15

OPE for deep inelastic scattering

15.1 Introduction

Deep-inelastic scattering (DIS) are classical QCD processes playing an important rôle in the understanding of perturbative QCD and of the nucleon structure function, where several structure functions $F_i(x, Q^2)[x$ (fraction of proton momentum) and Q^2 (squared of transfer momentum)] can be predicted and measured from different targets and beams and different polarizations. In the past DIS has been used for establishing the parton nature of quarks and gluons and QCD as a theory of strong interactions.

At present (as we shall see later on), DIS provide quantitative tests of QCD (measurements of quark and gluon densities in the nucleon, of $\alpha_s(Q^2), \ldots$). The theory of scaling violations for totally inclusive DIS processes are based on the operator product expansion (OPE) and renormalization group equation.

The OPE has been introduced by Wilson [222] and was proven by Zimmermann [223] in perturbation theory through the application of the BPHZ method. Let us consider the time-ordered product of two scalar fields:

$$\mathcal{T}\phi(x)\phi(0) \tag{15.1}$$

which we can write, using the Wick's theorem studied in Section (4.1), as:

$$\mathcal{T}\phi(x)\phi(0) = \langle 0|\mathcal{T}\left(\phi(x)\phi(0)\right)|0\rangle + :\phi(x)\phi(0): \tag{15.2}$$

The first term in the RHS is the scalar propagator:

$$\langle 0|\mathcal{T}\left(\phi(x)\phi(0)\right)|0\rangle = -i\Delta(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \frac{1}{p^2 - m^2 + i\epsilon} \simeq \frac{i}{(2\pi)^2}\frac{1}{x^2 - i0} + \cdots, \tag{15.3}$$

where $\cdots$ means less-singular terms. It is a c-number (unit operator) but singular for $x \to 0$, while the operator $:\phi(x)\phi(0):$ is regular. In general, any local operators $J(x)$ and $J'(y)$ can be expanded in a series of well-defined and regular operators $\mathcal{O}_i(x)$ multiplied with the c-number $C_i(x)$, the Wilson coefficients containing the singularity of the product $J(x)J'(y)$

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for $x = y$. This leads to the OPE or Wilson expansion:

$$J(x)J'(y) = \sum_{n=0}^{\infty} C_n(x-y)\mathcal{O}_n\left(\frac{x+y}{2}\right) \quad n = 0, 1, 2, \ldots . \tag{15.4}$$

15.2 The OPE for free fields at short distance

As an application, let us consider the neutral vector current:

$$J_\mu(x) =: \bar{\psi}(x)\gamma_\mu\psi(x) : \ , \tag{15.5}$$

which is a normal ordered product of two quark fields. Applying the Wick theorem studied in Part 1, one can write:

$$\begin{aligned} \mathcal{T}(J_\mu(x)J_\nu(0)) \ = \ & -Tr\{\langle 0|\mathcal{T}(\psi(0)\bar{\psi}(x))|0\rangle\gamma_\mu\langle 0|\mathcal{T}(\psi(x)\bar{\psi}(0))|0\rangle\gamma_\nu\} \\ & + : \bar{\psi}(x)\gamma_\mu\langle 0|\mathcal{T}(\psi(x)\bar{\psi}(0))|0\rangle\gamma_\nu\psi(0) : \\ & + : \bar{\psi}(0)\gamma_\nu\langle 0|\mathcal{T}(\psi(0)\bar{\psi}(x))|0\rangle\gamma_\mu\psi(x) : \\ & + : \bar{\psi}(x)\gamma_\mu\psi(x)\bar{\psi}(0)\gamma_\nu\psi(0) : \ , \end{aligned} \tag{15.6}$$

where the free propagator:

$$\langle 0|\mathcal{T}(\psi(x)\bar{\psi}(0))|0\rangle = -iS(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx}\frac{i}{\hat{p} - m + i\epsilon} , \tag{15.7}$$

