_s \sin\varepsilon_0 + \Delta\varepsilon_c) + i\frac{1}{2}(-\Delta\psi_c \sin\varepsilon_0 - \Delta\varepsilon_s) \tag{5}$$

$$a^- = a_r^- + i\,a_i^- = \frac{1}{2}(\Delta\psi_s \sin\varepsilon_0 + \Delta\varepsilon_c) + i\frac{1}{2}(-\Delta\psi_c \sin\varepsilon_0 + \Delta\varepsilon_s) \tag{6}$$

If the frequency λ of the nutation term is positive, then the term a^+ provides us with the prograde component, whereas a^- provides us with the retrograde component. If the frequency λ of the nutation term is negative, then the term a^+ provides us with the retrograde component, whereas a^- provides us with the prograde component.

It turns out that short period nutations are prograde, the retrograde components being less than 1 μas in absolute amplitude. In Table 3 we give the prograde components associated with the model REN 2000 greater than 2 μas. We give also the period with which these terms appear in a terrestrial reference frame. The quasi-diurnal terms get mapped into long-period polar motion, whereas the semidiurnal ones into prograde diurnal polar motion according to the equation $p = -Pe^{-i\Phi}$ (p is the polar motion).

Until now the computation of the diurnal and subdiurnal nutation was restricted to a rigid-Earth model. Some resonance effects with the Chandler frequency can be expected for the diurnal terms, since they appear as a long period polar motion in the terrestrial frame (around 1 month for the biggest term).

Generally the tidally induced nutation for a non rigid-Earth are deduced from those of a rigid Earth thanks to a transfer function involving geophysical properties. The transfer functions are constantly refined by taking account new processes as well improved values of geophysical parameters. We have used a transfer function computed recently by Mathews (1999). The prominent values for the circular components are reported in Table 3. Some strong decrease appears for some of the amplitudes, up to 60% of the "rigid" value.

4. Conclusion

The agreement between the values of the short period nutations of the three available series is remarkable at the level of 1μas. The nonrigidity of the Earth

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Table 3. REN2000 prograde nutations above 2 μas and corresponding values for a nonrigid Earth according to Mathews (1999). We give the in-phase a_r^+ (ip) and out-of-phase a_i^+ (op) terms defined hereabove. The unit is μas.

Argument						Period	Period	Rigid Earth		Non rigid Earth	
Φ	l_M	l_S	F	D	Ω	in space	in Earth	ip	op	ip	op
1	0	0	-1	0	0	1.03521	-27.20986	2.1	0.3	1.8	0.1
1	0	0	-1	0	-1	1.03505	-27.32087	13.5	1.7	11.0	0.8
1	1	0	-1	0	-1	0.99758	-3193.94631	7.9	1.0	7.7	0.9
1	1	0	1	-2	1	0.99216	193.68628	2.8	0.3	1.6	0.0
1	0	0	1	0	1	0.96215	27.32239	15.2	1.9	11.7	0.7
1	0	0	1	0	0	0.96201	27.20994	2.4	0.3	1.8	0.1
2	-1	0	-2	0	-2	0.52743	1.11951	2.0	-1.2	1.0	-0.8
2	0	0	-2	0	-1	0.51756	1.07596	1.9	-1.1	0.9	-0.7
2	0	0	-2	0	-2	0.51753	1.07583	10.0	-5.8	5.0	-4.0
2	0	0	-2	2	-2	0.50000	1.00275	4.1	-2.4	2.0	-1.6
2	0	0	0	0	0	0.49863	0.99725	-12.4	7.1	-6.0	4.9

affects mostly the semidiurnal nutations up to 6 μas. Astrometric determination of the Earth's orientation parameters as well as their interpretation require us to consider carefully these terms.

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