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ISOTOPES OF NEARLATTICES

ABU SALEH ABDUN NOOR

This thesis studies the nature of isotopes of a nearlattice. A nearlattice is a lower semilattice with the *upper bound property*, which says that any two elements possess a supremum whenever they have a common upper bound. The topic arose out of a study on the kernels, around a particular element n, of a skeletal congruence on a distributive lattice. Also, we found that the idea of an isotope was very fruitful in extending results on ideals of nearlattices to the n-ideals; that is, convex subnearlattices containing n.

Chapter 1 discusses ideals, join-partial congruences and other results on nearlattices which are basic to this thesis. Chapters 2 and 3 introduce the notions of standard element, neutral element and central element of a nearlattice. These concepts are essential for the further developments of Chapters 4 and 5. Also in Chapter 3, we discuss direct summands and multipliers of a nearlattice and generalize a number of results of [6] and [9].

Chapter 4 introduces the concept of an isotope. Two new types of elements arise; one is called *superstandard* and the other is *nearly neutral*. An element n of a nearlattice S is called *medial* if $m(x, n, y) = (x \wedge n) \vee (y \wedge n) \vee (x \wedge y)$ exists for all $x, y \in S$, while the nearlattice S is *medial* if each of its elements is medial. A *sesquimedial* element is a strengthening of a medial element, but in a medial nearlattice every element is sesquimedial. For a medial element n of a nearlattice S, we can form a new binary operation

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$a \cap b = m(a, n, b)$ on S. We show that for a medial and superstandard element n of a nearlattice S, $(S; \cap)$ is a semilattice. Moreover, when n is nearly neutral and sesquimedial, $(S; \cap)$ is in fact a nearlattice, which gives an extension of some of the results of [8] and [10]. What is more, we present converses of these results. We refer to $(S; \cap)$ as an *n-isotope*, S_n, or simply an *isotope* of S.

Chapter 4 also establishes the following fundamental results about a nearlattice S:

(i) for a nearly neutral and sesquimedial element n of S, the n-ideals of S are precisely the ideals of the isotope S_n;

(ii) when n is neutral and sesquimedial, the join-partial congruences of S are precisely the join-partial congruences of S_n;

(iii) if n is neutral and sesquimedial in S, then any neutral and sesquimedial element m has the same properties in S_n. Furthermore, the double isotope $(S_n)_m$ is precisely S_m.

Chapter 5 gives a characterization of the isotope S_n, when n is neutral and an upper element, in the sense that $x \vee n$ exists in S for all $x \in S$. Of course, every central element is upper. We show that, when n is a standard element of a lattice L, the associated isotope is a lattice if and only if n is central. This chapter also gives a nice generalization of a well known result of [3], namely, if a lattice L is boolean then the isotope L_n is also boolean and they are isomorphic. The generalization is concerned with a lattice with an involution. Chapter 6 is mainly concerned with the application of isotopes. Here, we extend a number of results on ideals of a nearlattice to its n-ideals. We discuss skeletal congruences of a distributive nearlattice and then extend the results of [1] to n-ideals. Chapter 7 contains the results of [2], which clarifies [7] on the ternary operation $k(a, b, c) = \big(a \vee (b \wedge c)\big) \wedge (b \vee c)$ on a modular lattice.

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