THE ACCELERATION OF GALACTIC COSMIC RAYS

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1. INTRODUCTION

Although it is clear that cosmic rays are of considerable astrophysical significance we know surprisingly little about their origin(s), their mode(s) of propagation and escape from the galaxy, and even of their general properties. Practically nothing is known of the composition and energy spectra of different species at energies below about 200 MeV/nucleon as a consequence of solar modulation, which has a significant effect on the observed spectra at energies up to about 10 GeV/nucleon. The energy spectra of ions (especially protons) appear to be well approximated by a power law (differential spectral index $\gamma \sim 2.6$) in the range 10^{10}-10^{15} eV/nucleon. At higher energies (up to $\sim 10^{20}$ eV/nucleon) both the spectrum and the composition of the ion component are less well known, however it is significant that the anisotropy, as evidenced from the sidereal diurnal variation, increases substantially, suggesting that perhaps an extragalactic origin must be considered (Wolfendale, 1977).

As far as the composition is concerned, it appears that there is a noticeable enhancement of the heavier primary ions relative to alpha-particles and (to a greater extent) protons, although the latter are still the dominant species at, say, a given energy/nucleon. One should note that relative abundances, instead of referring to the whole spectrum, are usually expressed on the basis of equal energy/nucleon, total energy, energy/charge or rigidity and the choice among these is largely a matter of taste. As far as the heavier ions are concerned, it seems to be the case that the composition is more or less consistent with what might be expected for the interstellar medium at the present epoch with due allowance for propagation of the primary beam through ~ 3 g/cm^2 after injection. Ignoring the effects of modulation, the total energy density of cosmic radiation in the solar neighbourhood is of the order of 10^{-12}ergs/cm^3 which is comparable to the energy density of the interstellar magnetic field and the total (thermal plus kinetic) energy density of the interstellar gas. This fact is presumably a consequence of the nature of the mechanisms of containment and escape of cosmic rays

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from the galaxy rather than to their sources and modes of acceleration. However, with this result and guessing the total volume occupied by galactic cosmic rays to be $2 \times 10^{67} cm^3$ we can use the "age" of 2×10^7 years deduced from the Be^{10} flux observed at (modulated) energies of ~ 100 MeV/nucleon (Garcia-Munoz et al., 1977) to obtain a total source power of 3×10^{40}ergs/sec.

Measurements of cosmic ray induced radioactivity in meteorites indicate that the average intensity of the cosmic radiation in the vicinity of the sun has remained constant to within a factor of 2 or 3 over the last 10^8-10^9 years (Forman and Schaeffer, 1979; Honda, 1979). This result does not exclude the possibility of short-lived variations, however, recent measurements of the amounts of Be^{10} in deep sea sediments suggest that there have been no significant enhancements during the last 2×10^6 years (Somayajulu, 1977). This may be an especially important result in view of the fact that there is some evidence that the sun is situated close to, or even inside, a very large supernova remnant which is of the order of 10^6 years old (Frisch, 1979, and private communication).

The situation with respect to electrons is rather less clear since their flux at a given energy is considerably less (~ 100 times) than that of protons and electrons are experimentally more difficult to measure. The spectral index in the range 1-10 GeV is similar to that of the protons and the bending that should result from synchrotron and inverse Compton losses gives an age which is roughly consistent with that obtained from Be^{10} in the cosmic radiation (Giler and Wolfendale, 1979). The positrons, which are clearly secondaries, provide in principle a better determination of both path length and age but again due to experimental difficulties the results are still somewhat uncertain.

Synchrotron emission associated with the interstellar magnetic field and inverse Compton scattering of starlight and microwave background radiation photons provide means of investigating the intensities of electrons throughout large regions of interstellar space, especially, in the case of synchrotron radiation, at energies below the range in which solar modulation effects become severe (e.g. Webber et al., 1979). As the observations refer to line-of-sight averages and involve several unknown factors such as the strength of the interstellar magnetic field and its variations, it is difficult to reach any very definitive results. However, it is clear from the distribution of non-thermal radio emissions in the galaxy that the cosmic ray electron flux is far from uniform. There are large variations, especially associated with supernova remnants and hence sources of energetic electrons may exist in which the present intensity is relatively high. Similar conclusions can be drawn from observations of the gamma ray emissions induced by protons (Kanbach, 1979).

2. ACCELERATION MECHANISMS

An excellent summary of the current view of various possible cosmic ray acceleration mechanisms has been given by Lingenfelter (1979). It seems likely, on the basis of the rather scanty evidence outlined above, that the most important energizing processes take place in the interstellar medium rather than in unusual objects such as supernovae, pulsar magnetospheres, flare stars, and the like. The mechanisms which are available in interstellar space are all dependent on mass motions of the medium, notably small-scale motions or waves (Fermi acceleration and magnetic pumping), and large scale motions involving shocks which are associated with supernovae, novae, expanding HII regions, stellar winds, and so on. In fact, these may all play a role but on energetic grounds at least the mass motions due to supernovae are likely to be the most important source. Shock waves provide the most direct and effective way of accelerating high energy particles and most of this review is concerned with this particular topic. However, the other processes are not negligible and may play a role at least in providing "seed" particles.

Of the possibilities which will not be discussed here in detail the supernova mechanism of Colgate and Johnson (1960, see also Colgate, 1979) is perhaps the most interesting. According to this idea the supernova shock wave, which tends to speed up on reaching the outer envelope of the star, eventually becomes relativistic. If this material were able to escape freely and diffuse into the interstellar medium without losing any significant amount of energy an energy spectrum could be achieved which is a power law not very different from that observed. There appears to be an essential difficulty, however: the particles must lose energy by doing work on the gas surrounding the star (either the pre-existing stellar wind or the interstellar medium) and there is no reason to believe that they retain their initial spectrum or even remain relativistic. Rayleigh-Taylor type instabilities might help overcome this difficulty to some extent but do not obviously save it.

