16

Parity violation in inclusive electron scattering

The measurement of parity violation in the scattering of longitudinally polarized electrons in inclusive deep-inelastic electron scattering from deuterium at SLAC is a classic experiment that played a pivotal role in the establishment of the weak neutral current structure of the standard model [Pr78, Pr79]. The measurement of parity violation in inclusive electron scattering from nuclear and nucleon targets $A(\vec{e}, e')_{\text{pv}}$, promises to play a central role in future developments in nuclear physics [Pa90]. In this chapter we use the previous results to develop a general description of this process.

Conservation of parity in the strong and electromagnetic interactions implies that there can be no difference in the cross section for the process $A(\vec{\mathrm{e}}, \mathrm{e}')$ upon reversal of the longitudinal polarization of the electron if the target is unpolarized and unobserved. This follows from general principles, for it would effectively imply a non-zero expectation value for the pseudoscalar quantity $\langle \boldsymbol{\sigma} \cdot \mathbf{k}_1 \rangle$. That the helicity-dependent lepton contribution to the cross section indeed vanishes with one photon exchange can be seen immediately from our preceeding analysis. Equation (15.2) states that a longitudinally polarized electron has an additional term in the response tensor of the form $h\,\varepsilon_{\mu\nu\lambda\rho}k_{2\lambda}k_{1\rho}$. When contracted with the response tensor for an unpolarized and unobserved hadronic target in Eq. (11.27), the result vanishes since the first expression is antisymmetric in the interchange of the indices μ and ν and the second is symmetric.

Parity violation necessitates the inclusion of the weak interaction. In addition to the exchange of a virtual photon, it is possible for an electron to exchange a $Z^{(0)}$, the heavy neutral weak vector boson with mass $M_Z = 91.19\,\text{GeV}$. The interaction takes place through the weak neutral current, which we now know is accurately described by the *standard model* of the electroweak interactions [Sa64, We67, Gl70, We72].

To start the discussion of parity violation, consider the scattering of

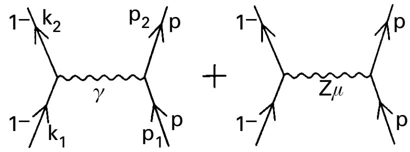

Fig. 16.1. Contributing Feynman diagrams (unitary gauge) for parity-violating asymmetry in scattering of longitudinally polarized electrons from point protons. Here $q = k_2 - k_1$.

a relativistic (massless) longitudinally polarized electron from a point proton. The contributing diagrams in the unitary gauge are shown in Fig. 16.1. The standard model is presented in detail in chapter 26 and [Wa95]. Here we simply anticipate that development and use the fact that the Feynman rules for the weak neutral current interaction of the standard model imply that the S-matrix is given by

$$\begin{aligned} S_{fi} = \frac{-(2\pi)^4 i}{\Omega^2}\delta^{(4)}(k_1+p-k_2-p')\Big\{&\bar{u}(k_2)(e\gamma_\mu)u(k_1)\frac{\delta_{\mu\nu}}{q^2}\bar{u}(p')(-e\gamma_\nu)u(p) \\ &+\bar{u}(k_2)\left[\frac{-g\gamma_\mu}{4\cos\theta_W}[(1-4\sin^2\theta_W)+\gamma_5]\right]u(k_1)\frac{(\delta_{\mu\nu}+q_\mu q_\nu/m_Z^2)}{q^2+m_Z^2} \\ &\times\bar{u}(p')\left[\frac{g\gamma_\nu}{4\cos\theta_W}[(1-4\sin^2\theta_W)+\gamma_5]\right]u(p)\Big\} \end{aligned} \tag{16.1}$$

At low energy one has $|\mathbf{q}|/M_Z \ll 1$, and the momentum-dependent terms can be neglected in the Z-propagator. Take the standard model values

$$\begin{aligned} e^2 &= 4\pi\alpha \\ \frac{g^2}{8m_Z^2\cos^2\theta_W} &= \frac{G}{\sqrt{2}} = \frac{1.024\times 10^{-5}}{\sqrt{2}\,m_p^2} \\ a &= -(1-4\sin^2\theta_W) \qquad ; \ \sin^2\theta_W = 0.2315 \\ b &= -1 \end{aligned} \tag{16.2}$$

