BULL. AUSTRAL. MATH. SOC. 58F03, 54H20
VOL. 71 (2005) [61-67]

EQUICONTINUITY OF A GRAPH MAP

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Let G be a graph, and $f : G \longrightarrow G$ be a continuous map with periodic points. In this paper we show that the following five statements are equivalent.

(1) f is equicontinuous.

(2) There exists some positive integer N such that f^N is uniformly convergent.

(3) f is S-equicontinuous for some positive integer sequence $S = \{n_1 < n_2 < \cdots\}$.

(4) $\Omega(x, f) = \omega(x, f)$ for every $x \in G$.

(5) $\sigma : \varprojlim\{X, f\} \longrightarrow \varprojlim\{X, f\}$ is a periodic map.

1. INTRODUCTION

Let N (respectively $\mathbf{Z}^+$) denote the set of positive integers (respectively nonnegative integers). Write $\mathbf{N}_n = \{1, 2 \ldots, n\}$ and $\mathbf{Z}_n = \{0, 1, \ldots, n\}$ for any $n \in \mathbf{N}$. For any compact metric space (X, d), let $C^0(X)$ be the set of all continuous maps from X to X. Suppose $f \in C^0(X)$, $x \in X$ and $r > 0$, write $B(x, r) = B(x, r, d) = \{y \in X : d(y, x) < r\}$, $O(x, f) = \{f^n(x) : n \in \mathbf{Z}^+\}$, $\omega(x, f) = \bigcap_{n=0}^{\infty} \overline{O(f^n(x), f)}$ and $\Omega(x, f) = \{y :$ there exist sequences $\{x_i\}$ in X and $\{n_i\}$ in N such that $x_i \longrightarrow x, n_i \to \infty$ and $f^{n_i}(x_i) \longrightarrow y\}$. $O(x, f)$ and $\omega(x, f)$ are called the orbit and the ω-limit set of x under f, respectively. For $n \in \mathbf{N}$, a point $x \in X$ is called a periodic point of f with period n (or an n-periodic point of f) if $f^n(x) = x$ and $f^k(x) \neq x$ for each $k \in [0, n) \cap \mathbf{N}$. x is called a fixed point of f if $f(x) = x$. If $x \in \omega(x, f)$, then x is called a recurrent point of f. Denote by $F(f)$, $P_n(f)$ and $R(f)$ the set of all fixed points, n-periodic points and recurrent points of f, respectively. Write $P(f) = \bigcup_{n=1}^{\infty} P_n(f)$. We use int$A$, ∂A, $\overline{A}$ and $\#A$ to denote the interior, boundary, the closure and the cardinality of a subset A of X, respectively. We also need the following definitions.

DEFINITION 1: Let $S = \{n_1 < n_2 < \cdots\}$ be a subsequence of N. $f \in C^0(X)$ is said to be S-equicontinuous if for any $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon)$ such that

Received 23rd August, 2004
This project was supported by NNSF of China (10361001, 10461001) and NSF of Guangxi (0447004).

$d(f^{n_k}(x), f^{n_k}(y)) < \varepsilon$ whenever $x, y \in X$ with $d(x,y) < \delta$ and $k \in \mathbf{N}$. If $S = \mathbf{N}$, then f is said to be equicontinuous.

DEFINITION 2: Let $f \in C^0(X)$ and $x \in X$. If there exists $y \in X$ such that $\lim_{n\to\infty} f^n(x) = y$, then f is said to be convergent at x. If there exists $N \in \mathbf{N}$ such that $X = F(f^N)$, then f is said to be a periodic map.

Let $\{X_i\}_{i=1}^{\infty}$ be a sequence of spaces, and $\{f_i\}_{i=1}^{\infty}$ a sequence of maps $f_i : X_{i+1} \longrightarrow X_i$, then the inverse limit of $\{X_i, f_i\}_{i=1}^{\infty}$, denoted by $\varprojlim\{X_i, f_i\}$, is the subspace of the Cartesian product space $\prod_{i=1}^{\infty} X_i$ given by $\varprojlim\{X_i, f_i\} = \{(x_1, x_2, \ldots) : f_i(x_{i+1}) = x_i \text{ for all } i \in \mathbf{N}\}$. When all the spaces X_i are the same space X and all the maps f_i are same map f, we denote the inverse limit by $\varprojlim\{X, f\}$ (see [**10**]).

