SOME REMARKS ON THE THEORY OF ROTATING STARS

L. Mestel
Astronomy Centre, University of Sussex

1. MIXING BY THE EDDINGTON-SWEET CIRCULATION

The principal question studied in Mestel (1953) was: under what conditions will the E-S circulation through the radiative envelope ensure that a Cowling-type main sequence star stays effectively homogeneous? The E-S velocities v_Ω are of order $(L/Mg)\overline{(\Omega^2 r/g)}$, so that the circulation time t_{circ} is $\simeq t(\text{Kelvin-Helmholtz})/\overline{(\Omega^2 r/g)}$, where the bar indicates a mean value. The circulation advects nuclear-processed material from the convective core, and so sets up horizontal variations $\Delta\mu$ in mean molecular weight μ. The condition of hydrostatic support requires corresponding horizontal temperature variations; the consequent local breakdown in radiative equilibrium yields a "μ-current" (analogous to the E-S "Ω-current") with velocities v_μ estimated roughly as $(L/Mg)\,(\Delta\mu/\mu)$. The total velocity v is the linear superposition $\tilde{v}_\Omega + \tilde{v}_\mu$, each constructed using first-order perturbation theory. The μ-distribution that fixes $\tilde{v}_\mu$ is itself determined by the circulation:

$$\partial\mu/\partial t = -\tilde{v}.\nabla\mu. \tag{1}$$

If the whole star is to be kept steadily mixed, then $\partial\mu/\partial t \sim L/M(c^2/125) \sim 1/t_{nucl}$. It is known that near the convective core the μ-current opposes the Ω-current, so that for steady mixing to be possible we require $v_\mu < v_\Omega$, whence from (1)

$$\Delta\mu/\mu \simeq r/(v_\Omega\, t_{nucl}) \simeq (t_{KH}/t_{nucl})/\overline{(\Omega^2 r/g)} \simeq 10^{-2}/\overline{(\Omega^2 r/g)}. \tag{2}$$

Note that in this problem the correct asymptotic form for $\Delta\mu/\mu$ varies inversely with $\overline{\Omega^2 r/g}$. "Rapid rotation" in this context means: $\overline{\Omega^2 r/g}$ large enough to keep $\Delta\mu/\mu < \overline{\Omega^2 r/g}$, so that the assumption $v_\mu < v_\Omega$ is vindicated. From (2), this yields

$$\overline{\Omega^2 r/g} > (t_{KH}/t_{nucl})^{1/2} \simeq 1/10. \tag{3}$$

(This should be contrasted with the estimate $\overline{\Omega^2 r/g} > t_{KH}/t_{nucl} \simeq 10^{-2}$ that follows from ignoring the μ-currents, and just comparing

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t_{circ} with t_{nucl}). The value (3) is high, but not so as to invalidate the use of linear perturbation theory over the bulk of the star; and in any case, the above meaning of rapid rotation is logically quite distinct from "requiring second-order terms".

The accurate treatment requires solution of the angle-dependent part of (1), subject to the appropriate boundary conditions $\Delta\mu = 0$ at the core surface and at the star's surface, and with $\partial\mu/\partial t$ again given by assuming steady mixing. The critical angular velocity is found to be given by

$$(\Omega^2 r/g)_{\text{core surface}} \simeq 1/30. \qquad (4)$$

(An extension of the analysis to take account of Legendre coefficients beyond the second will almost certainly increase this value somewhat). If a star could rotate more rapidly than this, then it would evolve up and to the left of the ZAMS. The Tassouls (1983) fail to find a critical angular velocity because they impose from the beginning $\Delta\mu/\mu \propto \overline{\Omega^2 r/g}$. They state that the horizontal μ-gradient is just a consequence of the rotational distortion of a zero-order radial μ-gradient, whereas in the present problem both gradients are determined by the circulation, and both decrease with increasing $\overline{\Omega^2 r/g}$. However, if the star rotates nearly uniformly, as is probable, then condition (4) implies that $\Omega^2 r/g$ at the stellar surface is near unity, simply because in a Cowling model the mean density within radius r decreases from core to surface by a factor ≃ 30. Thus it seems very unlikely that a star can in fact rotate at the necessary rate, but the limitation is set by the conditions at the stellar surface; in the region near the core where the μ-currents oppose the Ω-currents, linear perturbation theory would suffice. Similar results hold for shell-source stars (Mestel 1957).

