2

Magnetic materials

We saw in the previous chapter that magnetic fields are produced in a vacuum by currents in conductors. In this chapter, we will consider magnetic effects associated with matter. All materials have spin and orbital motion of charges at the atomic scale. For most materials, the random orientations of these internal currents tend to cancel out significant magnetic effects. However, in certain *magnetic materials*, such as iron or permanent magnets, these internal currents do not cancel, and there is a net external effect that also produces or enhances magnetic fields. In the field theory we have been discussing, the magnetic field B must be a continuous function of position. Thus, in magnetic materials, the macroscopic field B must be an average of the rapidly varying local fields surrounding the atoms in the material.[1]

2.1 Magnetization

When a magnetic material is placed in an external magnetic field, magnetic dipoles in the material set up internal fields that modify the applied field. We saw in Equation 1.10 that a current loop has an associated magnetic moment m. We define the *magnetization* vector M as the average magnetic moment per unit volume

$$\vec{M} = \sum_i \frac{\vec{m}_i}{V} = \frac{N_i I_i A_i}{V}\, \hat{n}_i \,, \tag{2.1}$$

where A_i is the area of loop i. The volume V must be large enough so the sum is statistically significant, yet small enough so that we can treat the variation of M as approximately continuous. For uniform magnetization, all the internal current loops cancel. However, as shown in Figure 2.1, there is still a net current around the surface of the material.

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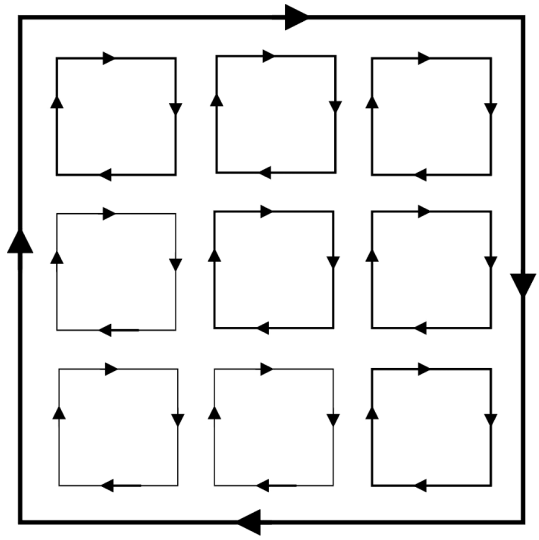

Figure 2.1 Magnetic moment loops in a uniformly magnetized material.

It follows that

$$M = \frac{I_m \sum_i A_i}{AL} = \frac{I_m}{L} = K_m,$$

where I_m is the Amperian loop current and K_m is the magnetization surface current density. In vector terms,

$$\vec{K}_m = \vec{M} \times \hat{n} \ . \tag{2.2}$$

Now consider the case when there is a nonuniform distribution of magnetization inside the volume, as shown in Figure 2.2. For the current loop on the left we have

$$M_z' = \frac{I'}{\Delta z},$$

while the loop on the right gives

$$M_z'' = M_z' + \frac{\partial M_z}{\partial y} \Delta y = \frac{I''}{\Delta z}.$$

Along the line AB, there is a net current

$$\frac{I_m}{\Delta z} = \frac{I'' - I'}{\Delta z} = \frac{\partial M_z}{\partial y} \Delta y. \tag{2.3}$$

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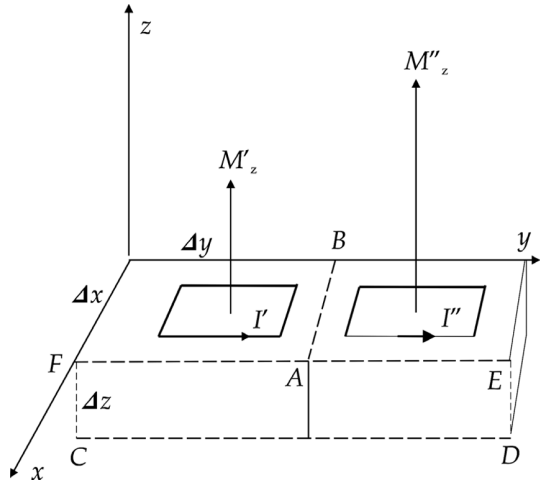

Figure 2.2 Magnetic moment loops in a nonuniform magnetized material.