is singular at *short distance* ($x \to 0$). Therefore, by inspecting Eq. (15.6), one can see that the first term is more singular than the second . . . , i.e. Eq. (15.6) is a typical example of an OPE. Relating the free fermion propagator to the scalar one:

$$S(x) = (i\hat{\partial} + m)\Delta(x) , \tag{15.8}$$

one can extract the leading singularity for $x \to 0$ from Eq. (15.3), which is quark mass independent. As the singularity behaves like x^2 (but not like x), it is on the light cone and called *light-cone singularity*. From the expression of the Fourier transform of the propagator:

$$\int dx\ e^{iqx}\frac{1}{(x - i\epsilon)^n} = 2\pi\frac{e^{i\frac{n\pi}{2}}}{\Gamma(n)}\theta(q)q^{n-1} , \tag{15.9}$$

one can see that the dominant contribution of the T-product of the two currents comes from the most singular part of the c-number coefficients. Therefore, near the light cone, one obtains [224]:

$$\begin{aligned} \mathcal{T}(J_\mu(x)J_\nu(0)) \ = \ & \frac{(x^2 g_{\mu\nu} - 2x_\mu x_\nu)}{\pi^4(x^2 - i\epsilon)^4} - \frac{x^\lambda}{(2\pi^2(x^2 - i\epsilon)^2} \\ & \times\left[i\sigma_{\mu\lambda\nu\rho}\mathcal{O}_V^\rho(x) + \epsilon_{\mu\lambda\nu\rho}\mathcal{O}_A^\rho(x)\right] + \mathcal{O}_{\mu\nu}(x) , \end{aligned} \tag{15.10}$$

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where $\mathcal{O}(x)$ are regular operators:

$$\begin{aligned}
\mathcal{O}_{\mu,V}(x) &= :\bar{\psi}(x)\gamma_\mu\psi(0) - \bar{\psi}(0)\gamma_\mu\psi(x): \;, \\
\mathcal{O}_{\mu,A}(x) &= :\bar{\psi}(x)\gamma_\mu\gamma_5\psi(0) + \bar{\psi}(0)\gamma_\mu\gamma_5\psi(x): \;, \\
\mathcal{O}_{\mu\nu}(x) &= :\bar{\psi}(x)\gamma_\mu\psi(x)\bar{\psi}(0)\gamma_\nu\psi(0): \;,
\end{aligned} \tag{15.11}$$

and:

$$\sigma_{\mu\lambda\nu\rho} = g_{\mu\lambda}g_{\nu\rho} + g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\nu\rho}\;. \tag{15.12}$$

We have used the relation:

$$\gamma_\mu\gamma_\lambda\gamma_\nu = (\sigma_{\mu\lambda\nu\rho} + i\epsilon_{\mu\lambda\nu\rho}\gamma_5)\gamma^\rho\;, \tag{15.13}$$

where $\epsilon_{\mu\lambda\nu\rho}$ is the totally anti-symmetric rank 4 tensor with the properties defined in Appendix D. Analogous expression can be derived for the current commutator:

$$\mathcal{T}[J_\mu(x),\, J_\nu(0)]\;, \tag{15.14}$$

by using:

$$\frac{1}{x^2 - i\epsilon} = \frac{\mathcal{P}}{x^2} + i\pi\delta(x^2)\;, \tag{15.15}$$

where $\mathcal{P}$ denotes principal value. Differentiating this expression, it is easy to obtain:

$$\frac{1}{(x^2 - i\epsilon)^n} - \frac{1}{(x^2 + i\epsilon)^n} = 2i\pi\frac{(-1)^{n-1}}{(n-1)!}\delta^{(n)}(x^2)\;. \tag{15.16}$$