Define $\sigma : \varprojlim\{X, f\} \longrightarrow \varprojlim\{X, f\}$ by

$$\sigma : ((x_0, x_1, \ldots)) = (x_1, x_2, \ldots),$$

which is called the one-sided shift map.

It is interesting to find some properties equivalent to equicontinuity ([**1**]). In [**3**], Blanchard, Host and Maass discussed topological complexity, and showed that a continuous surjection f of a compact metric space X is equicontinuous if and only if any finite open cover of X under f has bounded complexity.

On 1-dimentional spaces, one has some still finer results [**4, 5, 6, 7, 13**]. Sun in [**11, 12**] obtained necessary and sufficient conditions for equicontinuity of tree maps and σ-maps. In [**8**], Gu obtained necessary and sufficient conditions for equicontinuity of figure-eight map with a periodic point. Recently, Mai in [**9**] obtained the following theorem.

THEOREM A. *Let G be a graph and $f \in C^0(G)$ with $P(f) \neq \emptyset$. Then f is equicontinuous if and only if there exists $N \in \mathbf{N}$ such that $\bigcap_{n=1}^{\infty} f^n(G) = F(f^N)$.*

By a graph we mean a compact connected one-dimensional polyhedron. In this paper we shall find some new equivalent conditions of equicontinuous graph maps. Our main result is the following theorem:

THEOREM 2. *Let G be a graph and $f \in C^0(G)$ with $P(f) \neq \emptyset$. Then the following five statements are equivalent.*

(1) *f is equicontinuous.*

(2) *There exists $N \in \mathbf{N}$ such that f^N is uniformly convergent.*

(3) *f is S-equicontinuous for some subsequence $S = \{n_1 < n_2 < \cdots\}$ of $\mathbf{N}$.*

(4) *$\Omega(x, f) = \omega(x, f)$ for every $x \in G$.*

(5) *$\sigma : \varprojlim\{X, f\} \longrightarrow \varprojlim\{X, f\}$ is a periodic map.*

2. Equicontinuity and uniform convergence in $C^0(X)$

In this section we shall discuss the relation between equicontinuity and uniform convergence of continuous self-maps of a compact metric space.

Theorem 1. *Let X is a compact metric space and $f \in C^0(X)$. Then the following three statements are equivalent:*

(1) *f is uniformly convergent;*

(2) *$\bigcap_{n=1}^{\infty} f^n(X) = F(f)$ and f is equicontinuous;*

(3) *$\bigcap_{n=1}^{\infty} f^n(X) = F(f)$ and $\Omega(x,f) = \omega(x,f)$ for every $x \in X$.*

Proof: It is easy to see that $\Omega(x,f) \cup \omega(x,f) \subset \bigcap_{n=1}^{\infty} f^n(X)$ for every $x \in X$.

(2)⇒(1) Suppose $\bigcap_{n=1}^{\infty} f^n(X) = F(f)$ and f is equicontinuous. Let $x \in X$ and $a, b \in \omega(x,f)$, then $a, b \in F(f)$. Since f is equicontinuous, for any $\varepsilon > 0$, there exists $\delta > 0$ such that $f^n\big(B(u,\delta)\big) \subset B(f^n(u), \varepsilon/3)$ for every $u \in X$ and every $n \in \mathbf{N}$. Take $m \in \mathbf{N}$ such that $f^m(x) \in B(a,\delta)$, then $b \in \omega(x,f) = \omega(f^m(x),f) \subset B(a,\varepsilon)$. That is, $\{a\} = \omega(x,f)$, which implies that f is convergent at x.