If we integrate around the front face (*CDEF*), we find

$$\oint \vec{M} \cdot \vec{dl} = \left(M'_z + \frac{\partial M_z}{\partial y} \Delta y \right) \Delta z - M'_z \Delta z = \frac{\partial M_z}{\partial y} \Delta y \, \Delta z.$$

Therefore, using Equation 2.3 we find, in analogy with the Ampère law, that

$$\oint \vec{M} \cdot \vec{dl} = I_m, \tag{2.4}$$

where I_m is the effective number of internal amp-turns through the loop of integration. Applying Stokes's theorem, we can rewrite Equation 2.4 as

$$\int (\nabla \times \vec{M}) \cdot \vec{dS} = \int \vec{J}_m \cdot \vec{dS}.$$

The Amperian volume current density is then

$$\vec{J}_m = \nabla \times \vec{M}. \tag{2.5}$$

This implies that the volume current density vanishes for homogeneous materials. When magnetic materials are present, the Biot-Savart law must be generalized to

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{(\vec{K} + \vec{K}_m) \times \vec{R}}{R^3} \, dS + \frac{\mu_0}{4\pi} \int \frac{(\vec{J} + \vec{J}_m) \times \vec{R}}{R^3} \, dV,$$

where J and K without a subscript refer to the conduction current densities of free charges.

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2.2 Magnetic field intensity

Returning to the Ampère law, we take into account the effect of Amperian currents by writing

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 (I + I_m).$$

Using Equation 2.4, we have

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I + \mu_0 \oint \vec{M} \cdot \vec{dl}.$$

Combining the line integrals gives

$$\oint (\vec{B} - \mu_0 \vec{M}) \cdot \vec{dl} = \mu_0 I.$$

We can define an auxiliary vector H, known as the *magnetic intensity*, as

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}, \tag{2.6}$$

so that the magnetic flux density is

$$\vec{B} = \mu_0 (\vec{H} + \vec{M}). \tag{2.7}$$

In free space, this reduces to

$$\vec{B} = \mu_0 \vec{H}. \tag{2.8}$$

The vectors B and H describe different aspects of the same magnetic field. The advantage of working with this new vector is that the Ampère law for H

$$\oint \vec{H} \cdot \vec{dl} = I \tag{2.9}$$

only depends on the true conduction current that crosses the path of integration, despite the presence of any magnetic materials. Applying Stokes's theorem to Equation 2.9, we get

$$\int (\nabla \times \vec{H}) \cdot \vec{dS} = \int \vec{J} \cdot \vec{dS}.$$

Since the surface of integration is arbitrary, we find that H satisfies the differential equation

$$\nabla \times \vec{H} = \vec{J}, \tag{2.10}$$

where J is the conduction current density.

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We have introduced the vector H here as a useful mathematical artifact to take into account the averaged behavior of atomic currents in matter. However, there is a long history of trying to understand the physical meaning of H and of examining the distinction between vectors B and H.[2, 3]

2.3 Permeability and susceptibility

In homogeneous and isotropic materials, the magnetization is usually found to be proportional to the magnetic intensity, or

$$\vec{M} = \chi \vec{H}, \tag{2.11}$$

where the dimensionless coefficient χ is known as the *susceptibility*. Using this, we find that Equation 2.7 can be rewritten

$$\vec{B} = \mu_0 \vec{H} + \mu_0 \chi \vec{H} = \mu_0 (1 + \chi)\, \vec{H}.$$

It is useful to define the *permeability* for magnetic materials as

$$\mu = \mu_r \mu_0, \tag{2.12}$$

where μ_r is a dimensionless quantity known as the *relative permeability*. The susceptibility and the relative permeability are related by

$$\mu_r = 1 + \chi, \tag{2.13}$$

and the general relation between B and H can be written as

$$\vec{B} = \mu \vec{H}. \tag{2.14}$$

Materials where the directions of B and H are parallel are called *linear* materials.

