PART IV

THE ROTATION OF THE SUN

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THE ROTATION OF THE SUN*

(Review Paper)

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Abstract. The author's 1964 article in *Nature* on the sun's rotation is rediscussed in the light of new data. This article suggested that the sun might be oblate because of a gravitational quadrupole moment induced by a core rotating with a period < 2 days. Angular momentum lost from the core by molecular diffusion was assumed to be transferred to the solar wind which kept the sun's surface rotating slowly. The estimated solar wind torque was found to be in good agreement with the torque calculated from a solution of the diffusion equation.

Subsequent to the 1964 paper the oblateness was observed. Also the solar wind torque was 'observed' to be in good agreement with the early estimate. New observations discussed here seem to be important if the sun can be safely assumed to be a typical star and not an exception. It has been found by Kraft (1967) that very young solar type stars (Pleiades) are rotating with roughly the same angular velocity postulated for the solar core. As determined from observations on the Hyades in comparison with the Pleiades, the rotation of young solar-type stars is slowing at a rate consistent with a stellar wind torque equal to that of the solar wind acting on the outer 20–30% of the star by radius. The slowing of the rotation in young stars is accompanied by a depletion of lithium, but not berylium. This implies that only the outer 45% of the star by radius, or 5% by mass, is slowed by the solar wind. The rapid rotation of the inner 95% of the mass is sufficient to generate the observed oblateness. The rate of depletion of lithium, determined from observations on solar-type stars of various ages, is consistent with the rate of angular momentum loss assuming a reasonable model for the transport of angular momentum to the convective zone.

1. Introduction

In a brief note published in 1964 I suggested that the sun might have a rapidly rotating core. (Dicke, 1964.) This possibility was also discussed by Roxburgh (1964), Plaskett (1965) and Deutsch (1967). In the *Nature* article it was noted that such a rapidly spinning core would induce a gravitational quadrupole moment in the sun and that the resulting perturbation of Mercury's orbit could account for 5–10% of the classical 43″ arc/century excess motion of Mercury's perihelion. The resulting solar oblateness, as large as 6×10^{-5}, was considered measurable. In collaboration with H. Hill and H. M. Goldenberg, the design and construction of a special instrument had been launched a year earlier, in the spring of 1963. The first version of this telescope was put in operation during the summer of 1963. Two years were required to study the systematic errors of the instrument and to correct and improve its design. (See Figures 1, 2, and 3.) The first useful measurements, made during the summer of 1966 (Dicke and Goldenberg, 1967), gave an oblateness of $(r_{eq} - r_{pole})/r = (4.8 \pm .9) \times 10^{-5}$. (See Figure 4.) The measurements made during the summer of 1967 are not yet published, but they yield the same value for the oblateness with comparable precision.