Therefore:

$$\begin{aligned}
\mathcal{T}[J_\mu(x)J_\nu(0)] - \mathcal{T}[J_\mu(x)J_\nu(0)]^\dagger \equiv \epsilon(x_0)[J_\mu(x),\, J_\nu(0)]& \\
= -\frac{i}{3\pi^3}\delta^{(3)}(x^2)(x^2 g_{\mu\nu} - 2x_\mu x_\nu)& \\
- \frac{1}{\pi}x^\lambda\delta^{(1)}\left[i\sigma_{\mu\lambda\nu\rho}\mathcal{O}^\rho_V(x) + \epsilon_{\mu\lambda\nu\rho}\mathcal{O}^\rho_A(x)\right]& \\
+ \mathcal{O}_{\mu\nu}(x) - \mathcal{O}_{\nu\mu}(x)\;,&
\end{aligned} \tag{15.17}$$

where:

$$\epsilon(x_0) = \frac{x_0}{|x_0|}\;, \tag{15.18}$$

is the sign function.

15.3 Application of the OPE for free fields: parton model and Bjorken scaling

For simplicity, we consider the unpolarized process:

$$e + p \to e + X \tag{15.19}$$

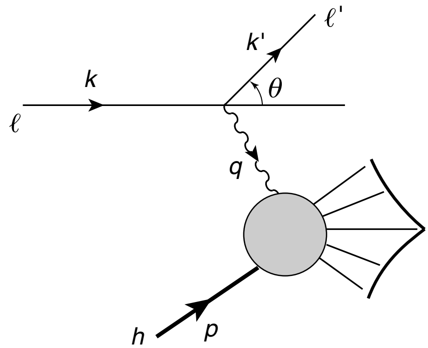

Fig. 15.1. Kinematics of the $e + p \to e + X$ process.

which we have anticipated in Section 2.3. Here, we shall derive explicitly the structure functions $W_{1,2}(Q^2, \nu)$ using OPE for free fields. The kinematics of the process is given in Fig. 15.1.

There are three independent kinematic variables:

$$s = (p+k)^2\,, \qquad q^2 = (k-k')^2, \qquad W^2 = (p+q)^2\,, \tag{15.20}$$

where k and k' are momenta of the initial and final electrons, p and q are respectively the proton and photon momenta. In the laboratory frame (proton rest frame) and neglecting the electron mass, one can rewrite:

$$\begin{aligned} s &= M_p(2E + M_p)\,, \\ q^2 &\equiv -(Q^2 > 0) = -4EE' \sin^2\frac{\theta}{2}\,, \\ W^2 &= M_p^2 + 2M_p(E - E') + q^2\,, \end{aligned} \tag{15.21}$$

where $E = k_0,\ E' = k_0'$ are the energies of the incident and scattered electrons in the proton rest frame, and θ is the scattering angle of the electron. The physical region is:

$$s \geq M_p^2\,, \qquad q^2 \leq 0\,, \qquad W^2 \geq (M_p + m_\pi)^2\,, \tag{15.22}$$

m_π being the pion mass. It is usual to introduce:

$$\nu \equiv p \cdot q = M_p(E - E')\,, \tag{15.23}$$

where ν/M_p is the energy transfer in the proton rest frame, in terms of which the physical region condition on W^2 reads:

$$2\nu + q^2 \geq m_\pi(2M_p + m_\pi)\,. \tag{15.24}$$

The inclusive differential cross-section of the unpolarized process can be written as:

$$E'\frac{d\sigma}{d^3k'} = \frac{1}{32(2\pi)^3}\frac{1}{k\cdot p}\sum_{\sigma,\sigma',\lambda}\sum_X (2\pi)^4\delta^4(p_X + k' - k - p)|\langle eX|T|eN\rangle|^2\,, \tag{15.25}$$

where σ', σ, λ are the spin components of the scattered, initial electrons and the target proton. The amplitude is:

$$\langle eX|T|eN\rangle = \bar{u}_{\sigma'}(k')(e\gamma_\mu)u_\sigma(k)\frac{1}{q^2}\langle X|(-e)J^\mu(0)|p,\lambda\rangle \,. \tag{15.26}$$