A second process involving supernova remnants, namely acceleration in pulsar magnetospheres, has been considered in some detail during the last ten years (see Arons, these Proceedings). Current ideas suggest however, that the particles accelerated in such regions are essentially all electrons and positrons and so it is clear that, as presently envisaged, the mechanism has nothing to do with cosmic rays. Of course the power associated with slowing down of the pulsar rotation should eventually appear as kinetic energy of the associated supernova remnant and may in turn be used to accelerate cosmic rays but the mechanisms involved would then be those discussed later in this paper.

A third mechanism which is sometimes invoked involves magnetic field line reconnection, i.e. stellar and galactic "flares". It is not easy to evaluate the effectiveness of this mechanism in view of our relative lack of knowledge of flare processes other than those occurring on the sun and in the earth's magnetosphere. One can say, however, that the maximum energy obtainable is determined by the voltage drop occurring along a magnetic "neutral" line which should exist in the flare site. This voltage drop is of the order of V_ABL, where V_A and B are the characteristic Alfvén speed and magnetic field strength, respectively, and L is the length of the neutral line. With typical values of these quantities appropriate to the earth's magnetosphere, the sun and the galaxy as a whole, one finds maximum energies of the order of 10^6eV, 10^{10}eV, and 10^{13}eV per charge, respectively. It seems doubtful that the total energy available in stellar flares is adequate to account for the cosmic radiation and even though large solar flares are quite efficient in producing high energy particles, one must remember that if the particles do not escape freely they loose energy by doing work on the surrounding medium. Galactic flares may not suffer from these disadvantages but on the other hand their existence is questionable, since there is no obvious reason why magnetic field line reconnection on a galactic scale should take place abruptly in a flare-like manner.

It is instructive in considering possible cosmic ray acceleration mechanisms to take account of the "ground truth" provided by in situ observations of particle acceleration made in the terrestrial and Jovian magnetospheres and in the interplanetary medium. Shock acceleration is commonly observed both in magnetospheric bow shocks and in interplanetary propagating shocks; acceleration associated directly with magnetic field line reconnection is observed in the tail of the earth's magnetosphere (Sarris and Axford, 1979, and references therein); adiabatic acceleration and deceleration occurs as a result of magnetospheric convection (Axford, 1969); semi-adiabatic acceleration is associated with the break-down of one but not all adiabatic invariants of particle motion in the presence of large-scale, time-varying electric fields (Schulz and Lanzerotti, 1974); stochastic acceleration due to interactions with waves appears to occur throughout the magnetosphere and possibly the magnetosheath; acceleration due to the presence of electric fields parallel to the magnetic field occurs in the auroral zone magnetosphere where electric currents become too large to be carried otherwise by the ambient plasma (Swift, 1979; Johnson, 1979); and finally, in the case of Jupiter, magnetospheric rotation and the effects of neutral particle pick-up are not insignificant acceleration mechanisms at least at low energies. All of these processes can be considered as candidates for cosmic ray acceleration and, except perhaps for the semi-adiabatic mechanisms mentioned, all may have a role to play. Probably, however, the processes which are most important are shock acceleration together with acceleration/deceleration effects due to compressions/expansions of the medium as a whole and also stochastic acceleration due to the presence of turbulence. In any case, it cannot be too strongly emphasized that the processes available to us should not be considered as being in any sense speculative or without observational foundation - indeed, any theoretical schemes we may construct should be tested against in situ observations available in interplanetary space, if they are to be taken at all seriously.

3. STOCHASTIC ACCELERATION

There are many possibilities for particle acceleration involving plasma/magnetohydrodynamic wave turbulence. For moderately energetic particles (i.e. those with speeds large compared with the characteristic Alfvén and magnetoacoustic speeds in the medium) the most interesting are those involving Alfvén wave turbulence (second order Fermi acceleration, Fermi 1949, 1954; Hasselmann and Wibberenz, 1968) and magnetoacoustic turbulence (magnetic "pumping", Thompson, 1955; Fisk, 1976). With suitable assumptions the source strength for these two mechanisms can be written

$$Q = \frac{\partial^2}{\partial T^2} (D_1 U) - \frac{\partial}{\partial T} (D_2 U/T) , \tag{1}$$

where U(T) is the cosmic ray number density in the kinetic energy range (T, T+dT). $D_1(T)$ and $D_2(T)$ are the sum of two components $D_A \propto \langle V_A^2 \rangle / \kappa$ and $D_S \propto \langle V_S^2 \rangle / \kappa$, where $\langle V_A^2 \rangle$ and $\langle V_S^2 \rangle$ are the mean square speeds due to Alfvén and acoustic mode turbulence, respectively, and κ is the spatial diffusion coefficient which is associated largely with pitch angle scattering. Note that both processes are stochastic in nature, involving a second derivative with respect to energy and therefore diffusion in energy space. One may regard the Fermi acceleration mechanism as one in which light test particles (i.e. cosmic rays) try to come into thermal equilibrium with a field of randomly moving heavy particles (i.e. Alfvén waves). The magnetic pumping mechanism may be regarded as a process in which acoustic waves are damped by the equivalent of heat conduction by cosmic rays.

