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Vincenzo Zappalà \({ }^{+}\), Paolo Farinella \({ }^{++}\)and Paolo Paolicchi \({ }^{+++}\)
+ Osservatorio Astronomico di Torino, Pino Torinese, Italy
++ Scuola Normale Superiore and Dipartimento di Matematica
    dell'Università, Pisa, Italy
+++ Osservatorio Astronomico di Brera, Merate, Italy
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#### Abstract

The outcomes of asteroidal catastrophic collisions are strongly affected by the target asteroid's gravity, since only the fragments escaping with initial velocities higher than the target's escape velocity are not reaccumulated into "rubble pile" remnants. This idea can be compared with the observational evidence on the properties of family asteroids in several ways : (1) the shape and spin period of the "reaccumulated" family asteroids will roughly fit the relationships valid for self-gravitating fluid bodies; (2) the relative velocities of the few escaping fragments arising from a breakup event marginally overcoming self-gravity will often have an anisotropic distribution, affecting the final distribution of orbital elements; (3) the amount of mass which in a given family escaped to "infinity" will be correlated with the target's size, since only for objects larger than $\sim 100 \mathrm{~km}$ self-gravity plays an important role. These predictions are discussed and compared with the available data.


In some recent papers (Farinella et al., 1981; Paolicchi et al., 1982; Farinella et al., 1982) we have suggested that the outcomes of asteroidal catastrophic collisions, at least for targets exceeding some critical size, should be strongly affected by the existence of a significant self-gravitational binding of the target's material. After the breakup event, only the fragments ejected with initial velocities higher than the target's escape velocity will eventually reach independent orbits; all the other fragments will be quickly reaccumulated into "rubble pile" remnant bodies, i.e., loose and internally fractured aggregates of material held together mainly by its own self-gravity and having very low mechanical strength (see also Davis et al., 1979, and Weidenschilling, 1981). These objects will tend to relax toward the equilibrium figures consistent with their spin rate and density; in order of increasing angular momentum of rotation, we will get oblate Maclaurin spheroids, triaxial Jacobi ellipsoids and 177

Darwin binary systems (Chandrasekhar, 1969). The critical size for which self-gravitational effects become important in determining the collisional response can be estimated by comparing the target's escape velocity $\mathrm{V}_{\mathrm{e}}$ with the typical velocity of the fragments ejected after a catastrophic impact ( $\mathrm{V}_{\mathrm{fr}}$ ); this latter at present can only be estimated by laboratory impact experiments such as those performed by Fujiwara and Tsukamoto (1980). Following the assumptions of Farinella et al. (1982), we have

$$
\begin{align*}
& \mathrm{v}_{\mathrm{e}} \simeq 1.2 \times 10^{-3}(\rho / 2.5)^{1 / 2} \mathrm{R} \mathrm{~cm} / \mathrm{s}  \tag{1}\\
& \mathrm{v}_{\mathrm{fr}} \simeq 2.1 \times 10^{-3}(\mathrm{E} / \mathrm{M})^{0.76} \mathrm{~cm} / \mathrm{s} \tag{2}
\end{align*}
$$

where $R$ and $\rho$ are the target's radius and density, while $E / M$ is the ratio between the projectile's kinetic energy and the target's mass (all these quantities must be expressed in cgs units). Typically the largest impacts endured by asteroids larger than $\sim 10 \mathrm{~km}$ occurred at velocities of the order of $5 \mathrm{~km} / \mathrm{s}$ and involved projectile objects of mass $10^{-3}$ to $10^{-2}$ times the mass of the target (Farinella et a1., 1982). This implies that for the majority of asteroids the most destructive collision occurred with an impact specific energy in the range $10^{8}$ to $10^{9} \mathrm{erg} / \mathrm{g}$. According to the evidence provided by laboratory experiments, this results always into an extensive fragmentation of the target, since most rocky materials are already ruptured at $E / M$ values of the order of $10^{7} \mathrm{erg} / \mathrm{g}$. Moreover, the critical size over which $V_{e}>V_{f r}$ (implying effective gravitational reaccumulation of fragments) comes out to be in the range 50 to 200 km , depending on the energy and geometry of the largest collision. This is a size range where many observational data on the asteroid physical properties are now available; since the previous conclusions are partially based on questionable extrapolations of small-scale laboratory experiments to bodies of asteroidal dimensions, it seems very interesting to try a check of the theoretical predictions with the data.

