Hence the following convenient rule: Multiply the given height in feet by $1\frac{1}{2}$ and extract the square root. The result gives the approximate distance of the horizon, in miles.

Example: What distance along the surface of the Earth can be seen from a height of 80 feet?

The working may be set down thus:

$$\begin{array}{r} 80 \\ \underline{40} \\ 120 \end{array}$$

$$\text{Distance} = \sqrt{120} = 11 \text{ miles, nearly.}$$

The converse problem is to find the height of an observer to whom an object on the Earth's surface at a given distance is just visible. This is easily solved by the following rule: Multiply the square of the distance in miles by $\frac{2}{3}$; the result is the required height in feet.

Thus, if the object is at a distance of 30 miles, the required height $= \frac{2}{3} \times 900$ feet

$= 600$ feet.

PETER RAMSAY

The Asymptotes of the Hyperbola.—The equation of the hyperbola usually presents itself to the student in the form $y = \dfrac{a}{x}$, in the elements of graphs, before the canonical form is reached in Analytical Geometry. Some of the properties of the curve are easily obtained from the above equation by the methods of elementary geometry; the object of this note is to illustrate the process. We shall use the equation in the form $xy = c^2$, and it will be convenient to arrange the discussion in numbered Theorems. The axes may be rectangular or oblique.

Theorem 1.

If PM, PN and $P'M'$, $P'N'$ are drawn parallel to the asymptotes Cy, Cx of a hypherbola from points P and P' on the curve, to meet Cx and Cy in M, M' and N, N' respectively, then MN', $M'N$ and PP' are parallel.

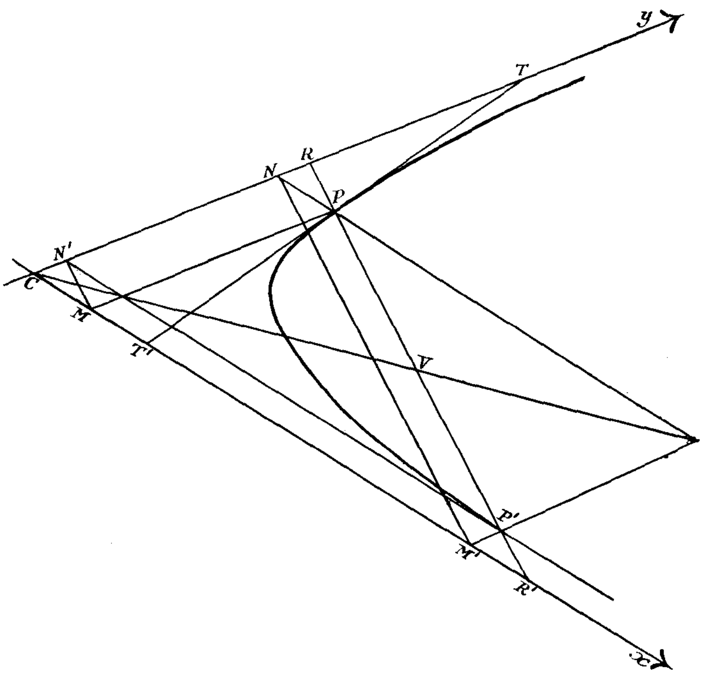
For, in the Figure,

$$MN' \text{ and } M'N \text{ are parallel}$$

if $$CM : CM' = CN' : CN,$$

that is, if $$CM \,.\, CN = CM' \,.\, CN';$$

and this is true, since P and P′ lie on the curve $xy = c^2$.



Again,

$$\|^m CMPN = \|^m CM'P'N',$$

since $$CM \,.\, CN = CM' \,.\, CN'.$$

But $$\|^m CMPN = 2\triangle NM'P,$$

and $$\|^m CM'P'N' = 2\triangle NM'P';$$

therefore $$\triangle NM'P = \triangle NM'P';$$

and therefore $$NM' \;\|\; PP'.$$

Hence MN′, M′N and PP′ are parallel.

https://doi.org/10.1017/S1757748900000761 Published online by Cambridge University Press

Theorem 2.

If the chord PP′ of a hyperbola meet the asymptotes in R and R′, then RP = R′P′.

Using the notation of Theorem 1, we see that MN′, M′N, PP′ are diagonals of parallelograms (shown in the figure) whose other diagonals form one and the same straight line (Euc. I, 43) through C.

Therefore if V is the middle point of PP′, CV bisects MN′.

But RR′ is parallel to MN′,

so that CV also bisects RR′,

and therefore RP = R′P′.

Theorem 3.

The locus of the middle points of parallel chords of a hyperbola is a straight line passing through the centre.

CV bisects both PP′ and the parallel M′N′.

As PP′ moves parallel to itself, so does MN′; but MN′ is constantly bisected by CV; therefore CV is the locus of the middle point of PP′.

Theorem 4.

The portion of a tangent intercepted between the asymptotes is bisected at the point of contact, and the tangent forms with the asymptotes a triangle of constant area.

Let P′ move into coincidence with P; then M′ coincides with M and N′ with N.

PP′ becomes the tangent at P. But the chord PP′ is parallel to MN′: therefore the tangent at P is parallel to MN.

Let the tangent at P meet the asymptotes in T and T′; then

$$PT = MN = PT'.$$

Again $$\triangle CTT' = \text{twice } \|^{m} CMPN$$
$$= \text{constant},$$

since CM . CN is constant and angle MCN is constant.

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