It is difficult to evaluate the effectiveness of these mechanisms as far as accelerating cosmic rays is concerned, as we have little information concerning the intensities of these modes of turbulence in the interstellar medium (see Jokipii, 1977; Cesarsky, 1980). Obviously, most of the energy available in the interstellar medium is involved in large scale mass motions which even if they are superficially chaotic cannot be regarded as representing turbulence in the sense implied above. Of course, such large scale motions may eventually degenerate into turbulence, so that on energetic grounds these stochastic processes should not be ruled out. However, if acceleration in connection with the large scale motions is very efficient as we will argue, one would expect that stochastic processes play only a secondary role. Nevertheless, since shock waves must inevitably be associated with large scale mass motions and hydromagnetic shocks can generate quite intense wave turbulence (as observed in interplanetary space) it seems quite likely that stochastic acceleration is important in producing "seed" particles from the high energy tail of the background plasma velocity distribution which could be ultimately accelerated to much higher energies by the shock acceleration mechanism.

As randomly moving shock waves interact with each other and the inhomogenieties that inevitably occur in the interstellar medium one would expect the strong shocks to degenerate into a random assembly of weak shock waves with associated expansions and compressions of the medium. The effect of such a situation on energetic particles has been considered by Bykov and Toptygin (1979) who show that it leads to acceleration in a manner similar to that expressed in equation (1) above. This is perhaps not surprising since weak shock waves are in every other sense essentially equivalent to sound waves.

4. GENERAL ASPECTS OF SHOCK ACCELERATION

Shock acceleration with regard to cosmic rays was first discussed theoretically by Parker (1958), Hoyle (1960), and Schatzmann (1963). Direct evidence for the occurrence of shock acceleration in interplanetary space was first found in the case of moderately energetic (1-100 GeV) cosmic rays by Dorman and his colleagues (see Dorman, 1963) in the form of a small ($\sim 1\%$) increase in intensity of cosmic rays observed by ground stations immediately before the occurrence of a geomagnetic sudden commencement. The latter is of course the signature of an interplanetary shock wave interacting with the earth's magnetosphere. Dorman and Freidman (1959) formulated a single reflection theory to account for these observations, which was the first of a series of papers treating this topic (see section 5).

Much clearer evidence for shock acceleration in interplanetary space was obtained by Axford and Reid (1962, 1963) from observations of polar cap absorption events using riometer networks which respond to the influence of solar energetic particles with energies of the order of 10 MeV. They found a number of cases in which large quasi-exponential increases of the solar energetic particle flux occurred prior to geomagnetic storm sudden commencements. Since a single reflection would be inadequate to explain the effect, these authors invoked multiple reflection on looped magnetic field lines which intersect a shock in two places or between two approaching shocks to obtain a many-fold increase in energy.

These ground-based observations were confirmed by in situ spacecraft observations, notably by Explorer 12 (Bryant et al., 1962) and later by numerous other US, Soviet, and German spacecraft. The observations are by now quite detailed in terms of mass, energy, directional and time resolution and show that in addition to the relatively slow, quasi-exponential increases mentioned above there are also short-lived, highly anisotropic bursts ("shock spikes") which occur near the time of but not necessarily coincident with the shock passage (Sarris and Van Allen, 1974). Perhaps the most impressive examples of shock acceleration in interplanetary space are those obtained by Helios (Richter and Keppler, 1977), and from Pioneers 10 and 11 (Barnes and Simpson, 1976). The latter show distinct double-peaked structures associated with forward-reverse shock pairs which occur as part of corotating interaction regions in the solar wind. The peaks are evidently the result of shock acceleration but the characteristic depression in the middle is probably the result of adiabatic deceleration in the expanding region between the two shocks (Skadron and Axford, 1976; Axford, 1977).

Isolated bursts and a generally structured distribution of energetic electrons was observed by several of the first spacecraft to pass out of the earth's magnetosphere on the sunwards side (Van Allen, 1959). These electron fluxes were considered to be part of the evidence for the existence of a bow shock in front of the magnetosphere and to be the result of acceleration in shock-associated turbulence (Axford, 1962). Jokipii and Davies (1964) interpreted the electrons found upstream of the shock as being the result of Fermi acceleration between the shock wave and scattering fields carried in the approaching solar wind (for a detailed discussion, see Jokipii, 1966). This suggestion, which is in principle similar to that of Schatzmann (1963) could not be fully worked out in the absence of suitable transport equations for the energetic electrons. However, it is essentially the concept involved in present-day scattering theories of shock acceleration as outlined in section 6 of this paper (see also Van Allen and Ness, 1967). In fact, it is possible that a third quite different mechanism may be responsible for many of the electron bursts seen upstream of the earth's bow shock, namely an electrostatic field in regions of the bow shock which are approximately tangential to the external interplanetary magnetic field and which increases the electron energy by a factor of the order of 10-40 times the solar wind proton energy of about 1 keV (e.g. Sonnerup, 1969). It would require only modest scattering in pitch angle for electrons which have transiently achieved such energies to escape upstream across the curved shock surface.

5. ACCELERATION BY LAMINAR SHOCKS

In the absence of pitch angle scattering energetic particles with large gyro-radii which traverse abrupt discontinuities in magnetic field strength and/or direction are found to conserve their magnetic moment, at least approximately (Parker, 1958; Alexeyev and Kropotkin, 1970). This has the immediate consequence for shocks propagating perpendicular to the upstream magnetic field that the increase in perpendicular kinetic energy (non-relativistic) is proportional to the increase in magnetic field strength and the parallel kinetic energy remains unchanged. The energy gain can be viewed as being the result of drift in the inhomogeneous magnetic field near the shock wave in the direction parallel or anti-parallel to the ambient electric field and in the case of strong shocks, amounts to a factor 4 for non-relativistic particles. This is sufficient to enhance the synchrotron emissivity per unit volume of the medium by a factor $\sim$ 500-1000 so it is clear that shocks are likely to produce significant inhomogenieties in the galactic non-thermal radio emission (cf. Hoyle, 1960; Chevalier, 1977).