The general considerations summarized above find a natural and intriguing application in the particular case of the asteroid dynamical families. These sets of objects, having independent heliocentric orbits with very close proper elements, are widely believed to be of collisional origin, i.e., to represent the remnants of the recent collisional disruption of a common parent asteroid. The physical and orbital properties of asteroids belonging to various families have been analysed by several investigators (Wiesel, 1978; Gradie et al., 1979; Ip, 1979; Fujiwara, 1982; and others) in order to reconstruct the properties of the parent body or the mechanism of collisional rupture. We now want to point out the remarkable fact that, by using the orbital data, it is possible to estimate the original relative velocities of the members of a given family : the typical velocity dispersion results to be in the range 0.1 to $0.3 \mathrm{~km} / \mathrm{s}$. This is within a factor two of the escape velocity of an object of radius $\sim 100 \mathrm{~km}$,
as many parent bodies of the asteroid families presumably were. In the case of families, therefore, we are probably observing just the outcome of the limiting case in which the target's self-gravitational barrier is marginally overcome by a part of the broken fragments, which can escape but retain only small relative velocities "at infinity", resulting eventually into a clustering of heliocentric orbital elements. In such conditions, it seems likely that the largest object resulting from the breakup is the core of the parent body covered by a "megaregolith", i.e., a "rubble pile" asteroid of the type described before. This outcome is produced because the low-velocity portion of the ejecta mass distribution conceivably has not sufficient kinetic energy to reach the escape velocity, and therefore falls back onto the principal remnant producing a partially reaccumulated central object within the family.

How can we compare this theoretical scenario with the observational evidence ? First of all, we have to face the difficulty that the exact population of each family is not univocally known : several authors have obtained significantly different family classifications (Carusi and Valsecchi, 1982), or at least their proposed memberships are often contrasting. For our present purposes, we have chosen to use the list of memberships and proper elements given by Williams (1979), limiting ourselves to 33 families having at least five numbered asteroids as members. This seems to us a reasonable compromise between the most restricting classifications, obtaining only the few numerous families originally discovered by Hirayama, and the broadest classifications for which the vast majority of asteroids belongs to some family. We decided to exclude from the analysis a few Williams' families for which the observed distributions of sizes and proper elements among the members suggest that some large interloper is present in the family itself. For instance, families No. 106, 113 and 138 have two or three largest members of comparable size, and this contrasts with the typical mass distribution arising from a collisional rupture (Fujiwara et al., 1977) and generally observed in the asteroid belt (Kresák, 1977). In a similar way, it seems very unlikely that Ceres really belongs to family No. 67 : no plausible physical mechanism could cause the escape from Ceres of a $150-\mathrm{km}$ sized fragment like 39 Laetitia; therefore, this family is considered without Ceres. Our sample of 33 families can then be analyzed from several points of view.

A first investigation can regard the photometrically determined rotational properties (spin period and lightcurve morphology; the maximum lightcurve amplitude is a rough indicator of the asteroid's triaxial elongation) for family asteroids of different sizes, and its main results have been already reported by Paolicchi et al., 1982, and Farinella et al., 1982. In brief, we have verified that for sizes larger than $\sim 100$ km the rotational properties of family asteroids show a clear correlation
between short periods and highly elongated shapes, which can be satisfactorily interpreted in terms of equilibrium shapes of self-gravitating "fluid" bodies (since the Jacobi triaxial figures can exist only for short rotational periods, ranging from $\sim 4$ to $\sim 6 \mathrm{hr}$ ). The non-family asteroids show the same type of correlation only for diameters larger than $\sim 200 \mathrm{~km}$, while in the size range from 100 to 200 km several objects having non-equilibrium shapes (for instance elongated but slow rotators) are certainly present (see also Farinella et al., 1981). This suggests that most inter-mediate-size family asteroids are indeed covered by deep layers of fragmented material whose global shape has roughly relaxed to the gravitational equilibrium figure; this structure appears at smaller sizes than for non-family objects because the breakup events generating families had to be always energetically close to the threshold for gravitational reaccumulation of fragments, since otherwise no clustering of orbital elements would be actually observed.