2.4 Types of magnetism

Normally the random orientations of atomic orbits and particle spins cause the associated magnetic moments in a material to cancel, so there is no net magnetization. However, when an external magnetic field is applied to the material, the electron orbital velocity increases for one direction of circulation and decreases for the opposite direction. This results in a small net magnetic moment that is present in all materials and is known as *diamagnetism*. The difference in frequency between the two orbital directions can be shown to be

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$$\Delta\omega = \pm\frac{eB}{2m_e},$$

where e is the electron charge and m_e is its mass.[4] The resulting net magnetic moment Δm is

$$\Delta m \simeq -\frac{e^2 r^2 B}{4m_e}, \tag{2.15}$$

where r is the radius of the atomic orbit. Note that the induced moment is *opposite* to the direction of the applied magnetic field. This effect is very small and is masked by larger effects in paramagnetic and ferromagnetic materials. Diamagnetic materials have $\chi < 0$ and $\mu_r < 1$, independent of temperature.

In *paramagnetic* materials, the magnetic moments from orbital motion and spins do not cancel, resulting in a small permanent moment. In an external magnetic field, torques tend to align the magnetic moments with the direction of the field. In these materials, the induced fields act to increase the magnitude of the applied field and $\mu_r > 1$. The degree of alignment is decreased by internal collisions, vibrations and thermal agitation inside the material. The resulting magnetization is a spin effect given by the Langevin equation

$$M(H) = Nm\left[\coth\frac{mH}{kT} - \frac{kT}{mH}\right], \tag{2.16}$$

where N is the number of atoms per unit volume, k is Boltzmann's constant, and T is the temperature.[5] Note that the dependence of M on the magnetic intensity H is temperature dependent and nonlinear in general. However, in the case where $\frac{mH}{kT} \ll 1$, the paramagnetic susceptibility is given by Curie's law

$$\chi = \frac{M}{H} = \frac{N\,m^2}{3kT}. \tag{2.17}$$

In certain crystalline materials where one of the electron shells is not filled, it is possible for one or more electrons to have unbalanced electron spins. In these *ferromagnetic* materials, it is possible to achieve a very high degree of magnetic alignment. Below a characteristic temperature known as the Curie temperature, coupling is possible between neighboring atoms, which can act together in regions known as *domains*. In an external magnetic field, the size of favorably oriented domains can grow. In addition, the magnetization directions in each domain tend to align with the external field. The dependence of B or M on H is very nonlinear in ferromagnetic materials. For example, we show a BH curve for a low-carbon steel alloy in Figure 2.3.[6] The flux density increases extremely rapidly for small values

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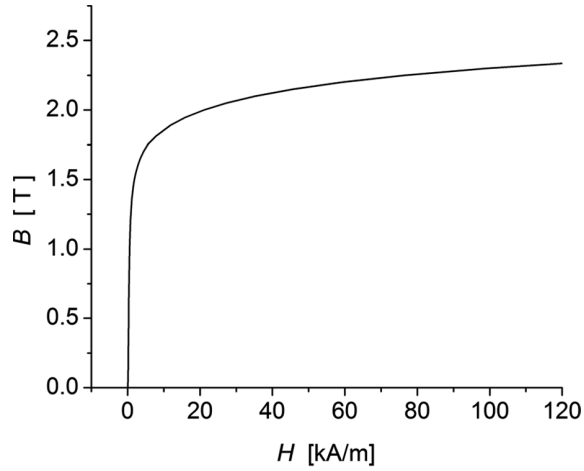

Figure 2.3 **A** *BH* curve for SAE 1020 low-carbon steel.