This leads to the expression of the cross-section as a convolution of the leptonic and hadronic tensors:

$$E'\frac{d\sigma}{d^3k'} = \frac{\alpha^2}{4(k\cdot p)q^4}L_{\mu\nu}W^{\mu\nu} \,, \tag{15.27}$$

where $\alpha = e^2/(4\pi)$ is the QED fine structure constant. The leptonic tensor is:

$$\begin{aligned} L^{\mu\nu} &= \frac{1}{4}Tr\{(\hat{k}+m_e)\gamma^\mu(\hat{k}'+m_e)\gamma^\nu\} \\ &= 4(k'^\mu k^\nu + k'^\nu k^\mu) + \left(2q^2+4m_e^2\right)g^{\mu\nu} \,. \end{aligned} \tag{15.28}$$

The hadronic tensor can be written as:

$$W_{\mu\nu} = \frac{1}{2\pi}\int d^4x\; e^{iqx}\frac{1}{2}\sum_\lambda\langle p;\lambda|J_\mu(x)J_\nu(0)|\lambda;p\rangle \,. \tag{15.29}$$

Using the property:

$$\int d^4x\; e^{iqx}\sum_\lambda\langle p;\lambda|J_\mu(0)J_\nu(x)|\lambda;p\rangle = 0 \,, \tag{15.30}$$

for physical process, which can be shown by using:

$$\begin{aligned} &\int d^4x\; e^{iqx}\sum_\lambda\langle p;\lambda|J_\mu(0)J_\nu(x)|\lambda;p\rangle \\ &= \sum_X(2\pi)^4\delta^4(q-p+p_X)\sum_\lambda\langle p;\lambda|J_\mu(0)|X\rangle\langle X|J_\nu(x)|\lambda;p\rangle \,. \end{aligned} \tag{15.31}$$

The assumption that $q-p+p_X=0$ in the physical region (Eq. (15.22)) would lead to the contradiction $q_0 \geq 0$. Therefore, one obtains:

$$W_{\mu\nu} = \frac{1}{2\pi}\int d^4x\; e^{iqx}\frac{1}{2}\sum_\lambda\langle p;\lambda|[J_\mu(x),\,J_\nu(0)]|\lambda;p\rangle \,. \tag{15.32}$$

Causality requires that the commutator vanishes for $x^2<0$, such that the integral is only non-zero for $x^2>0$. Using the optical theorem, one can relate the hadronic tensor to the absorptive part of the forward Compton scattering amplitude:

$$T_{\mu\nu} = \int d^4x\; e^{iqx}\frac{1}{2}\sum_\lambda\langle p;\lambda|\mathcal{T}J_\mu(x)J_\nu(0)|\lambda;p\rangle \,, \tag{15.33}$$

by the relation:

$$W_{\mu\nu} = \frac{1}{\pi}\mathrm{Im}T_{\mu\nu} \,, \tag{15.34}$$

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which corresponds to the discontinuity of $T_{\mu\nu}$ across the cut along the line $q_0 \geq 0$ in the complex q_0 plane:

$$\mathrm{Im}T_{\mu\nu} = \frac{1}{2i}[T_{\mu\nu}(q_0 + i\epsilon) - T_{\mu\nu}(q_0 - i\epsilon)] \,. \tag{15.35}$$

Using the general Lorentz decomposition, one can express $W_{\mu\nu}$ in terms of the invariants W_i (so-called structure functions) introduced in Section 2.3:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(Q^2, \nu) + \frac{1}{M_p^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right) W_2(Q^2, \nu)$$
$$+ i\epsilon_{\mu\nu\rho\sigma}\frac{p^\rho q^\sigma}{2M_p^2} W_3(Q^2, \nu) \,. \tag{15.36}$$

For unpolarized process, only $W_{1,2}$ are relevant. Then, the differential cross-section has the form:

$$\frac{d\sigma}{dQ^2 d\nu} = \frac{\pi\alpha^2}{4M_p E^2 \sin^4\theta E E'}\left\{2\sin^2\frac{\theta}{2} W_1(Q^2, \nu) + \cos^2\frac{\theta}{2} W_2(Q^2, \nu)\right\} \,. \tag{15.37}$$