Then

$$\begin{aligned} S_{fi} &= \frac{-(2\pi)^4 i}{\Omega^2}\delta^{(4)}(k_1+p-k_2-p')T_{fi} \\ T_{fi} &= -\frac{4\pi\alpha}{q^2}\Big\{\bar{u}(k_2)\gamma_\mu u(k_1)\bar{u}(p')\gamma_\mu u(p) - \frac{Gq^2}{4\pi\alpha\sqrt{2}}\bar{u}(k_2)\gamma_\mu[a+b\gamma_5]u(k_1) \\ &\qquad \times\bar{u}(p')\gamma_\mu\left[\frac{1}{2}(1+\gamma_5)-2\sin^2\theta_W\right]u(p)\Big\} \end{aligned} \tag{16.3}$$

This result is easily extended to point neutrons using the Feynman rules of [Wa95] through the replacement

$$
\begin{aligned}
T_{fi} &= -\frac{4\pi\alpha}{q^2}\left\{\bar{u}(k_2)\gamma_\mu u(k_1)\bar{u}(p')\gamma_\mu\frac{1}{2}(1+\tau_3)u(p)\right. \\
&\quad -\frac{Gq^2}{4\pi\alpha\sqrt{2}}\bar{u}(k_2)\gamma_\mu[a+b\gamma_5]u(k_1) \\
&\quad \left.\times\bar{u}(p')\gamma_\mu[(1+\gamma_5)\frac{1}{2}\tau_3 - 2\sin^2\theta_W\frac{1}{2}(1+\tau_3)]u(p)\right\}
\end{aligned} \tag{16.4}
$$

At this juncture one can redefine things so that the result is more general than for just point nucleons

$$
\begin{aligned}
S_{fi} &= \frac{-(2\pi)^4 i}{\Omega}\delta^{(4)}(k_1+p-k_2-p')T_{fi} \\
T_{fi} &= \frac{4\pi\alpha}{q^2}\left\{i\bar{u}(k_2)\gamma_\mu u(k_1)\langle p'|J_\mu^\gamma(0)|p\rangle\right. \\
&\quad \left.-\frac{Gq^2}{4\pi\alpha\sqrt{2}}i\bar{u}(k_2)\gamma_\mu(a+b\gamma_5)u(k_1)\langle p'|\mathscr{J}_\mu^{(0)}(0)|p\rangle\right\}
\end{aligned} \tag{16.5}
$$

Now these are single-nucleon matrix elements of the full electromagnetic and weak neutral current densities taken between exact Heisenberg states; for point nucleons, this expression reduces to Eq. (16.4).

The dimensionless ratio $Gq^2/4\pi\alpha\sqrt{2}$ forms the small parameter in these nuclear physics parity-violation calculations.

The first term in Eq. (16.5) leads to the electron scattering cross section derived in chapter 11

$$
\begin{aligned}
d\sigma &= \frac{4\alpha^2}{q^4}\frac{d^3k_2}{2\varepsilon_2}\frac{1}{\sqrt{(k_1\cdot p)^2}}\eta_{\mu\nu}W_{\mu\nu} \\
\eta_{\mu\nu} &= -2\varepsilon_1\varepsilon_2\frac{1}{2}\sum_{s_1}\sum_{s_2}\bar{u}(k_1)\gamma_\nu u(k_2)\bar{u}(k_2)\gamma_\mu u(k_1) \\
&= k_{1\mu}k_{2\nu}+k_{1\nu}k_{2\mu}-(k_1\cdot k_2)\delta_{\mu\nu} \\
W_{\mu\nu} &= (2\pi)^3\overline{\sum_i}\sum_f\delta^{(4)}(q+p'-p)\langle p|J_\nu^\gamma(0)|p'\rangle\langle p'|J_\mu^\gamma(0)|p\rangle(\Omega E_p) \\
&= W_1^\gamma(q^2,q\cdot p)\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right) \\
&\quad +W_2^\gamma(q^2,q\cdot p)\frac{1}{M_T^2}\left(p_\mu-\frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu-\frac{p\cdot q}{q^2}q_\nu\right)
\end{aligned} \tag{16.6}
$$

It is important to note that at this point we have again generalized the target response tensor to include the possibility of inelastic processes.