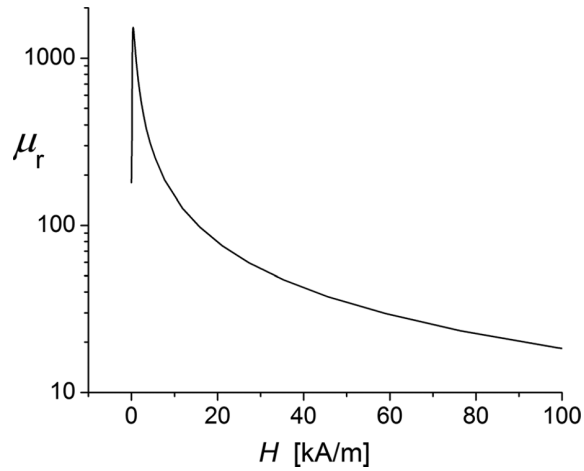

Figure 2.4 A μ*H* curve for SAE 1020 low-carbon steel.

of *H*. Then, as *H* increases further, the magnetization domains begin to saturate, and the curve starts to level out. Finally, for very large applied fields, the magnetization domains become completely saturated, and the growth in *B* is only due to the increase in the conduction current.

The relative permeability in a ferromagnetic material can be much larger than 1, as shown in Figure 2.4. The permeability in this example quickly reaches a maximum value ~1,525 for an excitation of 365 A/m. For larger excitations, the relative permeability decreases steadily.

If the current and thus *H* is cycled up and down in a ferromagnetic material, we find that a plot of *B* versus *H* has the characteristic shape illustrated in Figure 2.5.

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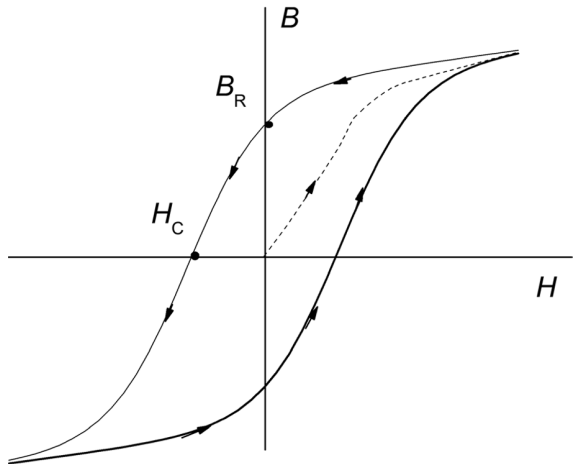

Figure 2.5 Hysteresis loop for a ferromagnetic material.

The material exhibits *hysteresis* because B is not a unique function of H. The value of B depends on the previous history of how H was varied. The dotted line shows the increase in B as H is increased, starting from an unmagnetized sample. After the initial excitation, $B(H)$ follows the arrows around the hysteresis loop. This effect arises because the domain boundaries don't completely return to their previous locations when H is reversed. The *remanence* or *remanent field* B_R is the value of B when H is returned to 0. The nonzero value for the remanence shows that the material can remain magnetic when the external driving current is turned off. This leads to the possibility of making permanent magnets,[1] so long as the temperature remains below the Curie temperature. The *coercivity* H_C is defined[2] as the value of H in the negative direction that is required to get $B = 0$. The *intrinsic coercivity* H_{Ci} is the reverse field required to remove the magnetization in a plot of M versus H.[7] The hysteresis loop is symmetric around the origin. Heat is generated for each cycle around the hysteresis loop.[3]

Magnetic materials with low values of coercivity are designated as "soft." In this case, it is easy for the magnetization to change direction as the external current changes, so these materials are suitable for *ac* operation.[8] A number of soft magnetic materials that have high permeabilities at low values of B are listed in Table 2.1. Also listed are the values of B corresponding to the maximum permeability, the coercivity, and the saturation value of B. The electrical resistivity ρ_e of the material is important for considerations of eddy current[4] losses in time-varying operations. Values for pure iron are also listed for comparison.