Coming back to the OPE of $W_{\mu\nu}$ given in Eq. (15.17) between two proton states, one can notice that the last term is less singular than the two former terms, such that we can neglect it to a first approximation. The first term can also be omitted as it corresponds to a disconnected diagram. Also noticing that the operators $\mathcal{O}$ are regular and finite for $x \to 0$, one can Taylor-expand the quark fields:

$$\psi(x) = \psi(0) + x^\mu[\partial_\mu\psi(x)]_{x=0} + \frac{1}{2!}[\partial_{\mu_1}\partial_{\mu_2}\psi(x)]_{x=0} + \cdots \tag{15.38}$$

and write:

$$\mathcal{O}^\rho_{V/A}(x) = \sum_0^\infty \frac{1}{n!} x^{\mu_1}\cdots x^{\mu_n}\mathcal{O}^\rho_{V/A,\mu_1\cdots\mu_n}(0) \,, \tag{15.39}$$

where:

$$\mathcal{O}^\rho_{V,\mu_1\cdots\mu_n}(x) = :[\partial_{\mu_1}\cdots\partial_{\mu_n}\bar\psi(x)]\gamma^\rho\psi(x) - \bar\psi(x)\gamma^\rho[\partial_{\mu_1}\cdots\partial_{\mu_n}\psi(x)]:$$
$$\mathcal{O}^\rho_{A,\mu_1\cdots\mu_n}(x) = :[\partial_{\mu_1}\cdots\partial_{\mu_n}\bar\psi(x)]\gamma^\rho\gamma_5\psi(x) + \bar\psi(x)\gamma^\rho\gamma_5[\partial_{\mu_1}\cdots\partial_{\mu_n}\psi(x)]: \tag{15.40}$$

For the unpolarized process which we discuss here, the operator $\mathcal{O}^\rho_A$ will not also contribute. One can express the matrix element:

$$\langle p|\mathcal{O}^\rho_{V,\mu_1\cdots\mu_n}(0)|p\rangle = \hat{\mathcal{O}}_n p^\rho p_{\mu_1}\cdots p_{\mu_n} + \text{terms containing } g_{\mu\nu} \,. \tag{15.41}$$

where $\hat{\mathcal{O}}$ is a Lorentz invariant constant reduced matrix element which depends on $p^2 = M_p^2$ and on quark masses. We have used the fact that the matrix element only depends on p_μ and is symmetric in the indices $\mu_1,\ \mu_2, \cdots\ \mu_n$. The terms containing $g_{\mu\nu}$ in Eq. (15.41) are of the form $p^\rho p^2 g_{\mu_1\mu_2}p_{\mu_3}\cdots p_{\mu_n}$ and so on, which are less singular in x^2 because $g_{\mu_1\mu_2}$

gives rise to x^2, and can therefore be neglected. Therefore, the relevant part of Eq. (15.17) for our process can be written as:

$$W_{\mu\nu} = -\frac{1}{2\pi^2}\sigma_{\mu\lambda\nu\rho}p^\rho \int d^4x\ e^{iqx}x^\lambda \epsilon(x_0)\delta^{(1)}(x^2)f(p\cdot x)\ , \tag{15.42}$$

with:

$$f(z) = \sum_{n=0}^{\infty}\hat{\mathcal{O}}_n\frac{z^n}{n!}\ , \tag{15.43}$$

where one can also notice that due to the form of $\mathcal{O}^\rho_{V,\mu_1\cdots\mu_n}$, $\hat{\mathcal{O}}_n$ vanishes for n even and the summation in Eq. (15.43) only runs for n odd. Taking the Fourier transform:

$$f(z) = \int_{-\infty}^{+\infty} d\zeta\ e^{iz\zeta}\mathcal{F}(\zeta)\ , \tag{15.44}$$

one can rewrite:

$$W_{\mu\nu} = -\frac{i}{2\pi^2}\sigma_{\mu\lambda\nu\rho}p^\rho\frac{\partial}{\partial q_\lambda}\int_{-\infty}^{+\infty} d\zeta\mathcal{F}(\zeta)\int d^4x\ e^{i(q+p\zeta)x}\epsilon(x_0)\delta^{(1)}(x^2)\ . \tag{15.45}$$