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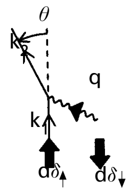

Fig. 16.2. Cross sections for right- and left-handed electrons.

From appendix D and chapter 15 we know that the following are projections for right- and left-handed (massless) Dirac electrons

$$P_{\uparrow} = \frac{1}{2}(1-\gamma_5) \qquad\qquad P_{\downarrow} = \frac{1}{2}(1+\gamma_5) \tag{16.7}$$

To calculate the cross sections for such particles (Fig. 16.2) one simply modifies $\eta_{\mu\nu}$ with the appropriate insertion of these projections and removes the average over the initial helicities[1]

$$\begin{aligned}
\text{for } d\sigma_{\uparrow}: \quad \eta^{\uparrow}_{\mu\nu} &= \ldots \overbrace{\left(\frac{1}{2}\right)}^{\text{omit}} \sum_{s_1}\sum_{s_2} \bar{u}(k_1)\ldots\frac{1}{2}(1-\gamma_5)u(k_1) \\
\text{for } d\sigma_{\downarrow}: \quad \eta^{\downarrow}_{\mu\nu} &= \ldots \sum_{s_1}\sum_{s_2} \bar{u}(k_1)\ldots\frac{1}{2}(1+\gamma_5)u(k_1) \\
\text{for } d\sigma_{\uparrow}-d\sigma_{\downarrow}: \quad \eta^{(-)}_{\mu\nu} &= \ldots \sum_{s_1}\sum_{s_2} \bar{u}(k_1)\ldots(-\gamma_5)u(k_1) \\
\text{for } d\sigma_{\uparrow}+d\sigma_{\downarrow}: \quad \eta^{(+)}_{\mu\nu} &= \ldots \sum_{s_1}\sum_{s_2} \bar{u}(k_1)\ldots(1)u(k_1)
\end{aligned} \tag{16.8}$$

Thus one now has either $(-\gamma_5)$ or (1) in the lepton trace. Since *all common factors cancel in the ratio* the asymmetry is given by

$$\mathscr{A} \equiv \frac{d\sigma_{\uparrow}-d\sigma_{\downarrow}}{d\sigma_{\uparrow}+d\sigma_{\downarrow}} = -\frac{Gq^2}{4\pi\alpha\sqrt{2}}\frac{\eta^{(1)}_{\mu\nu}W^{(1)}_{\mu\nu}+\eta^{(2)}_{\mu\nu}W^{(2)}_{\mu\nu}}{2\eta_{\mu\nu}W_{\mu\nu}} \tag{16.9}$$

Here

$$\begin{aligned}
\eta^{(1)}_{\mu\nu} &= -2\varepsilon_1\varepsilon_2\sum_{s_1}\sum_{s_2}\bar{u}(k_1)\gamma_\nu u(k_2)\bar{u}(k_2)\gamma_\mu(a+b\gamma_5)(-\gamma_5)u(k_1) \\
W^{(1)}_{\mu\nu} &= (2\pi)^3\overline{\sum_i}\sum_f \delta^{(4)}(q+p'-p)\langle p|J^{\gamma}_{\nu}(0)|p'\rangle\langle p'|\mathscr{J}^{(0)}_{\mu}(0)|p\rangle(\Omega E_p) \\
\eta^{(2)}_{\mu\nu} &= -2\varepsilon_1\varepsilon_2\sum_{s_1}\sum_{s_2}\bar{u}(k_1)\gamma_\nu(a+b\gamma_5)u(k_2)\bar{u}(k_2)\gamma_\mu(-\gamma_5)u(k_1) \\
W^{(2)}_{\mu\nu} &= (2\pi)^3\overline{\sum_i}\sum_f \delta^{(4)}(q+p'-p)\langle p|\mathscr{J}^{(0)}_{\nu}(0)|p'\rangle\langle p'|J^{\gamma}_{\mu}(0)|p\rangle(\Omega E_p)
\end{aligned} \tag{16.10}$$

[1] Note $d\sigma^{\uparrow}+d\sigma^{\downarrow} = 2d\sigma_{\text{unpolarized}}$.