[1] Permanent magnets are discussed in more detail in Chapter 9.
[2] Historically, this quantity is known as the coercive force.
[3] The energy loss in the hysteresis loop is discussed in Chapter 10.
[4] Eddy currents are discussed in more detail in Chapter 10.

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Table 2.1 *Selected soft magnetic alloys [9]*

Alloy	Initial μ_r	Max μ_r	B at max μ_r [T]	H_c [Oe]*	B_{sat} [T]	ρ_e [$\mu\Omega$-cm]
Sinimax	2,200	50,000	0.54	0.06	1.10	90
Monimax	3,000	60,000	0.62	0.06	1.45	80
16 Alfenol	4,000	80,000	0.35	0.044	0.80	153
Mumetal	20,000	100,000	0.20	0.30	0.65	60
1040 alloy	20,000	100,000	0.20	0.20	0.60	56
Supermalloy	55,000	300,000	0.40	0.006	0.68–0.78	65
Iron	150	5,000	0.80	1.00	2.14	10

* 1 Oe = 1 10^{-4} T/μ_0

2.5 Magnetic circuits

Ferromagnetic materials tend to concentrate magnetic flux. The permeability of iron can be thousands of times larger than that of free space. When analyzing "circuits" made up of coils and pieces of ferromagnetic materials, it is common to assume that all the flux goes through and is uniformly distributed inside the ferromagnetic material. A coil sets up Amperian currents in the material near the coil that continue to initiate further currents along the material.[10] Assume that we have an iron ring that is energized with a coil, as shown in Figure 2.6. From the Ampère law for H (Equation 2.9), we find that B inside the ring is

$$B_\phi = \frac{\mu N I}{2\pi R},$$

and the magnetic flux inside the ring is

$$\Phi_B = \left(\frac{\mu \pi r^2}{2\pi R}\right) N I. \tag{2.18}$$

This expression is only approximate because it ignores any leakage of the flux from the ring. Equation 2.18 resembles Ohm's law for circuits

$$I = \frac{V}{R_e},$$

where NI corresponds to the driving voltage and the resulting flux corresponds to the current. The term "analogous to the electrical resistance" R_e is called the *reluctance*, which we see can be expressed as

$$\mathcal{R} = \frac{L}{\mu A}, \tag{2.19}$$

where L is the path length in the material, and A is its cross-sectional area.

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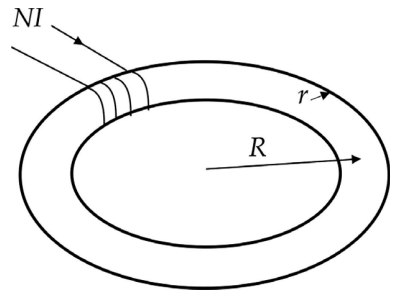

Figure 2.6 Magnetic circuit (Rowland ring).

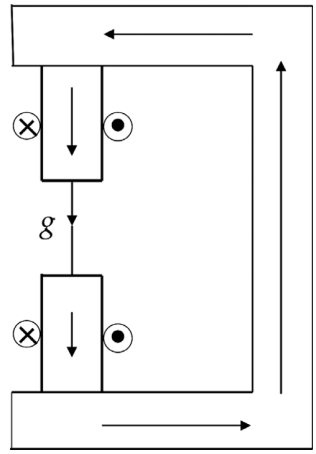

Figure 2.7 Magnetic circuit for a *C*-dipole magnet.