Using:

$$\mathcal{I}_n \equiv \int d^4x\ e^{iqx}\delta^{(n)}(x^2) = \frac{i\pi^2}{4^{n-1}(n-1)!}(q^2)^{n-1}\epsilon(q_0)\theta(q^2)\ , \tag{15.46}$$

one obtains:

$$\begin{aligned} W_{\mu\nu} &= \int_{-\infty}^{+\infty} d\zeta\mathcal{F}(\zeta)\big[-\big(p\cdot q+\zeta M_p^2\big)g_{\mu\nu}+2\zeta p_\mu p_\nu + p_\mu q_\nu + p_\nu q_\mu\big] \\ &\quad\times \epsilon(q_0+\zeta p_0)\delta\left(q^2+2\zeta p\cdot q+\zeta^2M_p^2\right)\ . \end{aligned} \tag{15.47}$$

In the Bjorken limit:

$$p\cdot q\to\infty, \qquad -q^2\to\infty \quad \text{and} \quad \zeta\equiv x = -q^2/(2p\cdot q) \quad \text{fixed}\ , \tag{15.48}$$

one can neglect $p^2 = M_p^2$, and deduce:

$$W_{\mu\nu} = \frac{1}{2}\mathcal{F}(x)\left(-g_{\mu\nu} - \frac{q^2}{(p\cdot q)^2}p_\mu p_\nu + \frac{p_\mu q_\nu + p_\nu q_\mu}{p\cdot q}\right)\ , \tag{15.49}$$

which one can rewrite in terms of $W_{1,2}$ defined in Eq. (15.36) with:

$$\begin{aligned} W_1(\nu,Q^2) &= \frac{1}{2}\mathcal{F}(x) \equiv F_1(x)\ , \\ \frac{\nu}{M_p^2}W_2(\nu,Q^2) &= \frac{x}{2}\mathcal{F}(x) \equiv F_2(x)\ , \end{aligned} \tag{15.50}$$

as given in Eq. (2.83) in terms of the Bjorken scaling function $F_{1,2}(x)$. This result shows that the assumption of free-field light-cone structure is equivalent to that of the parton model.

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15.4 Light-cone expansion in $\phi_6^3(x)$ theory and operator twist

For simplifying our discussions, we shall work in $\phi_6^3(x)$ theory with a mass m. The hadronic current is:

$$J(x) = \phi^2(x) , \tag{15.51}$$

and the OPE has the form given in Eq. (15.4). In the previous sections, we have used the OPE at short distance $x \to 0$, i.e. large q, such that we can neglect terms of order $p \cdot q$ compared with q^2. For instance, in this case, the tree level amplitude of a forward Compton scattering reads:

$$\frac{1}{(q+p)^2 - m^2} = \frac{1}{q^2} + \mathcal{O}\left(\frac{1}{q^4}\right) . \tag{15.52}$$

In deep inelastic scatterings, the light-cone region $x^2 \to 0$ corresponds to the Bjorken limit in Eq. (15.48). In this region, the tree level Compton amplitude reads:

$$\begin{aligned} F_0(q,p) &\equiv \frac{1}{(q+p)^2 - m^2} = \frac{1}{q^2} \frac{1}{1 + \frac{2p\cdot q}{q^2}} \\ &\simeq \frac{1}{q^2} - \frac{2p\cdot q}{q^4} + \frac{(2p\cdot q)^2}{q^6} + \cdots + \mathcal{O}\left(\frac{1}{q^4}\right) , \end{aligned} \tag{15.53}$$

which expresses that the dominant term of the amplitude in the Bjorken limit is due to an infinite number of 'composite operators'. This can be seen by taking the Fourier transform of Eq. (15.53):

$$\int d^6q \, e^{-iqx} F_0(q,p) \sim \frac{1}{x^4} + i\frac{px}{x^4} - \frac{(px)^2}{8x^4} + \cdots \tag{15.54}$$