In considering the reflection of particles from non-perpendicular shocks it should be noted that a particle must have a certain minimum kinetic energy T_m if it is required to move upstream:

$$T_m = \frac{1}{2} mV_1^2 \cot^2\theta_1 \ , \tag{2}$$

where the magnetic field line makes an angle θ_1 with respect to the shock surface and V_1 is the flow speed normal to the shock. For example, in the case of the shock wave terminating the supersonic solar wind near the ecliptic plane at a distance of 100 AU, say, $\theta_1 \sim 0.5°$ and $T_m \sim 10$ MeV/nucleon: galactic particles with lower energies are unable to penetrate into the supersonic region.

Assuming that particles interact with an oblique shock in such way that their magnetic moments are conserved one finds that for reflection to occur the incident particles must have a certain minimum energy T_1 and the reflected particles have a minimum energy T_2 in the shock frame:

$$T_1 = \frac{B_1}{(B_2 - B_1)} T_m \ , \qquad T_2 = (4 + \frac{B_1}{(B_2 - B_1)}) \ T_m \ , \tag{3}$$

where B_1 and B_2 are the magnetic field strengths ahead and behind the shock respectively. The transmitted particles tend to have a flat pitch angle distribution and there is no restriction on their energies, whereas reflected particles tend to be beamed along the magnetic field and must have energies greater than T_2. As in the case of perpendicular shocks, the acceleration can be regarded as being the result of drift in the direction of the electric field which occurs when particles interact with the inhomogeneous magnetic field at the shock front.

Numerous analytic and numerical studies have been made of energetic particle interactions with laminar shock waves (Dorman and Freidman, 1959; Parker, 1958, 1963; Shabansky, 1962; Wentzel, 1963, 1964; Hudson, 1965; Sonnerup, 1969; Alexeyev and, Kropotkin, 1970; Sarris and Van Allen, 1974; Chen and Armstrong, 1975; Pesses, 1979; Terasawa, 1979a). The results obtained must be correct if the particle gyroradius R_g is large compared with the shock thickness d, provided only that the scattering mean free path λ is very much larger than R_g. These are conditions which should certainly prevail in shock waves occurring in the interstellar medium and in the solar wind, so that this type of acceleration must be taken into account. If $R_g \sim d$ similar acceleration effects should occur but in this case the particles are likely to be affected by turbulent fields in the shock itself, so that the analysis would not be strictly valid.

One should note that a model based on purely laminar reflection and transmission of particles at shock waves cannot explain features such as the quasi-exponential increases which occur ahead of interplanetary shock waves. Such effects require that scattering be present, which in turn produces a qualitative change in the acceleration mechanism and can lead to acceleration to much higher energies. Attempts have been made to include scattering in numerical simulations of the laminar reflection process but in some cases the authors appear to have done this incorrectly (see, however, Terasawa, 1979b). It seems to be the case, nevertheless, that interplanetary shock spike events can be explained entirely on the basis of direct acceleration at the shock front itself without recourse to pitch angle scattering (Sarris et al., 1976).

6. ACCELERATION BY SHOCKS IN SCATTERING MEDIA

In order to understand the effects of scattering on the acceleration of energetic particles by shock waves it is necessary to have suitable transport equations describing the effects of convection and diffusion of particles in the scattering medium as well as energy changes due to various causes. The appropriate Fokker-Planck equation

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x}(VU) = \frac{\partial}{\partial x}\left(\kappa\frac{\partial U}{\partial x}\right) + \frac{1}{3}\frac{\partial V}{\partial x}\frac{\partial}{\partial T}(\alpha TU) + Q \;, \tag{4}$$

was given independently by Parker (1965) and Dolginov and Toptygin (1966). Here V is the speed of the scattering medium, Q represents energy gains and losses other than those due to compression and expansion, and $\alpha = (T+2T_0)/(T+T_0)$ where T_0 is the particle rest energy. The equation is written in its one-dimensional form appropriate to the treatment of plain shocks and accordingly κ is to be regarded as the coefficient for diffusion normal to the shock front. In order to make progress with the problem of shock acceleration it is necessary to use a pair of first order equations which are equivalent to (4)

$$\frac{\partial U}{\partial t} + \frac{\partial S}{\partial x} = -\frac{1}{3}V\frac{\partial^2}{\partial x\partial T}(\alpha TU) + Q \;, \tag{5}$$

$$S = V\left(U - \frac{1}{3}\frac{\partial}{\partial T}(\alpha TU)\right) - \kappa\frac{\partial U}{\partial x} \;, \tag{6}$$

obtained originally by Gleeson and Axford (1967). Here S is the particle current in (T, T+dT) and $C = (1 - (\partial(\alpha TU)/\partial T)/3U)$ is the "Compton-Getting" coefficient. In deriving these equations it is assumed that (1) the distribution function is nearly isotropic ($S/U \leqq 0.1\,v$, where v = particle speed); (2) the Compton-Getting transformation is linear ($V \ll v$); and (3) inertia effects are negligible ($\partial S/\partial t \ll v^2 S/3\kappa$). It should be noted that these assumptions can easily be violated in shock acceleration problems and conclusions should accordingly be drawn only with care. The equations are readily generalized to take into account anisotropic diffusion, three-dimensional flow, etc. Furthermore, it should be noted that the "adiabatic" acceleration/deceleration terms appearing in (4) and (5) are first order Fermi effects being in principle reversible and non-statistical in nature. However, if the velocity of the scattering medium has a random component with wave-lengths large compared with the scattering mean free path, it is possible, with suitable averaging, to derive the acceleration rate due to magnetic pumping given previously (Bykov and Toptygin, 1979).