A second type of analysis is based on the reconstruction of the mass distribution within each family, and on the assumption that the total observed mass corresponds roughly to the original mass of the parent body (PB). Of course a part of this mass could have been missed either due to physical reasons (if very small and/or high-velocity ejecta were produced by the catastrophic breakup), or due to the magnitude bias of the observations (mainly in the case of the outer-belt families), but for our statistical purposes these effects should not be very important and, in any case, for a typical mass distribution of fragments the contribution of the missing members to the total mass of the family is small. The asteroid masses have been obtained by using the diameters 1 isted by Bowell et al. (1979) and a mean density of $2.5 \mathrm{~g} / \mathrm{cm}^{3}$, or alternatively, when no diameter value was available, by estimating albedos via the known taxonomic types in the family and/or the position in the belt (i.e., by using an S-type 0.16 albedo for semimajor axes smaller than 2.6 AU , a C-type 0.037 albedo beyond 2.7 AU and the intermediate value 0.10 in the interval 2.6 to 2.7 AU ). Then the mass of the largest remnant (LR) asteroid in each family ( $M_{L R}$ ) can be compared with the mass of the parent body ( $M_{P B}$ ), and we can obtain a ratio $\delta$ which is closer to one for families whose mass is more concentrated in the largest object (and for which, presumably, the gravitational reaccumulation mechanism was more effective).

Before discussing the resulting values of $\delta$ and their correlations with other parameters characterizing the families, we think that it is useful to introduce a qualitative classification by dividing our sample of families into two main subsamples. Following Ip (1979), we can derive from the semimajor axis difference $\Delta$ a of each body with respect to the LR of its family the along-track component $\Delta V$ of the relative velocity when the two Keplerian orbits intersect :

$$
\begin{equation*}
\Delta V \simeq n \Delta a / 2 \tag{3}
\end{equation*}
$$

where $n$ is the orbital mean motion of the LR. $\Delta V$ can be identified with the ejection velocity after breakup, since the laboratory experiments have shown that in most cases the largest fragment moves very slowly with respect to the target. In this way we can plot the mass distribution of the fragments of each family versus the ejection velocity, in order to get some insight into the kinematics of the fragment ejection. In Figure 1 various examples of such plots are shown, where the vertical axis refers to the percentage of fragmental mass $M_{f r}\left(\equiv M_{P B}-M_{L R}\right)$ assigned to each $\Delta V$ bin of width $0.02 \mathrm{~km} / \mathrm{s}$ (the family numbers of Williams' classification are indicated in each case in the brackets). From the Figure it is clear that two broad categories can be readily identified : the "asymmetric" families, whose fragments are asymmetrically distributed on one side of the LR, and the "dispersed" families, which have their LR roughly at the center of the fragmental mass distribution. This classification could be made more quantitative by defining an "asymmetry parameter"

$$
\begin{equation*}
\mathrm{C} \equiv\left\langle\Delta \mathrm{~V}^{2}\right\rangle /\langle\Delta \mathrm{V}\rangle^{2}, \tag{4}
\end{equation*}
$$

where the mean values are weighted over the mass of the various fragments forming the family : for asymmetric families we have values of $C$ of the order of one, whilst for dispersed families $C \gg 1$. As remarked by Ip (1979), a similar conclusion follows when one uses more complex twoor three-dimensional mass vs. ejection velocity distributions, obtained by analysing the differences in proper eccentricity and inclination among the family members. In this case, however, a real reconstruction of the three-dimensional ejection velocity vector would need the knowledge of the PB's angular orbital elements (including true anomaly) at the time of breakup, and therefore some additional assumption must be introduced.

