Third Meeting, January 8, 1892.

John Alison, Esq., M.A., F.R.S.E., Vice-President, in the Chair.

On the smallest number of entries necessary in a table of logarithms to seven decimal places.

By Professor J. E. A. Steggall.

Taking seven figure logarithms it is required to find at what interval from n, a number of five figures, the next entry need be made, in order that any intermediate logarithm may be calculated by the method of proportional parts.

We have

$$\log(n+d) - \log n = \mu\left\{\frac{d}{n} - \frac{d^2}{2n^2} + \dots\right\} \quad \dots \quad \dots \quad (1),$$

and

$$\log(n+x) - \log n = \mu\left\{\frac{x}{n} - \frac{x^2}{2n^2} + \dots\right\} \quad \dots \quad \dots \quad (2),$$

also

$$\frac{x}{d}\{\log(n+d) - \log n\} = \mu\left\{\frac{x}{n} - \frac{xd}{2n^2} + \dots\right\}.$$

This latter quantity, which we add to $\log n$ in order to find log $(n+x)$ approximately, is got by calculation, and its error (in defect) from the true quantity, (2), is

$$\mu x\left\{\frac{(d-x)}{2n^2} - \frac{(d^2-x^2)}{3n^3} + \dots\right\};$$

the greatest value of this error is when $x = \frac{1}{2}d$, and it then is $\mu d^2/8n^2$.

In order that there may not be an error of a unit in the seventh place we must have $\mu d^2/8n^2 < \cdot 0000001$.

Whence $$d/n < \sqrt{\cdot 00000185}$$
or $$d < \cdot 00136n,$$

and varies from 13·6 to 136 in the ordinary course of the tables.

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For example, taking one of the most unfavourable cases

$$\log 99864 = 4{\cdot}9994090$$
$$\log 100000 = 5{\cdot}0000000$$

and therefore $$\log 99932 = 4{\cdot}9997045$$

which is correct to within a unit in the last figure.

In fact, the number of entries really required in a table is given by

$$(1{\cdot}00136)^n = 10,$$

whence

$$n = 1/{\cdot}00059024$$
$$= \text{about } 1700,$$

instead of 90000.

The table might be arranged

10000 to 20000	every	tenth	number	...	1000 entries
20000 „ 30000	...	twentieth	...	...	500 „
30000 „ 40000	...	fortieth	...	...	250 „
40000 „ 80000	...	fiftieth	...	...	800 „
80000 „ 100000	...	hundredth	...	...	200 „
					2750 „

If we apply the same method to a four figure table, all we require is that

$$\mu d^2/8n^2 < .0001,$$

or

$$d < \sqrt{.0019n},$$
$$d < \quad .044\,n.$$

Thus to obtain any logarithm to 4 decimal places we may, for example, work with only the logarithms of

104	135	170	208	270	348	448	580	760
108	140	175	216	280	360	464	600	790
112	145	180	224	290	375	480	625	800
116	150	185	232	300	390	500	650	830
120	155	190	240	312	400	520	675	865
125	160	195	250	324	416	540	700	900
130	165	200	260	336	432	560	730	930
								960

To reduce the error to a maximum of ·00005, we require about the number of entries following: that is, about 100 entries against about 60.

103	133	175	235	295	325	440	590	800
106	136	180	240	300	330	450	600	820
109	140	185	245	305	340	460	615	840
110	144	190	250	310	350	470	630	860
113	148	195	255	315	360	480	645	880
116	150	200	260	320	370	490	660	900
119	154	205	265	325	380	500	680	920
120	158	210	270	300	390	515	700	940
123	160	215	275	305	400	530	720	960
126	164	220	280	310	410	545	740	980
129	168	225	285	315	420	560	760	
130	170	230	290	320	430	575	780.	

In each case the smallest number of entries has not been taken but a series of numbers has been arranged so as to make the use of the proposed table rapid and simple. The smallest number of entries is given for the four figure table correct to ·0001 by the numbers

104	131	167	212	272	348	447	576	743
108	136	174	221	283	363	466	601	775
112	142	181	230	295	378	486	627	809
116	148	188	240	307	394	507	654	844
121	154	196	250	320	411	529	682	881
126	160	204	261	334	429	552	712	919
								959

or 55 entries in all; but it is clear that these entries are not so convenient as the somewhat larger number selected above.

On the Contact-Property of the Eleven-point Conic.

By R. E. Allardice, M.A.

It is sometimes the case that geometrical theorems, which are usually enunciated as properties of the triangle or the quadrilateral, may be stated more succinctly, and in a form that better suggests generalisation, as properties of the complete quadrilateral. Thus

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