Anticipating the use of the above equations in shock acceleration problems Gleeson and Axford (1967) gave the following "jump" conditions for the energetic particle density and current for a shock wave described as a rapid or discontinuous change of V with respect to x:

$$U_1 = U_2, \quad S_1 = S_2 , \tag{7}$$

where subscripts 1 and 2 refer to conditions ahead of and behind the discontinuity (shock), respectively. These conditions are obtained by integrating equations (5) and (6) across the transition, assuming uniform conditions on either side, that V, U, S, C, Q and $\partial U/\partial t$ are finite and that $\kappa > 0$. Note that although (7) refers to simple normal shock waves, a generalization to the case of oblique shocks with anisotropic diffusion coefficients is trivial. The procedure outlined is in fact invalid for the general case of oblique shock waves where the magnetic field lines are sharply kinked since we expect that $\lambda \gg d$ and the above equations may be invalid if used on such a scale. Furthermore, the possibility of reflection by the shock front itself is completely neglected; indeed, since particle inertia is neglected in the transport equations the requirement for a minimum energy for motion upstream and all the other considerations discussed in section 5 are also absent. Consequently, results obtained on the basis of the above equations and jump conditions should be considered as lacking an important physical element in all but the cases of perpendicular, parallel, and very weak oblique shocks where the magnetic field lines remain straight.

Consider a situation in which a shock wave is situated at $x = 0$ facing in the negative x- direction so that $V = V_1$ in $x < 0$ and $V = V_2$ in $x > 0$, with $V_1 \geqq V_2$. We assume that $U(T) \to U_1(T)$ as $x \to -\infty$ and U remains finite ($U \to U_2(T)$) as $x \to +\infty$. The appropriate solutions of equations (5) and (6) are

$$\begin{aligned} U(x,T) &= U_1(T) + \left[U_2(T) - U_1(T)\right] \exp \int_0^x (V/\kappa)dx , \quad x < 0 , \\ U &= U_2(T) , \quad x > 0 , \end{aligned} \tag{8}$$

where $U_2(T)$ is to be determined by requiring continuity of S(T) at $x = 0$:

$$\frac{1}{3} (V_1 - V_2) \frac{\partial}{\partial T} (\alpha T U_2) + V_2 U_2 = V_1 U_1 . \tag{9}$$

Hence

$$U_2(T) = \frac{3V_1}{\alpha(V_1 - V_2)} T^{-(\lambda+1)} \int U_1(T) T^{\lambda} dT , \tag{10}$$

where $\lambda = 3V_2/\alpha(V_1 - V_2)$.

The following are examples of the types of accelerated spectra that can be obtained:

$$\text{(a)}\quad U_1(T) = U_1\delta(T-T_1)\ ,$$
$$U_2(T) = (V_1\lambda/V_2)(U_1/T_1)H(T-T_1)(T/T_1)^{-(\lambda+1)}; \tag{11}$$
$$\text{(b)}\quad U_1(T) = U_1(T/T_1)^{-\mu}H(T-T_1)\ ,$$
$$U_2(T) = \beta\, U_1\left[(T/T_1)^{-\mu} - (T/T_1)^{-(\lambda+1)}\right] H(T-T_1)\ ; \tag{12}$$
$$\text{(c)}\quad U_1(T) = U_1(T/T_1)^{-(\lambda+1)}(\log_e T/T_1)^n H(T-T_1)\ ,$$
$$U_2(T) = (V_1\lambda/V_2)U_1(T/T_1)^{-(\lambda+1)}\ \frac{(\log_e T/T_1)^{n+1}}{(n+1)}\ H(T-T_1); \tag{13}$$

where H(x) is the Heaviside step function and $\beta = 1/(1-C_1(V_1-V_2)/V_1)$. The basic result (11) was obtained independently by Axford et al. (1977), Krimsky (1977), Blandford and Ostriker (1978), and Bell (1978). The most notable features of these solutions are that the pre-shock increase is exponential (if κ_1 is independent of x) with a scale length κ_1/V_1 and that a special power law appears with index $(\lambda+1)$, which is dependent only on the shock strength and not on the form of the diffusion coefficient.

A number of generalizations of this result are possible:

(a) The diffusion coefficient may be taken to be anisotropic and the shock oblique. One finds that in the case where the magnetic field makes a small angle with the shock surface the anisotropy can become very large and the transport equations may then cease to be valid.

(b) The effects of sources and sinks can be taken into account in certain special cases. For example, if $Q = -U/\tau(T)$ in $x > 0$ to simulate ionization and nuclear interaction loss for example, one finds that the effectiveness of the acceleration mechanism is drastically reduced if $\tau \leqq \kappa_2/V_2^2$ (Völk et al., 1979; Bulanov and Dogiel, 1979). The effects of losses due to synchroton and inverse Compton emission have been considered by Krimsky et al. (1979b) and the effects of acceleration by turbulence have been considered by Tverskoi (1978) and, in a Monte-Carlo treatment, by Scholer and Morfill (1975).

(c) Time variations, namely the inclusion of the term $\partial U/\partial t$ in (8), have been treated by Fisk (1971), Toptygin (1978) and Forman and Morfill (1979). It is found that the equilibrium spectrum is achieved only after a time of order $\kappa_1 \ln(T/T_1)/V_1^2$, which, for the more energetic particles, may be excessively long in comparison with other characteristic time scales so that the equilibrium spectrum is never achieved at high energies.