Its k-th term can be written as:

$$\frac{1}{x^4} x^{\mu_1} \cdots x^{\mu_k} \langle p | \mathcal{O}_{\mu_1 \cdots \mu_k} | p \rangle . \tag{15.55}$$

In general the OPE near the light cone has the form (*light-cone expansion*):

$$J(x)J'(0) = \sum_{i,k} C_k^{(i)}(x^2) x^{\mu_1 \cdots \mu_k} \mathcal{O}^{(i)}_{\mu_1 \cdots \mu_k}(0) , \tag{15.56}$$

where the index i specifies the type of composite operators. Identifying with Eq. (15.4), the coefficient functions are:

$$C_n(x) \equiv C_k^{(i)}(x^2) x^{\mu_1 \ldots \mu_k} . \tag{15.57}$$

In free-field theory, in order to match the mass dimension of both sides of Eq. (15.56), the coefficient function should behave as:

$$C_k^{(i)}(x^2) \sim (x^2)^{-(d_{J_o} + d_{J'_o} + k - d^{(i)}_{o,k})/2} , \tag{15.58}$$

where d_{J_o}, $d_{J'_o}$ and $d_o^{(i)}$ are canonical dimensions of the current J, J' and of the operator $\mathcal{O}^{(i)}_{\mu_1 \cdots \mu_k}$. This naïve power counting is valid for free-field theory as no other mass scale is

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present in the OPE. The index:

$$\tau_k \equiv d^{(i)}_{o,k} - k \equiv \text{dimension} - \text{spin} \tag{15.59}$$

which governs the strength of the singularity of the coefficient function is called *the twist* of the composite operator $\mathcal{O}^{(i)}_{\mu_1\cdots\mu_k}$ [225]. k is called the spin of the operator and d is its dimension. The operators of lowest twist dominate in the light-cone expansion. The scalar field ϕ, the fermion field ψ and the gauge field $G_\mu\nu$ have twist one. Taking the derivative of these fields cannot reduce the twist as the derivative increases the dimension by one unit but changes the spin by 1 or 0. Therefore, the minimum twist of an operator involving n fields is n. In the light-cone expansion the dominant operators have twist 2. In the presence of external field, the symmetric traceless tensors of rank k and twist 2 are e.g. of the form:

$$\begin{aligned}
\mathcal{O}^{(i)}_{s,\mu_1\cdots\mu_k} &= \phi^* D_{\mu_1}\cdots D_{\mu_k}\phi \ ,\\
\mathcal{O}^{(i)}_{f,\mu_1\cdots\mu_k} &= \frac{i^{k-1}}{k!}\{\bar\psi\gamma_{\mu_1}D_{\mu_2}\cdots D_{\mu_k}\psi + \text{permutations}\}\ ,\\
\mathcal{O}^{(i)}_{g,\mu_1\cdots\mu_k} &= 2\frac{i^{k-2}}{k!}Tr\{G_{\mu_1\alpha}D_{\mu_1}\cdots D_{\mu_k}G^{\alpha}_{\mu_k} + \text{permutations}\}\ .
\end{aligned} \tag{15.60}$$

where D_μ is the covariant derivative which is half the difference of the derivative acting to the right and to the left. In the presence of an external field the scale dimension counting does not hold. In the case of a theory with an UV fixed point, the scale invariance is recovered with the anomalous dimension, and the canonical dimensions are replaced by the scale dimensions d_J and $d^{(i)}_k$. Therefore, the light-cone singularity reads for $x^2 \to 0$ [222]:

$$C^{(i)}_k(x^2) \sim (x^2)^{-(d_J+d_{J'}+k-d^{(i)}_k)/2}\ . \tag{15.61}$$

In QCD, this expression will only be modified by logarithmic corrections as we shall see later on.