The Effect of Varying Helium and Hydrogen Layer Masses on the Pulsation Properties of White Dwarf Models

by
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1 Introduction

Currently, there are disagreements on the theoretical pulsation properties of DA white dwarf models. Winget and his group (cf Winget 1981, Winget and Fontaine 1982, Winget *et al.* 1981, 1982) use $0.6M\odot$ evolutionary white dwarf models, to find that $(T_{eff})_{blue}$ is sensitive to the mass of the hydrogen layer and very sensitive to the treatment of convective efficiency. However, $(T_{eff})_{blue}$ is relatively insensitive to the helium layer mass. Winget and collaborators also find that the hydrogen layer must be restricted to $10^{-8} > M_H/M_\star > 10^{-12}$ for $(T_{eff})_{blue}$ to occur at $\sim 10,500$K in models with ML1 convection (see below). More efficient (ML2) convection pushes the blue edge up to $\sim 12,200$K, which is still short of the observed blue edge of $\sim 13,000$K.

These conclusions are not in agreement with the results of Cox *et al.* (1987) use white dwarf models that have a mass of $0.6M\odot$, with hydrogen and helium layer masses of $10^{-4}M_\star$ each. These models violate the requirement that the helium layer mass must be at least 100 times greater than the hydrogen layer mass in order to avoid overlapping transition zones (cf. Acouragi and Fontaine 1980). In spite of the unphysical nature of the models, Cox et al. find a blue edge temperature for pulsational instability ($(T_{eff})_{blue}$) at $\sim 11,500$K for models with hydrogen layer masses of $10^{-4}M_\star$ and $10^{-8}M_\star$. Neither group considered the red edge of the instability strip, other than to note that convection-pulsation interactions would be important at lower temperatures.

The aim of this paper is to clarify the discrepancies in the theoretical results and refute some earlier erroneous conclusions made about ZZ Ceti stars. In this study, we use *evolutionary* models of static DA white dwarfs described by Winget (1981) and Winget and Fontaine (1982). These $0.6M\odot$ models have pure carbon cores with an overlying helium layer and an outer layer of pure hydrogen. For this study, we use a treatment of the Brunt-Väisälä frequency that is self-consistent in all regions of the model, including the composition transition zones. We neglect convection-pulsation interactions throughout. In this preliminary study, we examine the pulsation properties of $0.6M\odot$ white dwarf models in response to changes in the mass of the hydrogen layer and convective efficiency for a fixed helium layer mass of $10^{-2}M_\star$. In addition, we examined a model with a helium layer mass of $10^{-5}M_\star$ to see if $(T_{eff})_{blue}$ is sensitive to the helium layer mass.

We use two different numerical methods for the adiabatic analysis; one is the Newton-Raphson relaxation method described in Winget (1981), and the other is a Runge-Kutta integrator developed by Carl Hansen with a subroutine for interpolating extra shells if the resolution is insufficient. Kawaler (1986) describes this program and its advantages for computing adiabatic quantities. Only the Newton-Raphson method is available for computing the nonadiabatic results.

For future convenience, the quantities $M_\star$, M_H, and M_{He} are hereafter specified by the notation of the following example: 60208 refers to a $0.6M\odot$ sequence, with $M_{He}= 10^{-2}M_\star$, and $M_H= 10^{-8}M_\star$. The convective efficiency is specified in the following manner. ML1 convective efficiency refers to the mixing-length theory of Böhm-Vitense (1958), with the mixing length equal to one pressure scale height. ML2 convective efficiency refers to the mixing length theory of Böhm and Cassinelli (1971) with the mixing length equal to one pressure scale height. Finally, ML3 convective efficiency uses the same mixing-length theory of ML2, but the mixing length is twice the pressure scale height.

2 Determination of $(T_{eff})_{blue}$ and $(T_{eff})_{red}$

To determine $(T_{eff})_{blue}$ and $(T_{eff})_{red}$, we use linear interpolation to find the zero point of the growth rates for several modes (where possible). The resultant average value for $(T_{eff})_{blue}$ and $(T_{eff})_{red}$ along with the temperature of the nearest stable and unstable model are plotted versus the hydrogen layer mass in figure 1.

Our results confirm and supplement those of Winget (1981). We find $(T_{eff})_{blue}$ to be $\sim 10,500$K with ML1 convection for hydrogen layer masses between $10^{-12}M_\star < M_H < 10^{-8}M_\star$. We also find that for M_H greater than $10^{-8}M_\star$, the value of $(T_{eff})_{blue}$ drops to ~ 8000K and is insensitive to the treatment of convection. When M_H is $10^{-14}M_\star$, we find that the models are unstable between 16,400K and 12,400K, much hotter than the observed instability strip. The change in helium layer mass does not have any effect of the value of $(T_{eff})_{blue}$.

