

# PULSATIONS OF THE R CORONAE BOREALIS STARS

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## ABSTRACT

The radial pulsations of very luminous, low-mass models ( $L/M \sim 10^4$ , solar units), which are possible representatives of the R CrB stars, have been examined. These pulsations are extremely nonadiabatic. We find that there are in some cases at least one extra ("strange") mode which makes interpretation difficult. The blue instability edges are also peculiar, in that there is an abrupt excursion of the blue edge to the blue for  $L/M$  sufficiently large. The range of periods of the model encompasses observed periods of the Cepheid-like pulsations of actual R CrB stars.

## I. INTRODUCTION

It has been suggested by Wheeler (1978) that the hydrogen-deficient carbon (HdC) stars may be the progenitors of Type I supernovae. Among the HdC stars are the R Coronae Borealis (R CrB) stars, three of which exhibit, superposed on the large and erratic dimmings, Cepheid-like pulsations with periods close to  $40^d$  (King 1980).

The present calculations were initiated primarily to test Wheeler's (1978) suggestion by using pulsation theory to infer something about the masses of these stars. Our original goals have not been overly successfully realized. Yet, we can say that the masses seem to be around 1-2 solar masses. However, we have found, we think, some interesting and somewhat puzzling results, which are presented in §II.

The pulsations of models of R CrB stars are very nonadiabatic. The reason is that they have high luminosities ( $L$ ) and relatively low masses ( $M$ ). The ratio  $L/M$  is typically of order  $10^4$  (solar units), compared to  $\sim 10^3$  or less for Cepheids.

We have assumed that the models in their nonpulsating states are in hydrostatic equilibrium (Joss, Salpeter, and Ostriker 1973).

*Space Science Reviews* 27 (1980) 529-535. 0038-6308/80/0274-529 \$01.05.

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In §III we present some conclusions and some conjectures.

## II. RESULTS AND PROBLEMS

One of the most difficult and frustrating aspects of dealing with these very nonadiabatic pulsators is to know what "mode" the star is pulsating in. For these oscillations the amplitude  $|\xi| \equiv |\delta r/r|$  of the relative radial variation merely undergoes a dip at the "node" (i.e., what would be a node if the oscillations were perfectly adiabatic). Also, the phase of  $\xi$  may change gradually by a few radians over an appreciable range of radial distances around the "node" (instead of changing abruptly by  $\pm\pi$  radians at a node, as in the case of perfectly adiabatic oscillations). To make matters worse, there is often at least one extra mode; when this is the case, we have called such a mode a "strange" mode. The detailed properties of such a strange mode depend on the composition chosen for the model.

We have considered two compositions. One consists of 90% by mass of helium and 10% carbon (called HE9C1). The other consists of 98% helium and 2% heavier elements, in standard proportions (called Cox-Hodson 2 or CH2). The opacities and equation of state data have been obtained from the opacity library of Huebner *et al.* (1977), which gives information only for temperatures  $(T) \geq 12,000$  K. For  $T < 12,000$  K, special opacity and equation of state tables for HE9C1 have been prepared and used. For  $T < 12,000$  K and for CH2, the Stellingwerf (1975) opacity formula has been used. All of the linearized calculations reported here were carried out with a fully nonadiabatic code similar to that of Castor (1971).

