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CORRECTION TO 'CONJUGACY CLASS SIZES IN FINITE GROUPS'

AVINOAM MANN

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I. M. Isaacs pointed out to me that in [4] I misquoted a result of his from [2]. As a consequence, the proof of Theorem 1 in [4] is faulty. The other results in [4] are independent of Theorem 1. Below we provide a substitute for Theorem 1.

Recall that a conjugacy class of a finite group G is *minimal* if it has minimal size among the noncentral classes. An element of a minimal class is also called *minimal*. We denote by $M(G)$ the subgroup generated by all minimal elements. In [3] we proved that if G is nilpotent, then the nilpotency class of $M(G)$ is at most 3 (this is best possible). In [2] Isaacs simplified the proof, and extended the result to some wider families of groups, for instance, supersoluble ones. In Theorem 1 of [4] we stated that the same conclusion holds, assuming only that $M(G)$ is soluble and contains a normal subgroup N with abelian Sylow subgroups such that $M(G)/N$ is supersoluble. As explained above, the proof of that is faulty, though the first part, showing that under those assumptions $M(G)$ is itself supersoluble, is valid. We now show that the full conclusion holds, under one of two stronger assumptions: we require of G itself the structure postulated earlier for $M(G)$, or we require that $M(G)/N$ is nilpotent rather than supersoluble.

THEOREM 1. *The group $M(G)$ is nilpotent, of class at most* 3, *if one of the following holds:*
(i) *G is soluble, and contains a normal subgroup N with abelian Sylow subgroups such that G/N is supersoluble;*
(ii) *$M(G)$ is soluble and contains a normal subgroup N with abelian Sylow subgroups such that G/N is nilpotent.*

To prove Theorem 1(i) it suffices, by Theorem A of [2], to prove the next result.

PROPOSITION 2. *Let G be as in Theorem 1(i). Then G contains self-centralizing normal abelian subgroups.*

PROOF. Let F be the Fitting subgroup $F(N)$ of N, and let C be its centralizer $C_G(F)$ in G. Then F is an abelian normal subgroup of G, and $C \cap N = F$, therefore C/F is supersoluble. But $F \leq Z(C)$, therefore C itself is supersoluble. Let A be maximal among the normal abelian subgroups of G containing F. Then $A \leq C$. Let $D = C_G(A)$. Since $F \leq A$, then $F \leq A \leq D \leq C$. Moreover, C/F is G-isomorphic to CN/N, a normal subgroup of the supersoluble group G/N. If $D \neq A$, it follows that there is a normal subgroup E of G such that $A \leq E \leq D$ and E/A is cyclic. Then E is abelian, contradicting the maximality of A. Thus A is a self-centralizing normal abelian subgroup. □

We remark that the proof of Satz V.18.4 in [1] establishes that under the assumptions of the proposition, G is either abelian or contains a noncentral normal abelian subgroup.

PROOF OF THEOREM 1(ii). Write $M = M(G)$, and let F be the Fitting subgroup of N. Then F is abelian, and by Corollary 2 of [2], $F \leq Z_2(M)$, and in particular $F \leq Z_2(N)$. Since F is the maximal normal nilpotent subgroup of N, it follows that $F = N$. Now $N \leq Z_2(M)$ and M/N is nilpotent so M is nilpotent. It remains to prove the inequality for the class. Let $\gamma_n(M)$ be the nth term of the lower central series of M. By Theorem D of [2], $\gamma_5(M) = 1$, and therefore $[\gamma_2(M), \gamma_3(M)] = 1$. Let x be a minimal element, and write $H = C_G(x)\gamma_2(M)$. Then $|G : C_G(x)| = |G : H||H : C_H(x)|$, and if $y \in H$, then $|G : C_G(y)| \leq |G : H||H : C_H(y)|$. Thus, if $|H : C_H(y)| < |H : C_H(x)|$, then $|G : C_G(y)| < |G : C_G(x)|$, and $y \in Z(G)$.

Recall the identity $[x, yz] = [x, z][x, y]^z$. We take x as above and $y, z \in \gamma_2(M)$, and obtain $[x, yz] = [x, y][x, z]$, that is, the mapping $y \mapsto [x, y]$ is a homomorphism of $\gamma_2(M)$ into $\gamma_3(M)$. The image is the subgroup $K := [x, \gamma_2(M)]$, which is normal in H. Here $|K| = |H : C_H(x)|$, and if $1 \neq t \in K$, then all H-conjugates of t are nonidentity elements of K, therefore $|H : C_H(y)| < |K|$, and, by the previous paragraph, $t \in Z(G)$. Letting x range over all minimal elements, we see that $\gamma_2(M) \leq Z_2(M)$, which implies that $\gamma_4(M) = 1$ and M has class at most 3. □

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AVINOAM MANN, Einstein Institute of Mathematics, Hebrew University, Jerusalem 91904, Israel
e-mail: mann@math.huji.ac.il

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