Ip suggested that the asymmetric families arose because, after the catastrophic breakup, fragments were not scattered isotropically, but were ejected with some preferential direction. We agree that the asymmetry effect is connected with the mechanism of fragmentation, but in our opinion it is strongly amplified by the influence of the PB's self-gravity : when most fragments fall back and form a reaccumulated asteroid, it is highly probable that the small high-velocity fraction of escaping bodies is distributed anisotropically, and only these fragments are observed today as minor family members. If we use a reasonable initial velocity distribution of the ejected mass, it is easy to verify that an initially high $C$ value (corresponding to a nearly-isotropic explosion) is reduced by the reaccumulation mechanism by a factor of the same order as the ratio between the initially ejected mass and the mass of the non-reaccumulated fragments. Therefore, we expect that asymmetric families have LRs with a "rubble pile" structure, since the gravitational reaccumulation was


Figure 1. The Figure shows the fragmental mass vs. $\Delta V$ (ejection velocity) distributions for 10 Williams' families, 5 of the "asymmetric" and 5 of the "dispersed" type (see text). The dash-dotted vertical lines, corresponding to the zero of the $\Delta V$ axes, refer to the location of the largest member of each family (whose mass is not included in the distributions). The numbers in brackets identify the families as listed by Williams (1979).
very likely considerable. As a matter of fact, we find very frequently that the rotational properties of the LRs of asymmetric families are diagnostic of nearly-equilibrium figures, in the sense described earlier.

It is worthwhile to note that the mass distributions of asymmetric families often present a remarkable $\Delta V$ gap between the $L R$ and the main secondary concentrations of mass (see Figure 1). This fact can be explained in the following way. Let us assume that the initial mass vs. velocity distribution had the form

$$
\begin{equation*}
\mathrm{dM}=\mathrm{C}_{\mathrm{o}}^{-\alpha} \mathrm{d} \mathrm{~V}_{\mathrm{o}} \quad\left(\text { for } \mathrm{V}_{\mathrm{o}}>\mathrm{V}_{\min }\right) \tag{5}
\end{equation*}
$$

where $V_{o}$ is the ejection velocity after breakup, $d M$ is the fragmental mass ejected in the interval ( $\left.\mathrm{V}_{\mathrm{O}}, \mathrm{V}_{\mathrm{O}}+\mathrm{d} \mathrm{V}_{\mathrm{O}}\right), \mathrm{C}$ and $\alpha$ are constants and $\mathrm{V}_{\min }$ is a cutoff at small speeds (i.e., $\mathrm{d} M=0$ for $\mathrm{V}_{\mathrm{O}}<\mathrm{V}_{\text {min }}$ ). For crater ejecta, we have $\alpha \simeq 3$. At "infinity", that is after escaping the target's gravity, the velocity of a fragment will be

$$
\begin{equation*}
\mathrm{v}=\left(\mathrm{v}_{\mathrm{o}}^{2}-\mathrm{v}_{\mathrm{e}}^{2}\right)^{1 / 2} \tag{6}
\end{equation*}
$$

and the mass distribution, as a function of $V$ (the observable quantity) becomes :

$$
\begin{equation*}
d M=C V\left(V^{2}+V_{e}^{2}\right)^{-(\alpha+1) / 2} d V \equiv f(V) d V \tag{7}
\end{equation*}
$$

(provided $V_{e}>V_{\text {min }}$ ). Note that the function $f(V)$ approaches to zero for small values of $V$, and has a maximum for $V=V_{e} / \sqrt{\alpha}$. Due to the non-linear relation between $V$ and $V_{0}$, therefore, the probability of observing at "infinity" velocities much lower than $\mathrm{V}_{\mathrm{e}}$ is very small (this conclusion follows for every reasonable form of the mass vs. velocity distribution, and does not depend strictly on the use of Eq.(5)). For instance, for a target's escape speed $V_{e}=0.1 \mathrm{~km} / \mathrm{s}$, the only fragments with final velocities smaller than $0.02 \mathrm{~km} / \mathrm{s}$ will be those ejected initially with $\mathrm{V}_{\mathrm{o}}$ in the narrow range $0.1 \div 0.102 \mathrm{~km} / \mathrm{s}$. This effect could easily explain the observed $\Delta V$ gaps; moreover, it favours a concentration of mass for final velocities of the order of the escape speed, and this is also observed in most cases (as noted earlier).