(d) Various solutions have been given for the case of flow with spherical symmetry. These include: the familiar solar modulation problem with a shock (Fisk, 1969; Axford, 1972; see also Jokipii, 1968); a Forbush increase/decrease model with a shock wave moving outwards with constant speed (Fisk, 1969); a driven double shock/contact surface combination representing a corotating interaction region (Skadron and Axford, 1976); a Sedov blast wave solution (Krimsky et al., 1979a), and an accretion shock model (Cowsik and Lee, 1980). Some of the above results are in need of revision as part of the solution has been overlooked. A quite realistic model of a corotating interaction region has been treated by Fisk and Lee (1980) who have shown that the effects of adiabatic deceleration on both sides of an expanding shock wave in the solar wind are significant in shaping the spectrum, as is also the form of the diffusion coefficient, and that it is possible to account for the observed spectral form of an exponential in rigidity (see also Forman, 1980).

The essential conclusion of this type of analysis is that, provided scattering occurs, energetic particles can be accelerated very efficiently by shock waves without recourse to the laminar reflection mechanism discussed in section 5. Power law spectra can be achieved fairly easily and in the case of strong shocks ($V_1 \to 4V_2$) we obtain ($\lambda+1$) → 2 for relativistic particles ($\alpha \to 1$) which is rather close to the value 2.6 found for protons at energies of the order of 10 GeV or more (Krimsky, 1977; Blandford and Ostriker, 1978). In fact, not too much should be made of this result since more general analyses show that the spectrum obtained can be affected by such factors as adiabatic acceleration/deceleration, energy sources and sinks, time-dependence and non-linear effects (see section 7), all of which permit the diffusion coefficient (and especially its energy dependence) to have some influence on the spectrum of the accelerated particles. Perhaps the most important conclusion is that the acceleration is non-adiabatic (i.e. irreversible) in the sense that much more energy is given to energetic particles than would be achieved by adiabatic compression with the same compression ratio. This possibility, which was first noted by Hoyle (1960), contrasts with the common assumption that the energetic particles are simply compressed adiabatically (e.g. Newman and Axford, 1968; Chevalier, 1977).

7. THE SELF-CONSISTENT PROBLEM

If one calculates the pressure p_{c2} of cosmic rays behind a strong shock wave according to equation (11), it is found that it can become very large:

$$p_{c2} = \int_0^\infty \left(\frac{1}{3}\,\alpha U_2 T\right) dT \propto \left[\log_e T/T_1\right]_{T_1}^{\infty} .$$

In reality such a divergence would be easily supressed by effects such as time-dependence and energy losses, however it suggests that we should seriously take into account the effects of cosmic ray pressure on the background plasma flow. This problem has been considered by Axford et al. (1977) (see also Leer et al., 1976) for the case of a steady state one-dimensional flow, thus:

$$\rho V = \rho_1 V_1 = A_1 , \tag{15}$$

$$p + p_c + \rho V^2 = p_1 + p_{c1} + \rho_1 V_1^2 = A_2 , \tag{16}$$

$$\rho V \frac{d}{dx}\left[\frac{1}{2} V^2 + \frac{\gamma}{\gamma-1} \frac{p}{\rho}\right] = - V \frac{dp_c}{dx} , \tag{17}$$

where ρ is the plasma mass density, p and p_c the plasma and cosmic ray pressures, respectively, and V the speed of the plasma in the x direction. In addition we make use of the cosmic ray transport equation (4), integrated over energy and assuming $\kappa = \kappa(x)$ for convenience:

$$Vp_c - \kappa \frac{dp_c}{dx} + \frac{\alpha}{3} \int_{-\infty}^{x} \frac{dV}{dx} p_c \, dx = V_1 p_{c1} . \tag{18}$$

It is easily shown, using (15)-(17) that the background plasma behaves isentropically if there are no shock waves in the flow (i.e. p/ρ^γ = constant), and also that the total energy flux is constant:

$$\rho V(\tfrac{1}{2} \rho V^2) + \rho V(\tfrac{\gamma}{\gamma-1} p/\rho) + \int_0^\infty S(T)TdT = F_\kappa + F_T + F_c = A_3 . \tag{19}$$

Numerous solutions of the above set of equations for the case $\alpha = 2$ have been given by Leer et al. (1976) and Axford et al. (1977) with various initial conditions. To understand the nature of the solutions it is sufficient to consider the very simple case in which the plasma pressure is neglected everywhere (p = 0), so that ρ and p_c can be eliminated to yield

$$\kappa \frac{dV}{dx} = (1 + \alpha/6)(V_1 - V)(V_2 - V) , \tag{20}$$

where $V_2 = [(1+\alpha/3)A_2/(1+\alpha/6)A_1] - V_1$. Note that for $p_{c1} = 0$, $V_2 \rightarrow V_1/4$ if $\alpha = 2$, $V_2 \rightarrow V_1/7$ if $\alpha = 1$. We see immediately that the only solutions which are finite over the whole range of x and which begin with $V \rightarrow V_1$ as $x \rightarrow -\infty$ are such that V decreases smoothly and asymptotically to $V = V_2$ as $x \rightarrow \infty$, with a characteristic length scale of the order $L = \kappa/(V_1 - V_2)$, provided $V_1^2 \geq (1+\alpha/3)p_{c1}/\rho_1$ (i.e. the incident flow is super supersonic, since $1+\alpha/3$ is the specific heat ratio for the cosmic ray gas).

This result shows quite clearly that shock-like transitions are possible in which the pressure is provided entirely by the cosmic ray component and cosmic ray diffusion plays a role similar to that of heat conduction in ordinary shock waves with vanishing Prandtl number (see Illingworth, 1953, section 5). In this case, we find that the change in cosmic ray energy flux as a fraction of the initial kinetic energy flux is

$$(F_{c2} - F_{c1})/F_{\kappa 1} = 1 - (\alpha/(6+\alpha))^2 = 0.92\text{-}0.98 \ . \tag{21}$$

That is, the plasma kinetic energy can be converted to cosmic ray energy with 92-98% efficiency.