Choose $\{x_1, x_2, \ldots, x_k\} \subset X$ such that $\bigcup_{i=1}^{k} B(x_i,\delta) = X$. Then there exists $N \in \mathbf{N}$ such that

$$d\big(f^n(x_i), f^m(x_i)\big) < \varepsilon/3 \quad \text{for every } i \in \mathbf{N}_k \text{ and any } n > m > N.$$

For any $x \in X$, let $x \in B(x_i,\delta)$ for some $i \in \mathbf{N}_k$, then when $n > m > N$, we have

$$d\big(f^n(x), f^m(x)\big) < d\big(f^n(x), f^n(x_i)\big) + d\big(f^n(x_i), f^m(x_i)\big) + d\big(f^m(x), f^m(x_i)\big) < \varepsilon.$$

This implies f is uniformly convergent.

(2)⇒(3): See [**1**].

(1)⇒(2): Let $g(x) = \lim_{n\to\infty} f^n(x)$, then $g(x)$ is continuous. For any $\varepsilon > 0$, there exist $N \in \mathbf{N}$ and $\delta > 0$ such that

$$d\big(f^n(x), g(x)\big) < \varepsilon/3 \quad \text{for every } n > N \text{ and every } x \in X,$$

and

$$g\big(B(x,\delta)\big) \subset B(g(x), \varepsilon/3) \quad \text{for every } x \in X$$

and

$$f^i\big(B(x,\delta)\big) \subset B(f^i(x), \varepsilon) \quad \text{for every } i \in \mathbf{N}_N \text{ and every } x \in X.$$

Thus we have

$$f^n\big(B(x,\delta)\big) \subset B(f^n(x), \varepsilon) \quad \text{for any } n \in \mathbf{N}.$$

This implies f is equicontinuous.

Let $x \in \bigcap_{n=1}^{\infty} f^n(X)$, it follows from [**9**] that $x \in R(f)$. Then $x \in \omega(x,f) = \{x\}$, which implies $\bigcap_{n=1}^{\infty} f^n(X) = F(f)$.

(3)⇒(2) It only needs to be shown that f is equicontinuous. For given $x \in X$, let $x_n \to x$, $k_n \to \infty$, $f^{k_n}(x_n) \to a$ and $f^{k_n}(x) \to b$, then $a \in \Omega(x,f) = \omega(x,f)$ and $a, b \in F(f)$. Hence there exists $t_n \to \infty$ such that $t_n - k_n > n$ and $f^{t_n - k_n}\big(f^{k_n}(x)\big) = f^{t_n}(x) \to a$, which implies $a \in \omega(b,f) = \{b\}$. That is, f is equicontinuous. ∎

3. Proof of Theorem 2

Let G be a graph. For every $x \in G$, there exist a positive number $\varepsilon > 0$ and some n-star $X_n = \{z : z^n \in [0,1], z \text{ is a complex number}\}$ ([**2**]) such that for every $0 < \delta \leqslant \varepsilon$, there exists a homeomorphism $f : \overline{B(x,\delta)} \longrightarrow X_n$, such $B(x,\delta)$ are said to be a n-star-neighbourhoods of x. Write $V(x) = n$. If $V(x) \geqslant 3$, we call x a branched point of G. Let T ([**2**]) be a subtree of G and $a, b \in T$, we use $[a,b]_T$(or $[b,a]_T$) to denote the smallest connected subset of T containing a, b. Write $[a,b)_T = [a,b]_T - \{b\}$, $(a,b)_T = [a,b)_T - \{a\}$.

In what follows we let $B(G) = \{x_1, x_2, \dots, x_l\}$ be the set of all branched points of G, and $G - B(G)$ have p connected components. Put $u = V(x_1) + V(x_2) + \cdot + V(x_l) + 4p$ and $M = u!$. Let $S = \{n_1 < n_2 < \cdots\}$ be a subsequence of $\mathbf{N}$.

Lemma 1. *Let $f \in C^0(G)$ and $m \in \mathbf{N}$, then*

(1) *f is equicontinuous if and only if f^m is equicontinuous, and*

(2) *if f is S-equicontinuous, then $g = f^m$ is S_1- equicontinuous for some subsequence S_1 of $\mathbf{N}$.*

Proof: (1) See [**9**].