For all the above-mentioned reasons, it is plausible to expect that asymmetric families will show preferentially the marks of an effective gravitational reaccumulation, i.e., besides equilibrium-shaped LRs, also values close to one of the ratio $\delta \equiv M_{L R} / M_{P B}$. In Figure 2 the various families of our sample are represented in a diagram showing $\delta$ versus $M_{L R}$. Full circles correspond to asymmetric families, while open circles indicate dispersed families; some intermediate cases are also shown as halfcolored circles. We can note that asymmetric families lie always close to the line $\delta=1$, and this correlation is particularly strong for objects with nearly-equilibrium shapes. On the contrary, dispersed families are
 presumably triaxial equilibrium shapes. (Farinella et al., 1981).
Figure 2. For each family the mass ratio between the largest member and the parent body is shown versus (lower horizontal scale) and the diameter (upper scale) of the largest member. Open circles represent dispersed families, full circles asymmetric families and half-colored circles indicate intermediate cases. The numbered crosses correspond to the first four Hirayama families (as classified by Williams, 1979), while the full squares represent non-family asteroids having -

c
present throughout the range $0.1 \approx \delta \approx 0.8$. This trend clearly supports our interpretation that for asymmetric families the catastrophic breakup was just energetic enough to allow a very limited fragment escape, while most of the mass was gravitationally reaccumulated. In the case of dispersed families, on the other hand, the mass loss from the PB was much more pronounced, indicating a comparatively weaker effect of gravity.

The same conclusion is supported by another trend clearly evident in Figure $2: \delta$ is always $\approx 0.7$ for $L R$ diameters $D_{L R} \xlongequal[>]{\sim} 100 \mathrm{~km}$, and always $\lesssim 0.3$ for $D_{L R} \lesssim 30 \mathrm{~km}$. In the intermediate diameter range, a large dispersion of $\delta$ values is apparent. Correspondingly, asymmetric families are preferentially located at large diameters, while the opposite is true for dispersed families. These correlations can be easily interpreted by recalling the transition that we previously described between the response to catastrophic impacts of small asteroids, for which the target's gravity is not so important, and that of large asteroids, which are frequently subjected to gravitational reaccumulation of fragments. In this latter case (corresponding to $\mathrm{V}_{\mathrm{e}} \widetilde{\geqslant} \mathrm{V}_{\mathrm{fr}}$ ) a family, whenever formed, must have $\delta \simeq 1$ and, quite probably, an asymmetric appearance. In Figure 2 we have plotted as full squares also the non-family asteroids whose shapes have been interpreted as Jacobi equilibrium figures by Farinella et al. (1981): they are located obviously along the line $\delta=1$, but the fact that these objects are found preferentially at large diameters appears now as a natural extrapolation of the trend shown by family asteroids. The non-family equilibrium-shaped objects could be considered as "non-born" families, in the sense that they are probably outcomes of impact events causing fragmentation of the target but for which self-gravitation prevented any dispersion of mass, yielding finally a completely reaccumulated "rubble pile" body (Farinella et al., 1982). In general, the evidence provided by Figure 2 represents a strong indication that the transition towards gravity-dominated collisional outcomes does really occur : it appears to begin for diameters of about 50 km and to be almost completed at diameters $\sim 200 \mathrm{~km}$. This result compares satisfactorily with the theoretical estimates based on the results of laboratory impact experiments.