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The lepton traces have been evaluated in chapter 15. The result is[2]

$$\eta_{\mu\nu}^{(1)} = \eta_{\mu\nu}^{(2)} = -2(b\eta_{\mu\nu} + a\varepsilon_{\mu\nu\rho\sigma}k_{1\rho}k_{2\sigma}) \tag{16.11}$$

Thus in the numerator of Eq. (16.9) one needs $\eta_{\mu\nu}^{(1)}(W_{\mu\nu}^{(1)} + W_{\mu\nu}^{(2)})$ and

$$W_{\mu\nu}^{(1)} + W_{\mu\nu}^{(2)} = (2\pi)^3 \overline{\sum_i} \sum_f \delta^{(4)}(q + p' - p) \left[\langle p|J_\nu^\gamma(0)|p'\rangle\langle p'|\mathscr{J}_\mu^{(0)}(0)|p\rangle + \langle p|\mathscr{J}_\nu^{(0)}(0)|p'\rangle\langle p'|J_\mu^\gamma(0)|p\rangle\right](\Omega E_p) \tag{16.12}$$

Now separate the weak neutral current into its Lorentz vector and axial vector parts

$$\mathscr{J}_\mu^{(0)} = J_\mu^{(0)} + J_{\mu 5}^{(0)} \qquad ; \mathrm{V-A} \tag{16.13}$$

Since the asymmetry is already explicitly of order $Gq^2/4\pi\alpha\sqrt{2}$, one can then use the good parity of the nuclear states to write

$$W_{\mu\nu}^{(1)} + W_{\mu\nu}^{(2)} = W_{\mu\nu}^{\mathrm{int}} + W_{\mu\nu}^{\mathrm{V-A}} \tag{16.14}$$

Here the first term $W_{\mu\nu}^{\mathrm{int}}$ comes from $J_\mu^{(0)}$; it has the same general structure as $W_{\mu\nu}^\gamma$ in Eq. (16.6)[3]

$$W_{\mu\nu}^{\mathrm{int}} = W_1^{\mathrm{int}}(q^2, q\cdot p)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + W_2^{\mathrm{int}}(q^2, q\cdot p)\frac{1}{M_T^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right) \tag{16.15}$$

The second term in Eq. (16.14), coming from $J_{\mu 5}^{(0)}$, is a *pseudotensor*; the only pseudotensor that can be constructed from the two four-vectors (p_μ, q_μ) is[4]

$$W_{\mu\nu}^{\mathrm{V-A}} = W_8(q^2, q\cdot p)\frac{1}{M_T^2}\varepsilon_{\mu\nu\rho\sigma}p_\rho q_\sigma \tag{16.16}$$

Now combine these expressions with Eq. (16.11). The result follows from simple algebra and kinematics of the type carried out previously. The only non-zero terms are [see Eq. (11.35)]

$$\begin{aligned} 2\eta_{\mu\nu}W_{\mu\nu} &= 4\varepsilon_1\varepsilon_2[W_2^\gamma \cos^2\frac{\theta}{2} + 2W_1^\gamma \sin^2\frac{\theta}{2}] \\ -2b\eta_{\mu\nu}W_{\mu\nu}^{\mathrm{int}} &= (-b)4\varepsilon_1\varepsilon_2[W_2^{\mathrm{int}}\cos^2\frac{\theta}{2} + 2W_1^{\mathrm{int}}\sin^2\frac{\theta}{2}] \end{aligned} \tag{16.17}$$

[2] Note that the first term is symmetric in $\mu \leftrightarrow \nu$, while the second is antisymmetric.
[3] The proof of this result uses the fact that the current $J_\mu^{(0)}$ is conserved.
[4] Note that this expression is antisymmetric in $\mu \leftrightarrow \nu$.

