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New Approach for Solution of the Planet Transit Problem

Diana P. Kjurkchieva[1] **and Dinko P. Dimitrov**[2]

[1]Dept. of Astronomy, Shumen University, 9700 Shumen, Bulgaria;
email: `d.kyurkchieva@shu-bg.net`

[2]Institute of Astronomy, Bulgarian Academy of Sciences, Tsarigradsko shossee 72, 1784 Sofia
email: `dinko@astr0.bas.bg`

Abstract. We propose a new approach for a solution of the planet transit problem that is based on numerical calculation of integrals. The paper presents our method for the case of linear limb-darkening law and orbital inclination $i = 90°$, and illustrates the work of the code PTS written on the basis of our approach.

Keywords. methods: analytical, numerical, eclipses

There has been a sharp rise in the detections of transiting planets in recent years. This requires simple and easy modeling of the planet transits in order to determine the parameters of the planetary systems. The known codes for stellar eclipses are not applicable for this aim due to different reasons (non-effective convergence of the differential corrections for observational precisions poorer than 1/10 the depth of planet transit, etc.). New solutions of the direct problem for planet transits were proposed (Mandel & Agol 2002, Seager & Mallen-Ornelas 2003, Gimenez 2006, Pal *et al.* 2009, Kipping 2008). Their formulae contain special functions and as a result the derived results cannot be used directly to solve the inverse problem.

We propose a new approach for a solution of the planet transit problem that is based on numerical solution of integrals. Some of the derived analytical formulae as well as the code written on the numerical calculations can be applied to the inverse problem solution. This paper presents briefly our method for the case of a star with linear limb-darkening law and an inclination $i = 90°$ of the line-of-sight to the orbital plane.

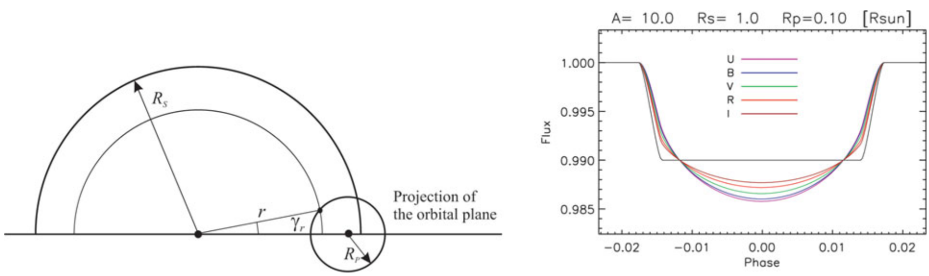

Figure 1. Left: Geometry of the transit; Right: UBVRI synthetic transits for a star with T=5000 K and $\log g$=4.5 obtained by the code PTS (the black curve corresponds to u=0)

We describe the relative decrease of the stellar normalized flux during the transit of a planet with radius R_p orbiting a star with radius R_s on a circle orbit with radius A by the expression

$$J(\varphi) = 1 - \frac{\int_{r_{min}(\varphi)}^{r_{max}(\varphi)} \left[1 - u + u\sqrt{1-(r/R_s)^2}\right] 2\gamma_r(\varphi) r dr}{\pi R_s^2(1-u/3)}. \tag{1.1}$$

The stellar area covered by the planet at phase φ is a sum of arc-like rings with areas $2\gamma_r(\varphi)rdr$ (Fig. 1, Left). We obtained analytical expressions for $\gamma_r(\varphi)$ and the extreme radii $r_{min}(\varphi)$ and $r_{max}(\varphi)$ of the stellar brightness' isolines covered by the planet.

Particularly, the phases of the outer and inner contact planet-star are

$$\varphi_1 = \frac{1}{2\pi}\left(\frac{\pi}{2} - \arccos\frac{R_s + R_p}{A}\right); \varphi_2 = \frac{1}{2\pi}\left(\frac{\pi}{2} - \arccos\frac{R_s - R_p}{A}\right) \tag{1.2}$$

The transit is partial in the phase ranges $[-\varphi_1, -\varphi_2]$ and $[\varphi_2, \varphi_1]$ while in the phase range $[-\varphi_2, \varphi_2]$ it is total.

The integral in eq. (1.1) has analytical solution only at the center of the transit ($\varphi = 0$)

$$J(0) = 1 - \frac{3(1-u)}{(3-u)}\frac{R_p^2}{R_s^2} + \frac{2u}{3-u}\left[1 - \left(1 - \frac{R_p^2}{R_s^2}\right)^{3/2}\right]. \tag{1.3}$$

The expressions (1.2-1.3) can be applied directly for the inverse problem' solution.

We made numerical solutions of the integral in eq. (1.1) at the phases of the planet transit. For this aim, we wrote a code PTS (Planet Transit Simulator) with input parameters: A, R_s, R_p and limb-darkening coefficient u. Figure 1 (Right) illustrates the result of our numerical solution of the direct problem of a planet transit.

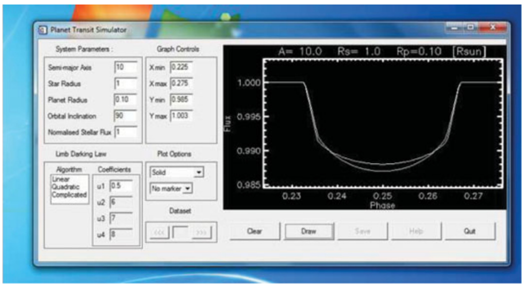

Figure 2. Graphical User Interface of the code PTS

The code PTS enables rapid and interactive calculation of planet transit light curves. The graphical possibilities (Fig. 2) make the code PTS user-friendly. Generalizations of our approach and code (for arbitrary limb-darkening law, arbitrary orbital inclination, flattened planets, etc.) as well as its application for a solution of the inverse problem by trials and errors and quantitative estimation of the fit quality are forthcoming.

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