Finally, in Figure 2 we have marked by numbered crosses four of the families originally discovered by Hirayama (the numbers refer to Williams' classification). They are all located in a peculiar region, at the boundary of the zones more populated by the other families, i.e., they have lower values of $\delta$ and/or larger values of $D_{L R}$ with respect to the general trend. This means that these families have small LRs though their PB mass was fairly large. Several hypotheses could explain this peculiar feature : (a) the rupture event possibly was more energetic than the average, due to a larger projectile mass or velocity (note also that Fujiwara, 1982, quotes the possibility suggested by experimental results that for the
same energy a larger projectile mass results into a more effective energy transfer to fragments); (b) possibly the catastrophic event was not a single one : after a first collision converting the PB into a "rubble pile" remnant, a second impact could have dispersed most of the mass (this could explain why the Themis family consists of a "core" of large objects surrounded by a "cloud" of smaller bodies, as noted by Gradie et al., 1979); (c) possibly these are not single families, but are composed by genetically different groups overlapping each other in the space of proper elements (as it seems the case for the Flora family; see Tedesco, 1979). At any rate, a more detailed analysis of the breakup event which generated the Eos, Themis and Koronis families has led Fujiwara (1982) to the conclusion that also in the case of these families the gravitational reaccumulation mechanism had to be of crucial importance.

Before concluding, we have to point out some problems and difficulties of the interpretation proposed in this paper, which in our opinion do not question its validity but certainly deserve further scrutiny. First of all, we have to be cautious in extrapolating all the small-scale experimental results on catastrophic collisions to impact events between bodies of asteroidal size. As we have seen, the experimental relationship between $V_{f r}$ and $E / M$ is consistent with the data on asteroid properties; on the other hand, Fujiwara and Tsukamoto (1980) found another empirical relationship between $E / M$ and the $M_{L R} / M_{P B}$ mass ratio ( $M_{L R} / M_{P B} \simeq 7 \times 10^{8}$ $(E / M)^{-1.34}$, with $E / M$ expressed in erg/g), which yields clearly inconsistent results. According to this relation, in fact, we should expect almost always a very high degree of fragmentation of the target, with typical values of $M_{L R} / \mathrm{MPB}$ of the order of $10^{-3}$. But even in the case of dispersed families (for which we have indications that self-gravity has not been very effective), we observe always $\delta \simeq 0.1$; the problem is not removed by the fact that obvious selection effects favour the discovery of families with $\delta$ not much lower than one. This discrepancy suggests that due to some unknown reason for the same impact energy the asteroids are more resistant than laboratory targets to collisional comminution, even if the broken fragments seem to be ejected with similar speeds in the two cases. The same problem appears for the Saturnian satellite Hyperion, if we assume that its irregular shape is due to a collisional breakup (Farinella et al., 1983), and this fact could indicate that some fundamental physical reason prevents the extrapolation of laboratory results. An alternative possibility could be that the discrepancy is due to a large structural difference between the solid laboratory targets and the asteroids, if the majority of them is converted into "rubble piles" before enduring a really catastrophic impact. Perhaps some comparative experiment on the collisional response of solid vs. fractured targets could test this hypothesis.

Different problems arise when we try to derive the three-dimensional
ejection velocities by using the differences in proper eccentricity and inclination among the asteroids of a given family. From a preliminary analysis, it seems that very often the contribution of eccentricity is dominant and while this could be understood in a single case (due to the possible anisotropy of the velocity field and to the unknown angular orbital elements of the PB at the time of breakup), when we "average" over our entire sample of families, the components of the relative velocities in different directions with respect to the original orbit should give comparable contributions. Possibly this problem is due to some bias implicit in the clustering method used to define family memberships. We note also that in some cases of asymmetric families such a bias could explain why the peak of the fragmental mass distribution is shifted at velocities higher than the PB's escape velocity, contrasting with the trend (described before) connected with the process of gravitational reaccumulation.

In conclusion, we can state that more extensive and detailed studies of the asteroid families, coupling physical and dynamical information, have the potential of providing new and significant insights into the physics and history of the asteroid collisional process. The main prerequisites at present appear to be : (1) a more univocal and reliable definition of the family memberships; (2) a better understanding of the rupture mechanisms for bodies of different structure and size; (3) an enlargement and improvement of the data base on the physical and rotational properties of the family asteroids.

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