The situation is illustrated in Figure 1, which shows  $\log P$  (period) versus  $\log T_e$  (effective temperature) for a model with  $M = 1 M_\odot$  (solar mass),  $L = 1.15 \times 10^4 L_\odot$  (solar luminosity), and the above two compositions. Shown with dashed lines are the linear adiabatic fundamental [F, labelled 0 (Ad)], first overtone [10, labelled 1 (Ad)], and second overtone [20, labelled 2 (Ad)] (almost the same for the two compositions). The corresponding nonadiabatic modes (indicated subsequently in the text by "F," "10," and "20") are more-or-less identifiable with their adiabatic counterparts, at least for the 10 and 20. The strange mode is clearly seen -- it is usually intermediate in period between the "F" and "10" modes, but its detailed characteristics obviously depend on composition. Such a strange mode was also found by Wood (1976), and called by him the "P<sub>1</sub>" mode. The third mode for the CH2 composition -- the mode that we have called "F" -- exhibits a very peculiar behavior indeed. The eigenfunctions for this mode have all the characteristics of an F mode at high  $T_e$ 's. Yet, at lower  $T_e$ 's its period becomes considerably less than what one would expect for a fundamental mode. In fact, the period in this mode becomes roughly constant and even diminishes slightly for the smaller  $T_e$ 's. Equivalently, one could say that the  $Q$ -value in the period-mean density relation becomes, for this composition, very much smaller than the

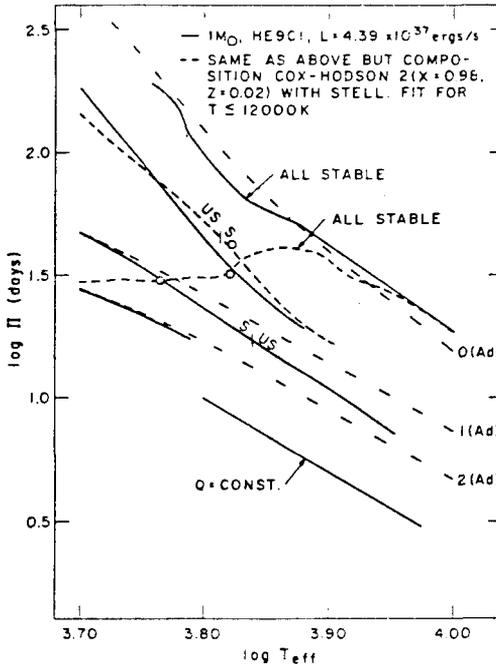


Fig. 1.  $\log \Pi$  (period in days) vs.  $\log T_e$  (effective temperature in K) for 1 solar mass models all having luminosity  $L = 1.15 \times 10^4 L_\odot$ , and the two compositions HE9C1 and CH2. S, stable; US, unstable.

adiabatic  $Q$ -value. (The solid line near the bottom of the figure is a constant- $Q$  line.) Could this mode actually be a second strange mode? Since the curve for this mode actually crosses the curves for several other modes, we have here several cases of degeneracy for purely radial modes.

On the other hand, for the HE9C1 composition the "F" mode at no  $T_e$ 's is ever very far away from the adiabatic F mode, at least for this luminosity. However, the above peculiar behavior of the "F" mode for HE9C1 seems to occur for about a 30% lower luminosity.

One might ask, "How do the eigenfunctions of two modes near a degeneracy differ?" Of the four cases that we have examined (all for CH2), two correspond to the near intersection of the "F" mode and the strange mode. The crossing point of these two curves is at  $\log \Pi \approx 1.55$ ,  $\log T_e \approx 3.83$ . These two cases are encircled in Figure 1, and are illustrated in Figures 2 and 3. The periods of these two modes are  $\approx 42^d$  (strange mode) and  $\approx 32^d$  ("F" mode). The other two cases (not illustrated) correspond to the intersection of the "F" mode and the "10" mode. This intersection point (encircled in Fig. 1) is near  $\log \Pi \approx 1.5$ ,  $\log T_e \approx 3.76$ .

