AN OPTIMUM DECONVOLUTION METHOD

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An optimum solution to a deconvolution problem has to fulfil three general criteria: (a) an explicit recognition of the smoothing nature of convolution; (b) a statistical treatment of noise, e.g., using the least-squares criterion; and (c) requiring the solution to conform to all our prior knowledge about it. In the usual least-squares method, one minimises a variance of 'residuals', or the departures of the observed data from the values expected according to the recovered solution. However, this condition does not lead to a stable solution in the case of deconvolution, since the only stable solutions are those conforming to a criterion of 'regularisation' or smoothness (see, e.g., Tikhonov and Arsenin 1977). In our method, the stability is achieved by minimising the variance of the second-differences of the solution simultaneously with the fulfilment of the least-squares criterion. Such a procedure was first used by Phillips(1962). However, the solution thus obtained is still unsatisfactory since it usually does not conform to our a priori information. When we seek the brightness distribution of an object, the most frequent violation of our prior knowledge is that of positiveness. This motivated us to develop an Optimum Deconvolution Method (ODM) which constrains the solution to satisfy prior knowledge while retaining the features of least-squares and smoothness criteria.

If prior knowledge is in the form of equalities, e.g., when the total intensity (area under the solution) is specified, it can be easily incorporated by using the Lagrange multiplier method of constrained minimisation. However, this is not applicable for positiveness or any specification of bounds on the solution since this leads to constraints in the form of inequalities. Such constraints are imposed in ODM by a new iterative algorithm, which minimises a weighted sum of squares of the departures of the solution from the specified bounds, simultaneously with the fulfilment of the criteria of least-squares and smoothness. The weights chosen depend on the degree of such departures in the previous iterations. The algorithm is found to converge rapidly, within 5 iterations in most of the several hundred applications tried by us for the Fresnel-diffraction curves produced by the lunar occultations of radio sources.

C. van Schooneveld (ed.), Image Formation from Coherence Functions in Astronomy, 287-290.

The method was extensively tested by using both the computer-simulated occultations and those of actual sources observed with the Ooty radio telescope. Restored profiles from these data obtained by using ODM were compared with those obtained by using the conventional method which was suggested by Scheuer(1962). The tests reveal that ODM often leads to an improvement in the attainable resolution by about a factor of two over the conventional method. In addition, it also gives a 'clean' solution masking the effects of noise on the recovered solution and hence leads to a more objective interpretation. A detailed description of the method and its application to lunar occultations has been given elsewhere (Subrahmanya 1979).

The usual purpose of seeking a nonclassical method like ODM is to improve our estimate of the size of a certain component of a source. For this, it is necessary to know the effective resolution with which the solution has been recovered. In a linear method, this effective resolution is always known independently of the solution to be recovered, e.g., as a property of the restoring function used. However, this is not the case in a nonlinear method like ODM or Maximum Entropy Method (MEM) which uses known properties of the solution before actually deriving it. In such a case, it becomes necessary to introduce an *a posteriori* definition of resolution, say, based on the statistical properties of the residuals. For this, we suggest the use of the mean zero-crossing interval of the residuals. For instance, if one examines the noise resulting from an observation with a Gaussian beam, the mean zero-crossing interval is about 1.9 times the half-power-width of the Gaussian beam.

We have seen that ODM can provide super-resolution and also a 'clean' output as compared to a classical method. The solution is optimum since it agrees statistically with the observed data and also conforms to all our prior knowledge about it. The main source of improvement over the conventional method is the incorporation of prior knowledge, just as in MEM. A few general remarks can be made by way of comparison of the two methods. Both are nonlinear methods. Both are able to use incomplete data like those with the absence of a few samples or the ignorance of phases of some Fourier components. They use the data as given and do not make any assumption on the unavailable data. The characteristics of 'super-resolution' and 'clean output' are also shared by both the methods. Since both are nonlinear methods, it may be worthwhile to compare their computational efficiencies. In our experience, ODM is particularly fast for a nonlinear method in lunar occultations, since the iterations generally converge within 3 to 5 iterations. On the contrary, an available figure for MEM is 8 to 40 iterations for the one-dimensional Fraunhofer diffraction problem studied by Frieden (1975). However, this cannot be used to make a categorical statement on the relative computational efficiency of the two methods, since they have only been used in different contexts.

Thus any fundamental difference between the two methods is to be expected only if the 'entropy' as used in MEM has a special sanctity apart from satisfying prior knowledge about the solution. Serious doubts have already been cast on this aspect and it is not clear at present whether the expressions being used for 'entropy' have the same significance as in information theory or thermodynamics(cf. Kikuchi and Soffer 1977). An essential difference in the usage is the definition based on a posteriori probability in MEM as opposed to that based on a priori probability in information theory. It is also possible that the existing schemes of MEM can be alternately viewed as merely numerical algorithms for incorporating positiveness. In this context, it may be noted that there is a numerical scheme for constrained minimisation called the 'logarithmic penalty-function method' (Fiacco and McCormick 1970) which is strikingly similar to MEM as being used by one class of workers (e.g. Ables 1974). Further work is needed to understand whether a concept like entropy is indeed pertinent to the problem or whether an arbitrary numerical algorithm which can efficiently incorporate prior knowledge is all that is relevant.

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DISCUSSION

Comment U.J. SCHWARZ
Is it really desirable to impose positiveness? This is not a question concerning this method only, but other methods too. The point is that statistically the estimate is not asymptotically equal to the true distribution, since the noise always gives a positive contribution via - essentially - a detection process.

Reply C.R. SUBRAHMANYA

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This is one of the reasons why I believe that it is desirable to incorporate prior knowledge explicity, with a provision to define the degree to which we know it. In ODM since an estimate of the uncertainty (σ) in total intensity (area) is known, the iterations for positiveness are terminated when the total contribution to the intensity from the negative points becomes $\sim 0.5\ \sigma$.

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