If continuous steady mixing by the E-S circulation is ruled out, how does a rapidly rotating star evolve? The original suggestion was that the circulation would continue in the radiative envelope, but would be deflected horizontally in a viscous boundary layer just outside a "μ-barrier" surrounding the central nuclear-processing region. However, it has proved difficult to construct such a model satisfying all the boundary conditions, and so it was later suggested alternatively that the circulation would be subject to "creeping paralysis"; the Ω-currents would steadily build up a μ-distribution that leads ultimately to $\underset{\sim}{v}_{\Omega} + \underset{\sim}{v}_{\mu} = 0$. The Tassouls in fact arrive at this picture from their analysis. In a rapidly rotating star, the associated radial μ-variations could have a significant effect on structure and so on position in the H-R diagram.

2. GENERAL COMMENT ON THE THEORY OF ROTATING RADIATIVE ZONES.

Studies of non-magnetic rotating radiative zones implicitly impose severe upper limits on the strength of any poloidal magnetic field B_p. It was early recognized that to keep the E-S circulation flowing in a chemically nearly homogeneous region, one needs something to off-set

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the departures from uniform rotation caused by advection of angular momentum by the circulation itself. Provided the Alfvén speed $B_p/(4\pi\rho)^{1/2} >> v_\Omega$, the non-uniform rotation will be kept small; also, over the bulk of the star one can expect $B_p^2/4\pi\rho << \Omega^2 r^2$, so that the implicit neglect of the magnetic disturbance to hydrostatic and thermal equilibrium is justified. It is because $v_\Omega << \Omega r$ that a field of less than 1 gauss can be simultaneously negligible for gross hydrostatic equilibrium and overwhelmingly dominant in determining the rotation field. Further, magnetic forces may be locally important for hydrostatic and thermal equilibrium. I agree that the standard E-S treatment is likely to yield large horizontal velocities in boundary layers, which in a non-magnetic star must be subject to locally large viscous forces. However, in a star with even a weak large-scale magnetic flux distribution, continuity plus flux-freezing can build up fields that locally exert forces comparable with the centrifugal force. For example in their approximate self-consistent steady-state models of rotating magnetic stars, Mestel and Moss (1977) and Moss (1983) find that in the low-density surface regions, the magnetic field satisfying the kinematic hydromagnetic equation (with finite resistivity necessarily retained) must be included in the hydrostatic and thermal equations.

In this Symposium we are especially interested in anything that can cause mixing of material and so affect the direction of stellar evolution. Over the long time-scales available, even a weak large-scale magnetic flux F in a rotating star may be significant. As an example, I refer to studies of the "Internal dynamics of the oblique rotator" (Mestel et al 1981; Nittmann and Wood (1981, 1983)). The motions enforced on the star can be analysed into the Eulerian nutation, of frequency $\omega \simeq \Omega(F^2/\pi^2 GM^2)$, and consequent divergence-free "$\underset{\sim}{\xi}$-motions" that preserve hydrostatic equilibrium. The radial amplitude $\xi_r \sim (\Omega^2 r/g)\lambda$, where λ is the local scale-height. Provided the $\underset{\sim}{\xi}$-motions persist, an oblique, rapidly rotating star may experience significant mixing, either directly through the oscillatory $\underset{\sim}{\xi}$-motions, or indirectly through the destruction of the μ-field that would kill off the E-S circulation.

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