(2) Let f be S-equicontinuous. Then by choosing a subsequence we can assume that there exists $r \in \mathbf{Z}_{m-1}$ such that $n_i = s_i m + r$ for any $i \in \mathbf{N}$. Since G is compact, for any $\varepsilon > 0$, there exists $\delta_1 > 0$ such that

$$d\big(f^i(u), f^i(v)\big) < \varepsilon \qquad \text{whenever } d(u,v) < \delta_1 \text{ and } i \in \mathbf{Z}_m.$$

Since f is S-equicontinuous, there exists δ such that

$$d\big(f^{s_i m + r}(u), f^{s_i m + r}(v)\big) < \delta_1 \qquad \text{whenever } d(u,v) < \delta \text{ and } i \in \mathbf{N}.$$

Hence

$$d\big(g^{s_i+1}(u), g^{s_i+1}(v)\big) < \varepsilon \qquad \text{whenever } d(u,v) < \delta \text{ and } i \in \mathbf{N}.$$

This implies that g is $S_1 = \{s_1 + 1, s_2 + 1, \dots\}$-equicontinuous. ∎

Lemma 2. *Let $f \in C^0(G)$, and $X = \bigcap_{n=1}^{\infty} f^n(G)$. If one of following two conditions holds,*

(1) f *is* S*-equicontinuous, or*

(2) $\Omega(x,f)=\omega(x,f)$ *for every* $x\in X$;

then for any given $m\in\mathbf{N}$, $X\subset\omega(f^m|_X)$.

PROOF: Let $g=f^m$. Since X is compact, we have $g(X)=X$ and X is a connected closed subset of G. For given $y_0\in X$, there exist points $y_1,y_2,\dots$ in X such that $g(y_n)=y_{n-1}$ for every $n\in\mathbf{N}$.

(1) If f is S-equicontinuous, then by Lemma 1 there exists a subsequence $S_1=\{s_1<s_2<\cdots\}$ of $\mathbf{N}$ such that g is S_1- equicontinuous. Therefore for any $\varepsilon>0$, there exists $\delta>0$ such that $d\big(g^{s_k}(u),g^{s_k}(v)\big)<\varepsilon$ whenever $d(u,v)<\delta$ and $k\in\mathbf{N}$. Since X is compact, there exists a subsequence $0<k_1<k_2<\cdots$ of $\mathbf{N}$ and $y\in X$ such that $y_{s_{k_j}}\longrightarrow y$. Then $d\big(g^{s_{k_j}}(y_{s_{k_j}}),g^{s_{k_j}}(y)\big)=d\big(y_0,g^{s_{k_j}}(y)\big)<\varepsilon$ for some $s_{k_j}\in\mathbf{N}$. Thus $y_0\in\omega(y,g)$.

(2) If $\Omega(x,f)=\omega(x,f)$ for every $x\in X$, then by choosing subsequence we can assume that there exists a subsequence $0<k_1<k_2<\cdots$ of $\mathbf{N}$ and $y\in X$ such that $y_{k_j}\longrightarrow y$ since X is compact. Thus $y_0\in\Omega(y,f)=\omega(y,f)=\bigcup_{i=0}^{m-1}\omega(f^i(y),g)$. ∎

LEMMA 3. *Let* $f\in C^0(G)$ *with* $P(f)\neq\emptyset$ *and* $X=\bigcap_{n=1}^{\infty}f^n(G)$. *If one of following two conditions holds,*

(1) f *is* S*-equicontinuous; or*

(2) $\Omega(x,f)=\omega(x,f)$ *for every* $x\in X$;

then $X=F(f^{\tau M})$. *Where* τ *is the smallest period of the periodic points of* f.

PROOF: Let $g=f^\tau$. Obviously $F(g^M)\subset X$. Now we show $X\subset F(g^M)$.

Assume on the contrary that $X-F(g^M)\neq\emptyset$. Take $p\in F(g)$ and let K be the connected component of $F(g^M)$ containing p, then K is a closed subset of X, $g(K)=K$ and $\partial K\cap\partial(X-K)\neq\emptyset$.