Example 2.1: C-shaped electromagnet
Consider an electromagnet with an air gap, as shown in Figure 2.7. From the Ampère law

$$N\,I = H_i L_i + H_g L_g, \tag{2.20}$$

where L is the mean length through the region, and the subscripts i and g refer to the iron and the gap. On the assumption there is no leakage flux, we have $B_i A_i = B_g A_g$. Substituting into Equation 2.20, we get

$$N\,I = \frac{B_i}{\mu} L_i + \frac{B_g}{\mu_0} L_g = \frac{B_g A_g}{A_i}\ \frac{L_i}{\mu} + \frac{B_g}{\mu_0} L_g,$$

so that

$$N\,I = B_g A_g \left[\frac{L_i}{\mu A_i} + \frac{L_g}{\mu_0 A_g}\right].$$

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Since $\mu \gg \mu_0$, we can neglect the reluctance in the iron for reasonable values of L and A and find that the field in the gap is

$$B_g \simeq \frac{\mu_0 N I}{L_g}. \tag{2.21}$$

2.6 Boundary conditions between regions with different μ

We now consider the constraints on the magnetic field at the boundary between two regions having different permeabilities. The pillbox construction in Figure 1.8 still applies in this case, so

$$B_{1n} = B_{2n}. \tag{2.22}$$

However, when dealing with permeable materials, Equation 1.30 for the tangential component of B is no longer accurate. In the present case, we can use the Ampère law for H for a path that encompasses both sides of the boundary to find that

$$H_{2t} - H_{1t} = K, \tag{2.23}$$

where we recall that K is the surface current density. If there is no surface current at the boundary, then the tangential component of H is continuous.

Consider a linear material with $K = 0$ and let θ be the angle between the magnetic field and the normal to the surface, as shown in Figure 2.8. Then we have from Equations 2.22 and 2.23

$$\mu_1 H_1 \cos\theta_1 = \mu_2 H_2 \cos\theta_2$$
$$H_1 \sin\theta_1 = H_2 \sin\theta_2.$$

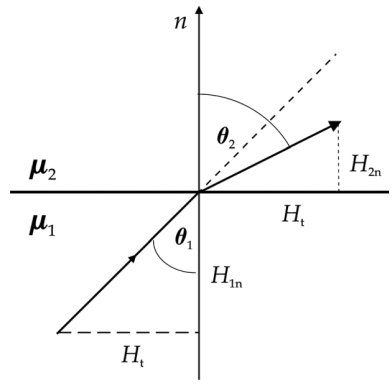

Figure 2.8 Refraction of $\boldsymbol{H}$ at the boundary between two permeable materials.

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Dividing these two equations, we find that

$$\frac{\tan\,\theta_1}{\tan\,\theta_2} = \frac{\mu_1}{\mu_2}. \tag{2.24}$$

If the field is incident from region 1 and μ is much larger in region 2 than region 1, then θ_1 will be smaller than θ_2. Thus, the field will make a larger angle with respect to the normal in the region with larger μ.

As a special case, consider the boundary between vacuum in region 1 and iron in region 2 when $K = 0$. Then, using Equation 2.23, we find

$$\frac{B_{t2}}{B_{t1}} = \frac{\mu_2}{\mu_1}.$$

It is often useful to make the approximation that μ_2 in the iron is infinite. Then the right-hand side of this equation is infinite, which demands that $B_{t1} = 0$. In this case, the field in the vacuum region would be perpendicular to the iron surface.

2.7 Method of images

Some magnetostatic problems with planar or spherical boundaries can be solved using the method of images.[11] In this method, the presence of an iron boundary is replaced with virtual currents, which, together with the currents from true conductors, reproduce the correct boundary conditions. Consider a current I in a region 1 with permeability μ_1 a distance d from the planar boundary with a region 2 of material with permeability μ_2, as shown in Figure 2.9. Let us designate case (a) to be the situation when the observation point P is in region 1. Then, to satisfy the boundary conditions, we assume there is a virtual current I' in region 2.