Our most significant finding is that with ML3 convection, $(T_{eff})_{blue}$ is $\sim 13,000$K for the 60210 and 60209 sequences, in excellent agreement with the observed blue edge and the results of Fontaine *et al.* (1984). This is in contrast with the result of Cox *et al.* (1987) who were unable to get $(T_{eff})_{blue}$ above $\sim 11,500$K. The red edge of these two sequences,

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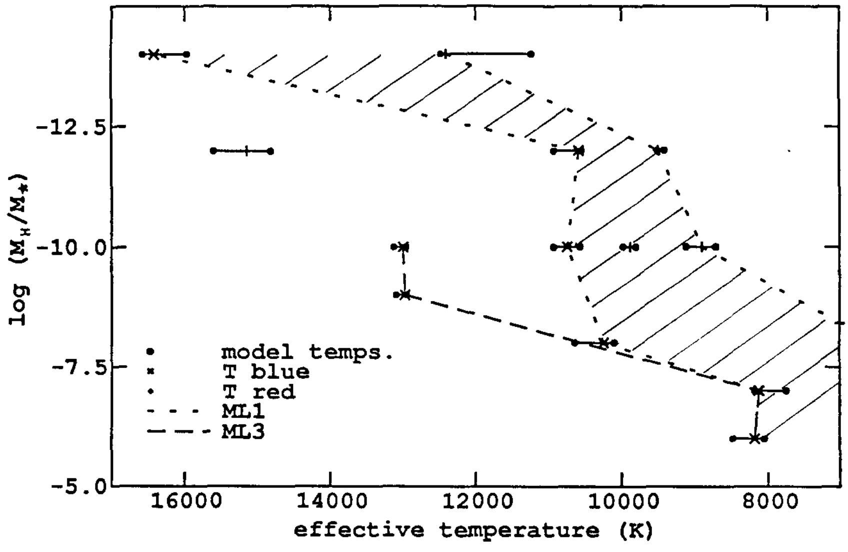

Figure 1: The value of $(T_{eff})_{blue}$ is plotted versus the hydrogen layer mass fraction, $\log(M_H/M_*)$. The hatched region shows the location of the g-mode instability strip.

$\sim 10,000$K, is about 1000K cooler than the observed red edge again suggesting that convection-pulsation interactions are important.

3 Mode Trapping

The rich spectrum of g-modes that are excited in theoretical models suggests that a filter mechanism is at work. Winget et al. (1981) proposed that mode trapping, which arises from a resonance between the wavelength of a g-mode oscillation and the thickness of the hydrogen layer could provide the answer. We find mode trapping in our models and prove that the presence of the H/He interface is vital for mode trapping to exist. In figure 2, we show the kinetic energies for $\ell = 2$ g-modes versus the period for various treatments of the Brunt-Väisälä frequency.

Notice that when the H/He transition zone is neglected or the Schwarzchild A criterion is used, the presence of mode trapping all but disappears. In contrast, using the self-consistent treatment of the Brunt-Väisälä frequency or neglecting the He/C transition zone produces a sharp minimum in the kinetic energy for $k = 6$. Since the kinetic energy of a g-mode is inversely proportional to the growth rate of that mode, the presence of mode trapping suggests that the trapped mode will be easier to excite, and will be preferentially driven at the expense of other modes.

4 Theoretical $\dot{\Pi}$ Values

We also computed theoretical values of rates of period change ($dP/dt \equiv \dot{\Pi}$) for representative g-modes of each sequence at various temperatures and find that the values range from $\sim 10^{-14}$ s/s to $\sim 10^{-16}$ s/s. In general, $\dot{\Pi}$ decreases with decreasing T_{eff} and $\dot{\Pi}$ increases with increasing radial order k for a given T_{eff}. Figure 3 shows the behavior of $\dot{\Pi}$ versus T_{eff} for representative values of k for the 60210 ML1 sequence, the 60210 ML3 sequence, and a 60410 ML1 sequence with a carbon core and discontinuous transition zones.

Where the two sequences overlap, there is good agreement between their values of $\dot{\Pi}$ at low values of k and gets worse with increasing k. Our values of $\dot{\Pi}/\Pi$ range from $\sim 5 \times 10^{-17}$ 1/s to $\sim 10^{-18}$ 1/s. These values are nearly independent of k for a given T_{eff} and decrease slightly with decreasing T_{eff}.

The best available observational values of $\dot{\Pi}$ are for R548 with an upper limit of $\dot{\Pi} < 9.6 \times 10^{-15}$ s/s (Tomaney 1987) and for G117-B15A with an upper limit of $\dot{\Pi} < 1.25 \times 10^{-14}$ s/s (Kepler et al. these proceedings). Due to the short pulsation periods, the g-modes of these stars have low values of k suggesting that the theoretical $\dot{\Pi}$ values are most likely a few times 10^{-15} s/s. This implies that observational detection of $\dot{\Pi}$ for these stars is still a few years away.