CLAIM 1. $g\big(\partial K\cap\partial(X-K)\big)\subset\partial K\cap\partial(X-K)$.

PROOF OF CLAIM 1: Assume on the contrary that there exists $a\in\partial K\cap\partial(X-K)$ such that $g(a)\notin\partial K\cap\partial(X-K)$. Then we can choose a neighbourhood U of a such that $U\cap(X-K)\neq\emptyset$ and $g\big(U\cap(X-K)\big)\subset K$. Thus $U\cap(X-K)\not\subset\omega(g|_X)$ since $g(K)=K$, which contradicts Lemma 2. Claim 1 is proven. ∎

Take $a_0\in\partial K\cap\partial(X-K)$. Let s be the period of a_0 under g and V_i be a k_i-star-neighbourhood of $g^i(a_0)(i\in\mathbf{Z}_{s-1})$. We can assume $k_0=\min\{k_i:i\in\mathbf{Z}_{s-1}\}$. Choose $0<\delta_1<\delta_2<\cdots<\delta_{k_0+2}$ such that

$$g^s\big(B(a_0,\delta_i)\big)\subset B(a_0,\delta_{i+1})\subset V_0\qquad(i\in\mathbf{N}_{k_0+1}).$$

CLAIM 2. If there exist $y\in B(a_0,\delta_{k_0+1})$ and $k\in\mathbf{N}$ such that $\{g^{is}(y):i\in\mathbf{Z}_k\}\subset B(a_0,\delta_{k_0+1})$ and $y,g^{ks}(y)$ is contained in same connected component L of $B(a_0,\delta_{k_0+1})-\{a_0\}$, then $g^{ks}(y)\in(a_0,y]_L$.

PROOF OF CLAIM 2: Suppose that $y_0 = y \in (a_0, g^{ks}(y))_L$. Then there exist points $y_1, y_2, \dots$ in L such that $y_n \in (a_0, y_{n-1})_L$ and $g^{ks}(y_n) = y_{n-1}$ ($n \in \mathbf{N}$). Let $y_n \to v \in F(g^{ks})$, then $y_0 \in \Omega(v, f) - \omega(v, f)$ and $d(g^{ksn}(y_n), g^{ksn}(v)) = d(y_0, v) > 0$ for any $n \in \mathbf{N}$, which implies that g^{ks} is not S-equicontinuous for any subsequence S of $\mathbf{N}$. A contradiction. Claim 2 is proven. ∎

CLAIM 3. Let $y \in B(a_0, \delta_1)$, then $O(y, g^s) \subset B(a_0, \delta_{k_0+1})$.

PROOF OF CLAIM 3: Assume on the contrary that $O(y, g^s) \not\subset B(a_0, \delta_{k_0+1})$. Let B_1 be the connected component of $B(a_0, \delta_{k_0+1}) - \{a_0\}$ containing y and $r_1 = \min\{i : g^{is}(y) \notin B_1\}$. It follows from Claim 2 that $\{y, \dots, g^{(r_1-1)s}(y)\} \subset B(a_0, \delta_1)$. Let B_2 be the connected component of $B(a_0, \delta_{k_0+1}) - \{a_0\}$ containing $g^{r_1 s}(y)$ and $r_2 = \min\{i : g^{is}(y) \notin B_2 \cup B_1\}$. Again it follows from Claim 2 that $\{y, \dots, g^{(r_2-1)s}(y)\} \subset B(a_0, \delta_2)$. Continuing on, we inductively define $0 = r_0 < r_1 \leqslant r_2 \leqslant \dots \leqslant r_{k_0}$ and the connected components $B_1, B_2, \dots, B_{k_0}$ of $B(a_0, \delta_{k_0+1}) - \{a_0\}$ such that

(i) $r_j = \min\{i : g^{is}(y) \notin \bigcup_{\lambda=1}^{j} B_\lambda\}$ for every $j \in \mathbf{N}_{k_0}$;

(ii) B_j be the connected component of $B(a_0, \delta_{k_0+1}) - \{a_0\}$ containing $g^{r_{j-1}s}(y)$ for every $j \in \mathbf{N}_{k_0}$;

(iii) $\{y, \dots, g^{(r_j-1)s}(y)\} \subset B(a_0, \delta_j)$ for every $j \in \mathbf{N}_{k_0}$.