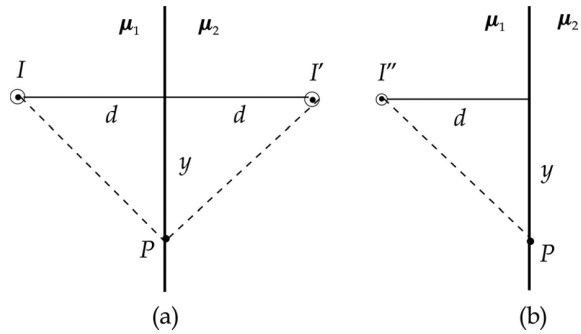

Figure 2.9 Image currents near a plane boundary.

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We designate case (b) to be the situation when the observation point is inside the iron in region 2. In this case, there is no conduction current in the observation region. We assume the resultant field is due to a virtual current I'' in region 1. By symmetry, the original current and the two image currents lie on a line perpendicular to the boundary. We assume that the currents all flow in the same direction and that the distances of the currents from the boundary are equal and look for a solution for the magnitude of the currents. Consider a point P along the boundary at a distance y from the line connecting the currents. In case (a), the boundary conditions are

$$H_t^{(a)} = \frac{d}{2\pi(d^2 + y^2)} \quad (I - I')$$

$$B_n^{(a)} = \frac{\mu_1 y}{2\pi(d^2 + y^2)} \quad (I + I'),$$

whereas for case (b) we have

$$H_t^{(b)} = \frac{d}{2\pi(d^2 + y^2)} \; I''$$

$$B_n^{(b)} = \frac{\mu_2 y}{2\pi(d^2 + y^2)} \; I''.$$

Both cases must give the same solution for any position y along the boundary. Thus, we obtain the two equations

$$I - I' = I''$$
$$\mu_1 I + \mu_1 I' = \mu_2 I''.$$

Solving these two equations for the unknown magnitudes of I' and I'', we find the solution [12]

$$I' = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} \; I \tag{2.25}$$

and

$$I'' = \frac{2\mu_1}{\mu_1 + \mu_2} \; I. \tag{2.26}$$

In the special case (a) when region 1 is vacuum and region 2 is infinitely permeable iron, Equation 2.25 reduces to $I' = I$.

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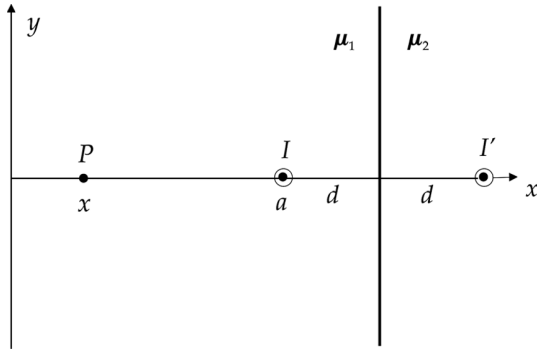

Figure 2.10 Line current near a planar iron slab.

Example 2.2: Field enhancement in a planar iron slab
Consider a filamentary conductor a distance d away from an infinite slab of iron. We use the method of images and replace the iron slab with a virtual filament, as shown in Figure 2.10.

Let us examine the field for locations P along the line perpendicular to the boundary. Assume the observation point is at x and the filament is located at a. Then the boundary is located at $a + d$, and the image current is at $a + 2d$. The field at P is

$$B_y(x) = -\frac{\mu_0 I}{2\pi(a-x)} - \frac{\mu_0 I'}{2\pi(a+2d-x)}.$$

Assuming $\mu_1 = \mu_0$ and $\mu_2 = \mu_r \mu_0$ and using Equation 2.25, we find that

$$B_y(x) = -\frac{\mu_0 I}{2\pi}\left[\frac{1}{a-x} + \left(\frac{\mu_r - 1}{\mu_r + 1}\right)\frac{1}{a+2d-x}\right].$$