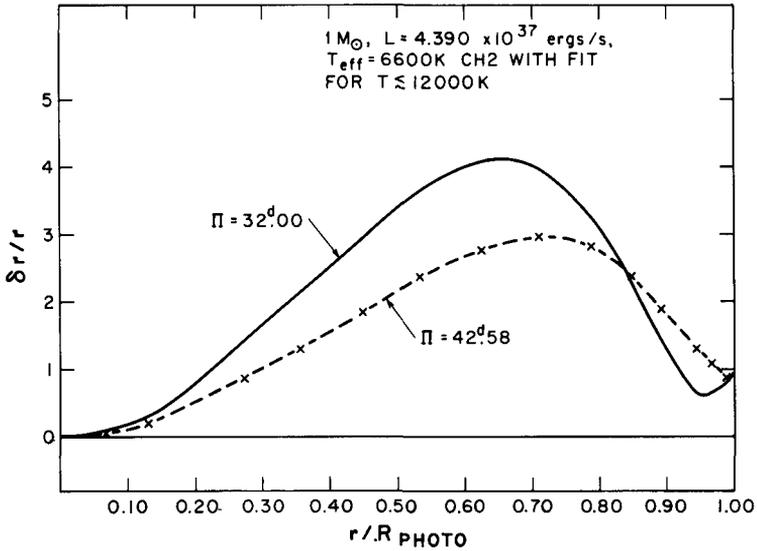


Fig. 2.  $|\delta r/r|$  vs.  $x = r_0/R_0$  for the two cases indicated in Fig. 1 by the right-most two circles.

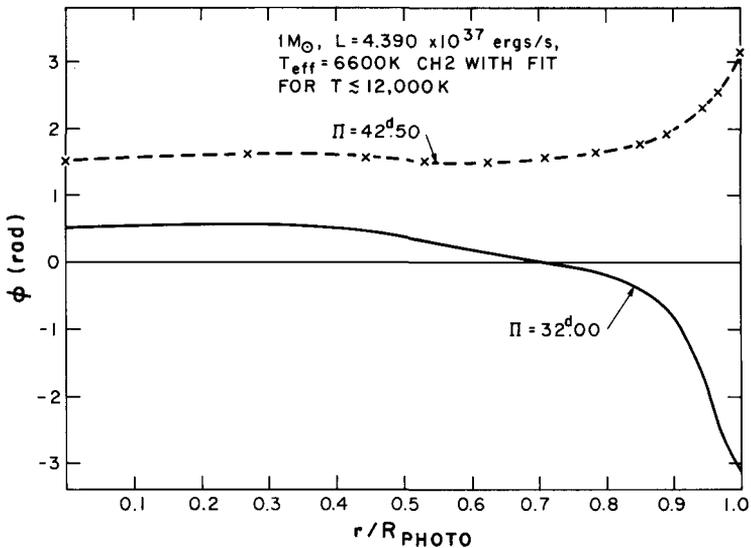


Fig. 3. Phase vs.  $x$  for the two cases considered in Fig. 2 (see text for an explanation of the phase).

In Figure 2 are plotted the magnitudes of the relative radial variation,  $|\delta r/r|$ , versus  $x = r_0/R_0$  (fractional radial distance) for the above two modes. In Figure 3 are shown the phases for these two modes, versus  $x$ . The phase is in each case normalized to be exactly  $\pi$  at the surface. The phases would be  $\pi$  throughout the star for an adiabatic F;  $\pi$  down to the node and zero interior thereto for an adiabatic I0; etc. The appearance of the eigenfunctions for the other two cases examined is qualitatively the same as illustrated above.

Finally, the blue edges of the instability regions exhibit a very peculiar behavior, which we think is due ultimately to the extreme nonadiabaticity of the pulsations. In Figure 4 is shown the blue instability edge on a Hertzsprung-Russell (H-R) diagrams for  $M = 1 M_{\odot}$  and HE9Cl. For  $L \lesssim 10^4 L_{\odot}$ , the blue edge is at "normal"  $T_e$ 's  $\sim 6000$ - $7000$  K and has a slope roughly parallel to that of the Cepheid instability strip. However, at around  $L \sim 10^4 L_{\odot}$  the blue edge shifts rather abruptly to higher  $T_e$ 's, around  $10,000$  K. For comparison, also shown is the blue edge for the same composition, but for  $M = 2 M_{\odot}$ . The approximate location of R CrB is also shown (Schönberner 1975).