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and

$$
\begin{aligned}
(-2a\varepsilon_{\mu\nu\rho\sigma}k_{1\rho}k_{2\sigma}) & \left[W_8(q^2, q\cdot p)\frac{1}{M_T^2}\varepsilon_{\mu\nu\alpha\beta}p_\alpha q_\beta\right] \\
&= -\left(\frac{4a}{M_T^2}W_8\right)(k_1\cdot p\, k_2\cdot q - k_1\cdot q\, k_2\cdot p) \\
&= -\left(\frac{2a}{M_T^2}W_8\right)q^2\, p\cdot(k_1+k_2) \\
&= \left(\frac{2a}{M_T}W_8\right)4\varepsilon_1\varepsilon_2\sin\frac{\theta}{2}\left(q^2\cos^2\frac{\theta}{2}+\mathbf{q}^2\sin^2\frac{\theta}{2}\right)^{1/2} \qquad (16.18)
\end{aligned}
$$

The ERL is assumed with $q = k_2 - k_1$, and the results are written in the laboratory frame. The last line follows from the following manipulations in that frame

$$
\begin{aligned}
q^2\cos^2\frac{\theta}{2}+\mathbf{q}^2\sin^2\frac{\theta}{2} &= \mathbf{q}^2 - q_0^2\cos^2\frac{\theta}{2} \\
&= \varepsilon_2^2+\varepsilon_1^2-2\varepsilon_1\varepsilon_2\cos\theta-(\varepsilon_2-\varepsilon_1)^2\cos^2\frac{\theta}{2} \\
&= (\varepsilon_1+\varepsilon_2)^2\sin^2\frac{\theta}{2} \qquad (16.19)
\end{aligned}
$$

The final result is

$$
\begin{aligned}
\left[\frac{d\sigma_\uparrow - d\sigma_\downarrow}{d\sigma_\uparrow + d\sigma_\downarrow}\right]\left[W_2^\gamma\cos^2\frac{\theta}{2}+2W_1^\gamma\sin^2\frac{\theta}{2}\right] &= \frac{Gq^2}{4\pi\alpha\sqrt{2}} \\
\times\Bigg\{b\left[W_2^{\text{int}}\cos^2\frac{\theta}{2}+2W_1^{\text{int}}\sin^2\frac{\theta}{2}\right] & \\
-a\left(\frac{2W_8}{M_T}\right)\sin\frac{\theta}{2}\left(q^2\cos^2\frac{\theta}{2}+\mathbf{q}^2\sin^2\frac{\theta}{2}\right)^{1/2}\Bigg\} & \qquad (16.20)
\end{aligned}
$$

Several features of this result are of interest:

- This is the general expression for the parity-violating asymmetry in relativistic polarized electron scattering from a hadronic target arising from the interference of one-photon and one-Z exchange (Fig. 16.1).[5]
- The left hand side is the product of the asymmetry $\mathscr{A}$ [Eq. (16.9)] and the basic $(\mathrm{e}, \mathrm{e}')$ response [Eqs. (16.6) and (16.17)].

[5] Additional contributions to the parity-violating asymmetry can arise from parity admixtures in the nuclear states coming from weak parity-violating nucleon–nucleon interactions. These contributions are generally negligible, except perhaps at very small q^2 [Se79, Dm92].

- The characteristic scale of parity violation in nuclear physics from the process $(\vec{\mathrm{e}}, \mathrm{e}')$ is set by the dimensionless parameter $Gq^2/4\pi\alpha\sqrt{2}$ appearing on the right hand side.

