LETTER TO THE EDITOR

Dear Sir:

It is well known that when the distribution of independent intervals (with unit mean) between claims is other than exponential the pure premium for the company's claims, each "expected" to amount to one monetary unit, is the so-called renewal function. Its derivative is the renewal intensity (the pure premium rate at epoch t) and only asymptotically does this become unity. It is of interest to see how Thorin and Wikstad's (1973) c, the "gross risk premium per unit of time", implies variable risk loadings on the corresponding pure premium rates. This is all the easier to do because Bartholomew (1973) has chosen to provide explicit forms for the renewal intensities of:

- (i) two-term mixed exponential distributions, and
- (ii) Pareto distributions.

Thorin and Wikstad's (loc. cit.) renewal densities of interclaim intervals were

$$k(t) = 0.25 \times 0.4e^{-0.4t} + 0.75 \times 2e^{-2t}$$
(1)

and

$$k(t) = 1.5 (1 + 2y)^{-2.5}$$
⁽²⁾

respectively, and using Bartholomew's relations (7.6) and (7.8) for the corresponding renewal intensities we obtained the following results.

Claim epoch t	Renewal intensity at epoch t corresponding to:	
	(1)	(2) (approx.)
2	1.121	1.554
4	1.024	1.403
6	<u>,</u> 1.005	1.330
8	1.001	1.285
10	1.000	1.254
20	1.000	1.176

In the Thorin and Wikstad article c is given the five values 0.90 (.05) 1.10 for (1) in two of their tables, and the seven values 1.05 (.05) 1.30, 2.00 in another. We see that the pure premium rate is larger than unity until the epoch of the eighth expected claim; thereafter c-1 is the constant risk loading in the gross premium—and this is negative in some cases. As for (2) the six chosen values of c are 1.05 (.05) 1.30. The rate of risk loading is thus negative for most of these c-values until ten or more claims have been expected and some have negative risk loadings even after 20 expected claims. These early variations in the risk loading which is commonly thought of as being constant (at c-1) are not, perhaps, very serious but they occur whenever the premium is paid from a claim epoch unless the distribution of intervals between claims is exponential.

Yours very truly,

HILARY L. SEAL

References

BARTHOLOMEW, D. J., (1973). Stochastic Models for Social Processes. Wiley, London.

THORIN, O. and WIKSTAD, N., (1973). Numerical evaluation of ruin probabilities for a finite period. ASTIN Bull. 7, 137-153.