Hence $g^{r_{k_0}s}(y) \in B(a_0, \delta_{k_0+1})$ since $g^s(B(a_0, \delta_{k_0})) \subset B(a_0, \delta_{k_0+1})$, which contradicts the definition of r_{k_0}. Hence $O(x, g^s) \subset B(a_0, \delta_{k_0+1})$. Claim 3 is proven. ∎

CLAIM 4. $\omega(B(a_0, \delta_1) \cap (X - K), g) \subset F(g^M)$.

PROOF OF CLAIM 4: Let $y \in B(a_0, \delta_1) \cap (X - K)$, it follows from Claim 3 that $O(y, g^s) \subset B(a_0, \delta_{k_0+1})$.

If $g^{is}(y) \in K$ for some $i \in \mathbf{N}$, then $\omega(y, g^s) \subset F(g^M)$ and $\omega(g^i(y), g^s) = g^i(\omega(y, g^s)) \subset g^i(F(g^M)) \subset F(g^M)$, which implies $\omega(y, g) \subset F(g^M)$.

If $O(y, g^s) \cap K = \emptyset$, then it follows from Claim 2 that $\#(\omega(y, g^s)) = r \leqslant k_0$ and $\omega(y, g^s) \subset F(g^{sr})$. Thus $\omega(y, g^s) \subset F(g^M)$ and $\omega(y, g) \subset F(g^M)$. Claim 4 is proven. ∎

Let $y \in B(a_0, \delta_1) \cap (X - K)$, it follows from Lemma 2 that there exists $x \in X$ such that $y \in \omega(x, g)$. Choose $m \in \mathbf{N}$ such that $g^m(x) \in B(a_0, \delta_1) \cap (X - K)$, then $y \in \omega(x, g) = \omega(g^m(x), g)$. By Claim 4 we have $y \in F(g^M)$. Hence $B(a_0, \delta_1) \cap (X - K) \subset F(g^M)$, which implies $B(a_0, \delta_0) \cap (X - K) \subset K$, a contradiction. Lemma 3 is proven. ∎

PROOF OF THEOREM 2. (1)⇔ (2) is from Theorem A, Theorem 1 and Lemma 1. (1)⇔ (3,4) is from [**1**, Theorem 2.3], Theorem A and Lemma 3.

(1)⇒(5) Suppose f is equicontinuous. It follows from Theorem A that $\bigcap_{n=1}^{\infty} f^n(G) = F(f^N)$ for some $N \in \mathbf{N}$. Let $x = (x_0, x_1, \dots) \in \varprojlim\{X, f\}$, then for given $i \in \mathbf{Z}^+$, we have $x_i = f^n(x_{i+n})$ for all $n \in \mathbf{N}$. Thus $x_i \in \bigcap_{n=1}^{\infty} f^n(G) = F(f^N)$, which implies $\sigma^N(x) = x$ for all $x = (x_0, x_1, \dots) \in \varprojlim\{X, f\}$.

(5)⇒(1) Suppose there exists $K \in \mathbf{N}$ such that $\sigma^K(x) = x$ for all $x \in \varprojlim\{X, f\}$. Let $y \in X = \bigcap_{n=1}^{\infty} f^n(G)$. Since $f(X) = X$, there exist points $y_1 = y, y_2, \ldots$ in X such that $f(y_{i+1}) = y_i$ for all $i \in \mathbf{N}$, thus $x = (y_1, y_2, \ldots) \in \varprojlim\{X, f\}$, $\sigma^K(x) = x$, which implies $y \in F(f^K)$. By Theorem A we know that f is equicontinuous since $\bigcap_{n=1}^{\infty} f^n(G) \subset F(f^K)$. $\square$

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