- The parameter b characterizes the lepton axial-vector weak neutral current [Eq. (16.2)]; its coefficient here arises from the interference of the vector part of the weak neutral and electromagnetic hadronic currents [Eqs. (16.12), (16.13), and (16.15)]

$$\begin{aligned} W_{\mu\nu}^{\mathrm{int}} &= (2\pi)^3 \overline{\sum_i} \sum_f \delta^{(4)}(q+p'-p) \Big[\langle p|J_\nu^\gamma(0)|p'\rangle\langle p'|J_\mu^{(0)}(0)|p\rangle \\ &\quad + \langle p|J_\nu^{(0)}(0)|p'\rangle\langle p'|J_\mu^\gamma(0)|p\rangle \Big] (\Omega E_p) \\ &= W_1^{\mathrm{int}}(q^2, q\cdot p)\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \\ &\quad + W_2^{\mathrm{int}}(q^2, q\cdot p)\frac{1}{M_T^2}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right) \end{aligned} \tag{16.21}$$

- The parameter a characterizes the lepton vector weak neutral current [Eq. (16.2)]; its coefficient here arises from the interference of the axial vector part of the weak neutral and electromagnetic hadronic currents [Eqs. (16.12)–(16.14) and (16.16)]

$$\begin{aligned} W_{\mu\nu}^{\mathrm{A-V}} &= (2\pi)^3 \overline{\sum_i} \sum_f \delta^{(4)}(q+p'-p) \Big[\langle p|J_\nu^\gamma(0)|p'\rangle\langle p'|J_{\mu 5}^{(0)}(0)|p\rangle \\ &\quad + \langle p|J_{\nu 5}^{(0)}(0)|p'\rangle\langle p'|J_\mu^\gamma(0)|p\rangle \Big] (\Omega E_p) \\ &= W_8(q^2, q\cdot p)\frac{1}{M_T^2}\varepsilon_{\mu\nu\rho\sigma}p_\rho q_\sigma \end{aligned} \tag{16.22}$$

- The three response functions on the right hand side of Eq. (16.20) can be separated by varying the electron scattering angle θ at fixed $(q^2, q\cdot p)$.[6]

- The parity violation arises from the interference of the transition matrix element of the electromagnetic and the weak neutral currents. If the electromagnetic matrix elements have been measured, then *parity violation in* $(\vec{\mathrm{e}}, \mathrm{e})$ *and* $(\vec{\mathrm{e}}, \mathrm{e}')$ *provides a measurement of the matrix elements of the weak neutral current in nuclei at all* q^2.

We give one example [Wa84, Wa95]. Consider elastic scattering from a 0^+ target (Fig. 16.3a). Then from Lorentz covariance and current con-

[6] This is known as a *Rosenbluth* separation.

Fig. 16.3. Example of parity-violating asymmetry in scattering from (a) $J^\pi = 0^+$, and (b) $(J^\pi, T) = (0^+, 0)$ target.

servation the transition matrix elements of the electromagnetic and weak neutral currents must have the form[7]

$$
\begin{aligned}
\langle p'|J_\mu^\gamma(0)|p\rangle &= \left(\frac{M_T^2}{EE'\Omega^2}\right)^{1/2} F_0^\gamma(q^2)\frac{1}{M_T}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right) \\
\langle p'|J_\mu^{(0)}(0)|p\rangle &= \left(\frac{M_T^2}{EE'\Omega^2}\right)^{1/2} F_0^{(0)}(q^2)\frac{1}{M_T}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right) \\
\langle p'|J_{\mu 5}^{(0)}(0)|p\rangle &= 0
\end{aligned}
\tag{16.23}
$$

The last relation follows since it is impossible to construct an axial vector from only two four-vectors (p_μ, q_μ).

Insertion of these relations in the defining equations yields

$$
\begin{aligned}
W_1^{\text{int}} &= W^{\text{A}-\text{V}} = 0 \\
\mathscr{A} &= \frac{Gq^2}{4\pi\alpha\sqrt{2}} b \frac{2F_0^{(0)}(q^2)}{F_0^\gamma(q^2)}
\end{aligned}
\tag{16.24}
$$

Hence

$$
\mathscr{A} = -\frac{Gq^2}{2\pi\alpha\sqrt{2}} \frac{F_0^{(0)}(q^2)}{F_0^\gamma(q^2)} \tag{16.25}
$$

This expression allows one to measure the ratio of the weak neutral current and electromagnetic form factors — the latter measures the distribution of electromagnetic charge in the 0^+ target, and the former the distribution of weak neutral charge.