We suspect that for the lower luminosities we are seeing the conventional "F" blue edge. However, at the higher luminosities the blue edge that we are seeing is probably that of the shorter-period strange mode. If the period were to become rather abruptly shorter at these higher luminosities, then it can be shown that an abrupt shift of the blue edge to the blue is exactly what would be expected for instability due to an envelope ionization mechanism.

### III. CONCLUSIONS

We have examined the radial oscillations of models of essentially helium stars of large luminosity and small mass, i.e.,  $L/M \sim 10^4$  (solar units). Such oscillations may be represented in nature by the pulsations of certain R CrB stars.

While our original goal of inferring something about the masses of these highly luminous pulsators has not been overly successfully realized, we have found some curious and, so far, baffling results and some bizarre behavior for these very nonadiabatic oscillations. For example, we have found at least one additional ("strange") nonadiabatic mode among the lower pulsation modes for sufficiently large  $L/M$ . Examination of the eigenfunctions of the strange mode and of other modes does not appear to help much in the mode identification problem.

The blue instability edges on an H-R diagram for these models behave very peculiarly (see Fig. 4). While the interpretation of these results is not clear yet, it does appear that instabilities in some mode(s) or other(s) exists throughout a rather large region in the H-R diagram encompassing the location of the R CrB stars.

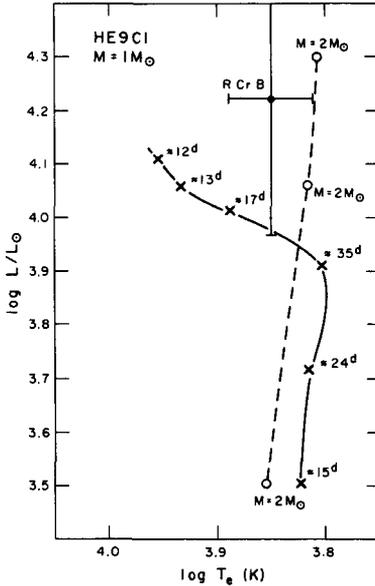


Fig. 4. Blue instability edge for  $M = 1 M_{\odot}$  and the composition HE9C1. Numbers alongside the curve give the corresponding periods, in days.

Finally, we wonder if such extremely nonadiabatic oscillations as found here, in particular as exemplified by the "strange" mode, have any bearing on the Mira variables. After all, it is known (e.g., Langer 1971; Keeley 1970a,b) that the pulsations of these stars are very nonadiabatic.

We are grateful to H. Saio for helpful conversations. This work was supported in part by NSF Grant No. AST79-22254, through the University of Colorado.

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## DISCUSSION

KEELEY: Some of the things you are saying remind me very much of calculations I did a number of years ago in studying long period variables, particularly the peculiar shape of the fundamental eigenfunctions.

J. COX: In fact, I was wondering if these peculiar modes have anything to do with Mira variables.

KEELEY: I don't know. These particular ones seem to be similar objects, very luminous with very long periods.

STARFIELD: What is the mass of R Cor Bor?

KING: Somewhere between  $0.8 M_{\odot}$  and  $2.0 M_{\odot}$ . We did not solve that problem.

A. COX: I want to make a remark. Your eigenfunctions frequently get very large as you go into the model. I believe you said four or more in some cases. I think that the reason for this is that you have a negative density gradient because you are so near the Eddington luminosity. You may even exceed it in local regions. Is the eigenfunction large because you have very little mass so that you can swing it around wildly?

J. COX: I don't know. I suppose that is a possibility.

A. COX: It might be interesting to plot the density on the same scale. I will bet that it will be a minimum at the maximum of the eigenfunction.

KEELEY: What do the adiabatic eigenfunctions look like? Can you match them up with the nonadiabatic ones?

J. COX: The adiabatic ones look normal.

KEELEY: Art may be right. The long period variable stars tend to have a broad density inversion. Mine always had larger amplitude there.