In the limit when $\mu_r \to \infty$ we find that $B_y(a+d) = 0$, as it should at the surface of the iron. We define the iron *enhancement factor E(x)* to be the ratio of the field at x with the iron present to the field at x from the conductor by itself. In the limit $\mu_r \to \infty$, the enhancement factor is

$$E(x) = 1 + \frac{a-x}{a+2d-x}.$$

We show the dependence of the enhancement factor on x in Figure 2.11. The enhancement is greater than 1 in the region to the left of the filament and then becomes less than 1 in the region between the filament and the iron boundary.

It is also possible to use the method of images to solve problems with more complicated arrangements of planar surfaces. For example, a line current between two parallel iron boundaries can be solved using an infinite series of image

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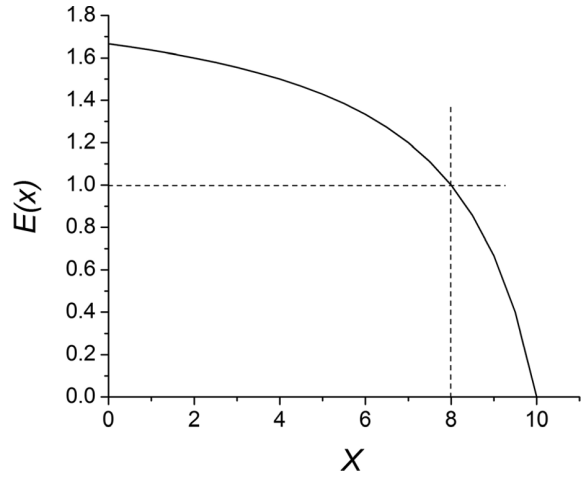

Figure 2.11 Planar iron enhancement factor for $a = 8$ and $d = 2$.

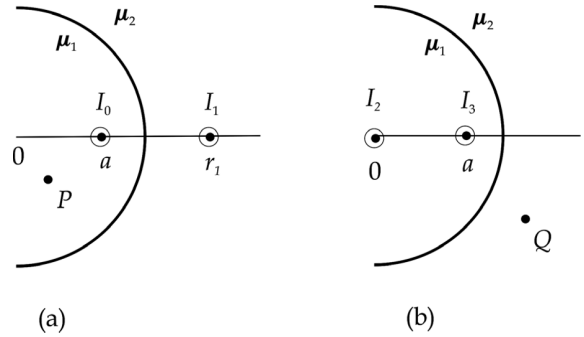

Figure 2.12 Images for a line current near a circular boundary.

currents,[13] and a line current near the corner of two perpendicular iron planes can be solved using three image currents.[5]

The method of images is also useful when considering a line current near a circular boundary surface. Consider a line current I_0 at radius a in a region with permeability μ_1 near a circular boundary of radius R of a region with permeability μ_2, as shown in Figure 2.12.

In case (a), the magnetic field at a point P inside the aperture of the magnet ($r < a$) can be written as the sum of the fields due to the conduction current I_0 at $r = a$ and the image current I_1 at $r = r_1$ inside region 2, where

[5] This is discussed in Chapter 5, Section 5.13.

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$$I_1 = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} I_0 \tag{2.27}$$

$$r_1 = \frac{R^2}{a}. \tag{2.28}$$

In case (b), the magnetic field at a point Q inside region 2 ($r > R$) can be written as the sum of the field from two image currents in region 1, I_2 at $r = 0$ and I_3 at $r = a$, where

$$I_2 = \frac{\mu_2 - \mu_1}{\mu_2 + \mu_1} I_0 \tag{2.29}$$

$$I_3 = \frac{2\mu_1}{\mu_2 + \mu_1} I_0. \tag{2.30}$$

The justifications for these statements come from the solution of the corresponding boundary value problem, which we will discuss in Chapter 4, Section 4.2.

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