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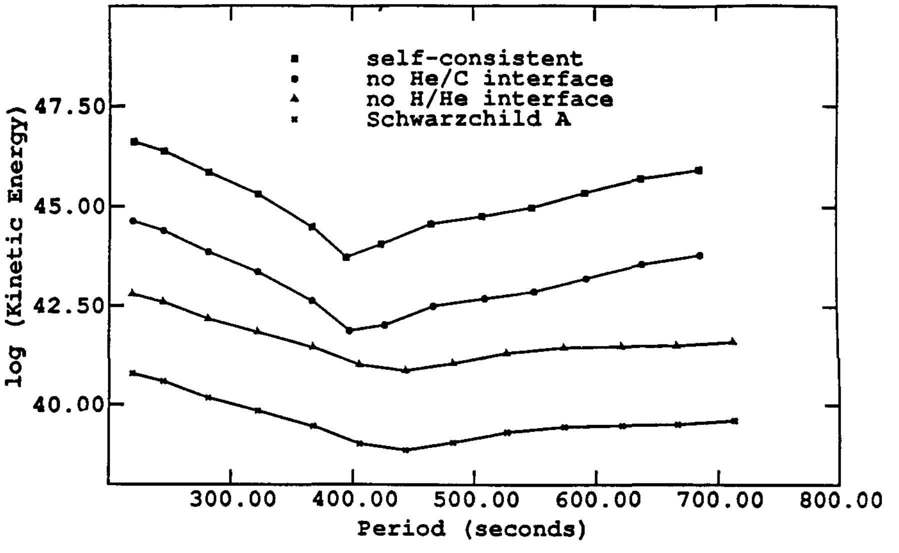

Figure 2: A plot of log(kinetic energy) versus the period of $\ell = 2$ g-modes for a 60510 model ($T_{eff} = 9141$K).

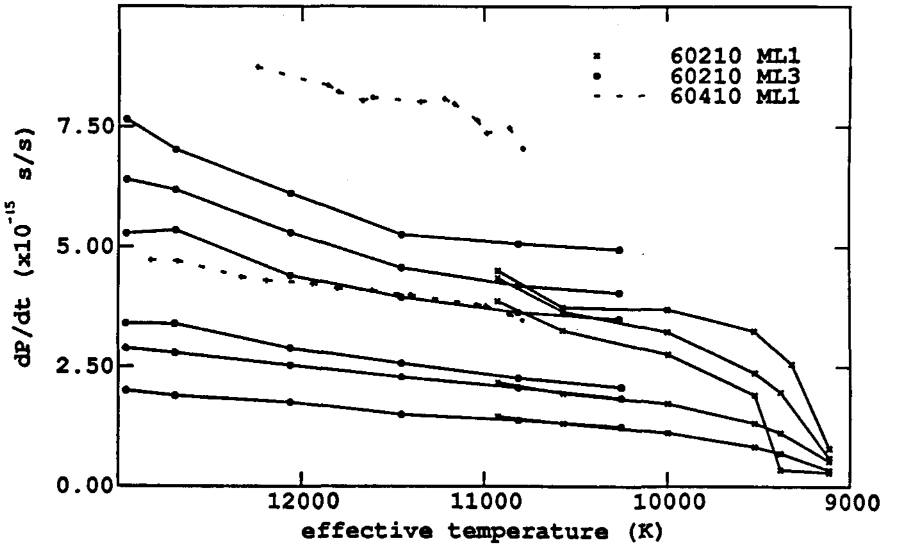

Figure 3: $\dot{\Pi}$ versus T_{eff} for $\ell = 2$ g-modes of the 60210 ML1 sequence (x), the 60210 ML3 sequence (•), and the 60410 ML1 (C/O) sequence (+).

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5 Conclusions

We present results of a linear adiabatic and nonadiabatic analysis of several sequences of evolutionary white dwarf models. These preliminary results allow us to conclude the following:

1. Efficient (ML3) convection is required for the theoretical blue edge to match the observed blue edge. Contrary to the results of Cox et al. (1987), we find pulsational instabilities exist for models up to 13,000K, but only for hydrogen layer masses of $10^{-9}M_\star$ and $10^{-10}M_\star$. However, if M_H is $10^{-7}M_\star$ or greater, the blue edge drops to $\sim$ 8,000K, independent of convective efficiency.

Therefore, this detailed reinvestigation contradicts the conclusions of Cox *et al.* (1987) of white dwarf models with thick hydrogen layers being able to pulsate with temperatures appropriate for the ZZ Ceti instability strip.

2. Matching the observed value of $(T_{eff})_{red}$ must wait until convection-pulsation interactions are included.

3. We find that mode trapping occurs when the Brunt-Väisälä frequency is computed in a self-consistent manner and demonstrate the H/He transition zone is responsible for this phenomena. We agree with Winget et al. (1981) that this a likely filtering mechanism for selecting which unstable g-modes will be excited to detectable amplitudes, although nonlinear calculations are required to verify this.

4. Our theoretical calculations of $\dot{\Pi}$ range from $\sim 10^{-14}$ s/s to $\sim 10^{-16}$ s/s. In addition, our $\dot{\Pi}/\Pi$ values range from $\sim 5 \times 10^{-17}$ 1/s to $\sim 10^{-18}$ 1/s. The theoretical values are still lower than the observed upper limits for R548 and G117-B15A. This should change with a few more observing seasons worth of data.

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