Now suppose that, in addition, the target has isospin $T = 0$ (Fig. 16.3b). Then only isoscalar operators can contribute to the matrix elements. In the *nuclear domain* of (u, d) quarks and antiquarks, the only isoscalar piece of the weak neutral current in the standard model arises from the electromagnetic current itself, and hence in this case (see chapter 26)

$$
J_\mu^{(0)} \doteq -2\sin^2\theta_W J_\mu^\gamma \tag{16.26}
$$

This implies

$$
F_0^{(0)}(q^2) = -2\sin^2\theta_W F_0^\gamma(q^2) \tag{16.27}
$$

[7] Hermiticity of the current implies that the form factors, as defined here, are real.

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The ratio of form factors is then the constant $-2\sin^2\theta_W$ at all q^2 — a truly remarkable prediction![8] Insertion of this equality in the expression for the asymmetry leads to [Fe75]

$$\mathscr{A} = \frac{Gq^2}{\pi\alpha\sqrt{2}}\sin^2\theta_W \tag{16.28}$$

Several comments are of interest:

- It is important to note that this result holds *to all orders in the strong interactions* (QCD);
- This expression is linear in q^2 with a coefficient that depends only on fundamental constants;
- It can be used to measure $\sin^2\theta_W$ in the low-energy quark sector, complementing other measurements of this quantity;
- It can be used to test the remarkable prediction in Eq. (16.27) that holds in the nuclear domain.

A measurement of this parity-violating asymmetry for elastic scattering from ^{12}C at $q = 150$ MeV has been carried out in a tour de force experiment at the Bates Laboratory [So90]. Take

$$q = 150\ \text{MeV} \qquad \sin^2\theta_W = 0.2315$$
$$\alpha^{-1} = 137.0 \qquad G = \frac{1.024\times 10^{-5}}{m_p^2}$$
$$\mathscr{A} = 1.868\times 10^{-6} \tag{16.29}$$

Then, with an electron beam polarization P_e, one has [So90, Mo90]

$$\begin{aligned}\mathscr{A}P_e &= 0.691\times 10^{-6} && ;\ \text{theory}\ (P_e = 0.37)\\ \mathscr{A}P_e &= 0.60\pm 0.14\pm 0.02\times 10^{-6} && ;\ \text{experiment}\end{aligned} \tag{16.30}$$

The first error is statistical. Note that the systematic error, the key to these experiments, has been reduced to 2×10^{-8}. This experiment provides the prototype for the next generation of electron scattering parity-violation studies.

Consider next the *extended domain* of (u, d, s, c) quarks and their antiquarks. The standard model then has an additional isoscalar term in the weak neutral current (see chapter 26)

$$\delta\mathscr{J}_\mu^{(0)} = \frac{i}{2}[\bar{c}\gamma_\mu(1+\gamma_5)c - \bar{s}\gamma_\mu(1+\gamma_5)s] \tag{16.31}$$

[8] This result depends on the assumption of isospin invariance that is broken to $O(\alpha)$ in nuclei.

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Table 16.1. Quark sector used in discussion of parity-violating deep-inelastic electron scattering from the nucleon (see chapter 26).

	u	d	s
$Q_i^{(0)}$	$1/2-(4/3)\sin^2\theta_W$	$-1/2+(2/3)\sin^2\theta_W$	$-1/2+(2/3)\sin^2\theta_W$
$Q_i^{(05)}$	$1/2$	$-1/2$	$-1/2$

This leads to an additional contribution $\delta F_0^{(0)}$ in the form factor in Eq. (16.27); the asymmetry for elastic scattering of polarized electrons on a $(0^+,0)$ nucleus such as ^{4}He then takes the form

$$\mathscr{A} \;=\; \frac{Gq^2}{\pi\alpha\sqrt{2}}\sin^2\theta_W\left[1-\frac{\delta F_0^{(0)}(q^2)}{2\sin^2\theta_W F_0^\gamma(q^2)}\right] \tag{16.32}$$