It is found that for highly supersonic shocks the transition is smooth as in the above example. In general, however, a plasma shock must be included in the transition using the usual Rankine-Hugoniot conditions and also requiring that p_c and F_c are continuous in accordance with equation (17) (Drury and Völk, these Proceedings).

To obtain the cosmic ray spectrum and current and thereby close the loop self-consistently we must next solve the transport equations (4) and (5) using the form V(x) obtained in the above manner. The result is presumably not very sensitive to the precise form of V(x) but only to the length scale of the transition (L). Furthermore we may permit κ to be energy dependent in this second step since the κ which determines L must be defined by the particles which provide most of the pressure. Analytic solutions of the equations are unfortunately difficult to obtain. However, the results to be expected from a smooth transition, for example, are obvious: (1) low energy particles ($\kappa << (V_1-V_2)L$) do not diffuse readily and are simply adiabatically compressed; (2) very high energy particles ($\kappa >> (V_1-V_2)L$) should see the transition as a discontinuity in flow speed and should accordingly be accelerated more or less as in the non-self-consistent case treated in section 6; (3) a 'special' spectrum may develope and, since there is a length scale in the problem, its form will depend on κ as well as V_1 and V_2. It is unfortunately not easy to solve the one-dimensional transport equation analytically with V a function of x and κ a function of T (and x), however attempts are in progress. Blandford (1980) has carried out a perturbation analysis for the case $p_c << \rho_1 V_1^2$ (presuming a shock wave to be present) and finds that the spectrum tends to flatten at high energies, which is not inconsistent with the above remarks. Eichler (1979) has argued that the incident kinetic energy flux is divided equally between relativistic (~ 1 GeV) particles and plasma thermal energy, however, this is in contradiction to the existence of smooth transitions as shown above.

8. CONCLUSIONS

The essential conclusion of this review is that provided they are scattered effectively and provided energy losses are not too severe, cosmic rays can be very efficiently accelerated by shock waves in the interstellar medium. Indeed, in favourable circumstances, most of the kinetic energy associated with the shock wave can be converted to cosmic ray energy. The provisos are significant since the Alfvén waves which are necessary to scatter the particles can only be produced effectively by the same shock waves and by strong anisotropies in the cosmic rays themselves. In the cooler regions of the interstellar medium (which in any case are regions which do not contain strong shocks) since Alfvén waves tend to be strongly damped by ion-neutral collisions and cosmic ray energy losses due to ionizing collisions and nuclear interactions are also relatively important, one would expect little acceleration to occur. We are therefore directed towards the hot, low density, fully-ionized regions (i.e. large HII regions and supernova remnants) which occupy most of the interstellar medium by volume. In such regions, ionization losses are very low and Alfvén waves are effectively damped only by viscosity, Landau damping, non-linear decay, expansion of the medium, and as a result of second order Fermi acceleration of cosmic rays. In particular, the very low density, high temperature (10^5-10^6K) interstellar medium is especially important since shock waves should tend to speed up and strengthen in such regions and, depending on the geometry of the situation, may propagate over large distances without too much attenuation. [See Cesarsky (1980) for a review of the problem of Alfvén wave generation and damping in the interstellar medium.]

In considering the behaviour of cosmic rays in the galaxy, one should note that during the time $\tau_c \sim 2\times10^7$ years, which is of interest, some 5×10^5 supernova occur, each of which expands to a radius of the order of 100 parsecs in a period of 10^6 years or so. This means that every point in the interstellar medium is passed by a rather strong shock wave ($\sim$100 km/sec) about ten times during the period in question, excluding reflections. From the point of view of the cosmic rays therefore, the interstellar medium is a violent place in which they are continually accelerated or decelerated or accelerated again, with of course a strong net acceleration because the effect is statistically irreversible (Ostriker, 1979). One should also note that the cosmic rays, at least in the low density medium, must continually be transported in one direction or another by mass motions and indeed produce these mass motions in part as a result of their own contribution to the pressure of the medium.

If one adopts the above picture of the interstellar medium, noting of course that it is complicated by the existence of stellar winds and expanding HII regions, there appears to be no strong reason to doubt that galactic cosmic rays are accelerated by shock waves with some assistance from second order Fermi effects and magnetic pumping. Furthermore, the cosmic rays play a major role in the energy balance and dynamics of the medium, and to some extent control their own destiny. That is, cosmic rays are in no sense to be regarded as passive riders on a more or less static interstellar magnetic field which scatters them occasionally so that they eventually by chance find their way out of the galaxy. Instead, the cosmic rays leave the galaxy because they are forced out and because they force their own way out. As a result of this process, a galactic halo is formed and eventually a galactic wind in which the cosmic rays are energetically dominant (Johnson and Axford, 1971). Shock waves which pass into the halo should tend to strengthen as they move into regions of rapidly decreasing density and in doing so further heat the halo gas and accelerate the cosmic rays in a manner analogous to the process suggested by Biermann (1948) for heating the solar corona.

It has been argued occasionally that the diffusion coefficient in the interstellar medium, at least in the direction normal to the plane of the galaxy, should lie in the range 10^{27}-10^{28}cm²/sec since the cosmic ray scale height is of the order of 10^2-10^3 parsecs and the escape time $2x10^7$ years. Such large diffusion coefficients, if typical, would tend to make all but nearly perpendicular shocks ineffective in accelerating cosmic rays since the time scale for acceleration by shock waves with speeds of the order of 100 km/sec would be at least 10^{13}-10^{14}sec, which is too long. In fact, this diffusion coefficient has little physical significance beyond the two parameters used in deriving it as it contains the implicit assumption that the interstellar medium is more or less static, non-convective, and lacking the highly dynamic mass motions which are implied in the description given above. If it means anything at all, this diffusion coefficient is related to the supersonic turbulent diffusion of the hot interstellar medium as a whole rather than to the diffusion of the cosmic ray gas it carries along with it. Except in the outer regions of the halo, where the cosmic rays tend to separate out, the diffusion coefficient for the cosmic ray gas relative to the plasma could be very much smaller.