The additional weak neutral current form factor comes from the vector current in Eq. (16.31), and is expected to arise predominantly from the much lighter strange quarks. Hence one has a direct measure of the *strangeness current* in nuclei. The total strangeness of this nucleus must vanish in the strong and electromagnetic sector, and hence $\delta F_0^{(0)}(0)=0$; however, just as with electromagnetic charge in the neutron, there can be a strangeness *distribution*, which is determined in this experiment.

The *quark–parton model* predictions for parity violation in deep-inelastic scattering from the nucleon follow directly from the previous analysis. Go back to the intermediate step in Eq. (14.34) and identify in the quark response tensor

$$Q_i^2[p'_\mu p_\nu+p'_\nu p_\mu-(p\cdot p')\delta_{\mu\nu}]\rightarrow\frac{Q_i^2}{4}\text{trace}\,[\gamma_\nu(\gamma_\rho p'_\rho)\gamma_\mu(\gamma_\sigma p_\sigma)] \tag{16.33}$$

In the response tensor arising from the interference of the electromagnetic and vector weak neutral currents, one has instead

$$\begin{aligned}\frac{Q_iQ_i^{(0)}}{4}\text{trace}\,[\gamma_\nu(\gamma_\rho p'_\rho)\gamma_\mu(\gamma_\sigma p_\sigma)+\gamma_\nu(\gamma_\rho p'_\rho)\gamma_\mu(\gamma_\rho p_\rho)] \;&=\\ 2Q_iQ_i^{(0)}[p'_\mu p_\nu+p'_\nu p_\mu-(p\cdot p')\delta_{\mu\nu}]&\end{aligned} \tag{16.34}$$

Here $Q_i^{(0)}$ is the weak neutral charge of the quarks, shown for the first few quarks in Table 16.1. The arguments proceed precisely as those following Eq. (14.34), with the result that the following combinations of response functions are predicted to satisfy Bjorken scaling

$$\begin{aligned}2W_1^{\text{int}}(\nu,q^2) \;&\rightarrow\; H_1(x)=2\sum_i Q_iQ_i^{(0)}f_i(x)\\ \left(\frac{\nu}{m}\right)W_2^{\text{int}}(\nu,q^2)\;&\rightarrow\; H_2(x)=xH_1(x)=2x\sum_i Q_iQ_i^{(0)}f_i(x)\end{aligned} \tag{16.35}$$

https://doi.org/10.1017/9781009290616.019 Published online by Cambridge University Press

For the interference term between the axial vector and electromagnetic currents, the corresponding replacement in Eq. (16.33) is

$$\frac{Q_iQ_i^{(05)}}{4}\text{trace}\,[\gamma_\nu(\gamma_\rho p'_\rho)\gamma_\mu\gamma_5(\gamma_\sigma p_\sigma)+\gamma_\nu\gamma_5(\gamma_\rho p'_\rho)\gamma_\mu(\gamma_\sigma p_\sigma)] \quad = \\ -2Q_iQ_i^{(05)}\varepsilon_{\mu\nu\rho\sigma}p'_\rho p_\sigma \tag{16.36}$$

Hence a repetition of the arguments following Eq. (14.34) allows one to conclude that the following combination must scale

$$-\left(\frac{\nu}{m}\right)W_8(\nu,q^2)\to H_8(x)=2\sum_i Q_iQ_i^{(05)}f_i(x) \tag{16.37}$$

Here $Q_i^{(05)}$ are the axial vector couplings of the quarks, also shown for the first few quarks in Table 16.1. Note that if Q_i and $f_i(x)$ are known from DIS through the electromagnetic interaction, then the parity violation measurements allow one to determine the weak neutral current couplings of the quarks.[9]

We will return to the subject of parity violation in the discussion of applications and future directions.

[9] Parity violation in DIS from the nucleon is further analyzed in [Ka78].