The question of seed particles is an important but difficult question in view of our lack of knowledge of the microscopic properties of the interstellar medium. A number of suggestions for favouring the medium and heavy nuclei relative to hydrogen and helium have been made but it is difficult to find critical tests for any of them except perhaps by observing what happens in the interplanetary medium. One suggestion is that the ionization potential is the significant parameter since this would clearly discriminate against hydrogen and helium (Cassé et al., 1975). If this were the case, however, one would have to rely on the acceleration taking place in relatively cool and low density HI regions which, on the basis of the discussion given here, is most unfavourable since Alfvén waves damp rapidly, ionization losses are relatively important and if the shock waves are strong they would fully ionize the medium in any case. A second suggestion uses the condensation of solids as the filter, namely by invoking sputtering from interstellar grains following passage by a shock wave (Meyer et al., 1979; Cesarsky et al., these Proceedings). The sputtered atoms would have speeds of the order of the shock speed and hence, on becoming ionized, would have some chance of being selectively accelerated. An obvious difficulty with this idea is that helium and neon would be essentially absent.

Since we have advocated the hot intercloud regions of the interstellar medium as being the most likely sites for acceleration it is perhaps more reasonable to seek a seeding mechanism which is consistent with the properties of such a hot plasma. Thus we suggest that the parameters mass (A) and mass per charge (A/Z) could lead to the favoured injection of heavier species into the cosmic radiation. Since it is likely that in a collision-free shock ions tend initially to have the same energy per nucleon rather than the same total energy, the heavy ions are immediately favoured. Ions with large A/Z ratios receive further favourable treatment since their gyro-periods are larger and they therefore tend to resonantly interact with lower frequency components of the shock-induced turbulence where more wave power is available. This hypothesis, or variants of it, could in principle be put to test in the interplanetary medium (e.g. Hamilton et al., 1979). The existence of the anomalous component is perhaps some evidence in its favour since the particles possibly have large A/Z (Fisk et al., 1974; Klecker, 1977) and may result from a combination of stochastic (Fisk, 1976) and shock acceleration (Axford et al., 1977). In addition of course it must be recognized that the shock acceleration mechanism is also selective to the extent that the diffusion coefficient plays a role as it must in the non-linear interaction described in section 7. At a given energy per nucleon heavy ions tend to have higher rigidities and larger diffusion coefficients than protons; consequently, the heavy particles tend to "see" the transition differently and may be preferentially accelerated (cf. Eichler, 1979). It should of course be remembered that seed particles produced by stochastic acceleration in a shock transition must have at least the minimum energy T_m (equation (2)) if they are to be subsequently accelerated in the upstream region.

As far as electrons are concerned, once they are sufficiently relativistic they should behave more or less like protons and be almost as easily accelerated. However, the seeding processes for electrons are quite another matter since they respond to a quite different frequency domain in shock-induced electromagnetic turbulence and also to electrostatic fields in the shock, so that a separate discussion is required. However, it is clear from observations of energetic electrons associated with the interplanetary shocks that there is no principle difficulty in accelerating electrons (Armstrong and Krimigis, 1976).

We have emphasized expanding supernova remnants as the most likely means of accelerating cosmic rays, because more than enough energy is available ($\sim 10^{42}$ergs/sec) and because they can interact with the hot intercloud medium on a very large scale. However, there are other sources of mass motion in the interstellar medium which are not insignificant from the point of view of energetics, namely novae, expanding HII regions and stellar winds. Novae produce about 10^{39}ergs/sec. In the case of HII regions, the energy available is $\sim 6\text{x}10^{41}\eta$ ergs/sec, where η is the efficiency (about 1%) with which the energy of ionizing photons is converted to kinetic energy (Kahn and Dyson, 1965). There are of course shock waves around HII regions and cosmic rays must be affected by them (Newman and Axford, 1968); however the shock speeds are relatively low (10-50 km/sec) and they tend to occur in relatively dense HI regions where the conditions for efficient cosmic ray acceleration are unfavourable. Stellar winds would be negligible contributors of kinetic energy (~ 3×10^{37}ergs/sec) if the solar wind were a typical example; however, it appears that the distribution of stellar wind energy fluxes is very non-uniform and is dominated (~ 10^{41}ergs/sec?) by the same O and B stars which are responsible for the large HII regions (Cassé and Paul, 1979). On energetic grounds alone one could therefore not rule out stellar winds as a significant cause of cosmic ray acceleration (Cassé and Paul, 1979). Dorman (1979) has made this point in a somewhat different way by noting that solar modulation is in fact an acceleration mechanism since the solar wind must do work against the cosmic ray pressure gradient; one can readily show that if the typical stellar wind region were only about ten times larger than the 100 AU expected in the case of the sun the necessary power required to maintain the galactic cosmic radiation would be available. Dorman's calculation is in fact conservative since he neglects the enhancement due to acceleration at the shock wave terminating the supersonic wind.

Finally, it should be noted that intergalactic acceleration of cosmic rays by shocks is a perfectly feasible proposition and may account for the highest energy cosmic rays we observe. Inverse Compton and adiabatic expansion losses must of course be overcome but there is no shortage of seed particles and intergalactic shocks, at least those associated with radio galaxies, certainly exist. On this basis, one is entitled to ask whether galactic winds modulate the intergalactic cosmic radiation and also contribute to their acceleration.

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