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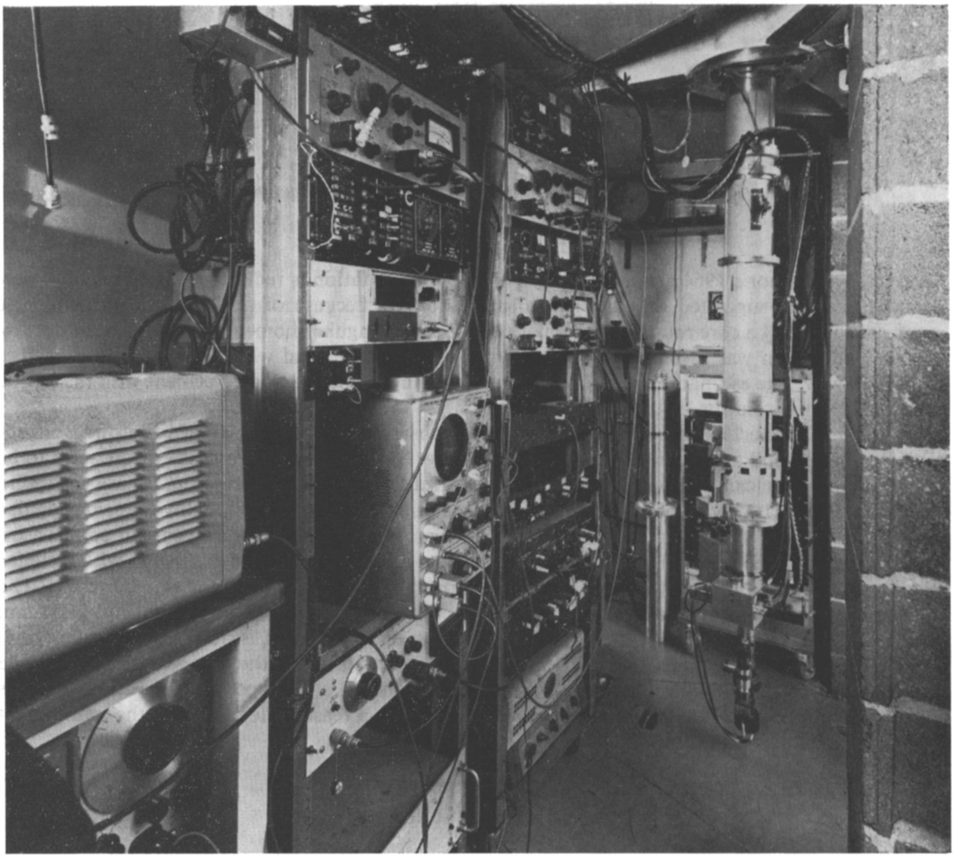

Fig. 1. The solar oblateness telescope located on the grounds of the Princeton Observatory. This instrument was designed by H. Hill, H. M. Goldenberg and the author. As the picture clearly shows, the ratio of number of pieces of electronic equipment to telescope aperture in inches is probably greater for this instrument than any other in existence (reprinted courtesy American Philosophical Society).

I do not propose to discuss the observations here. This will be the subject of a full treatment by Goldenberg and me. Rather I shall return to the old publication in *Nature*. It may seem strange to devote space to a paper that is over 5 years old, but much of this story has not been told and some of it now requires updating.

The note in *Nature* was extremely condensed and I had intended immediately to follow it with several detailed papers, but this was precluded by the press of the observational program. One of these papers was finally written and is soon to be published (Dicke, 1970a).

This paper discusses the effects of surface stresses on the sun's shape and brightness distribution. This contribution to oblateness and equatorial brightening is explicitly calculated. The constraint imposed on the theory by the lack of equatorial brightening is examined. The effects on the oblateness of observed magnetic and velocity fields are

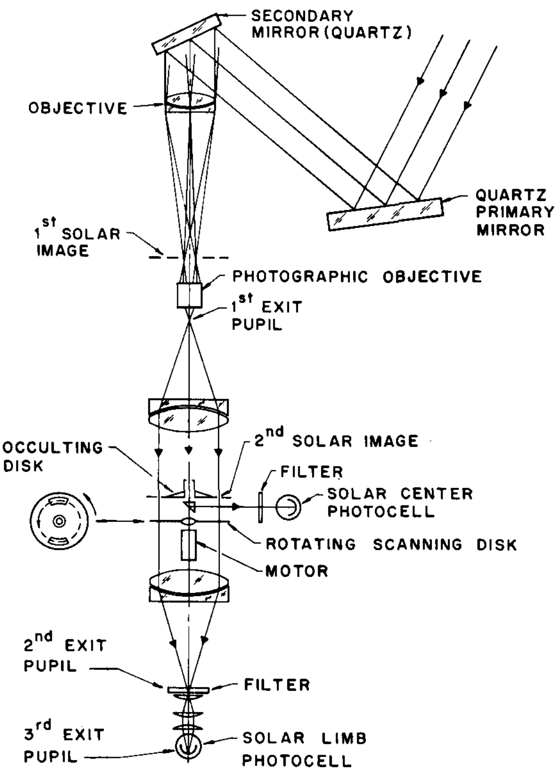

Fig. 2. The optical system of the oblateness telescope. The first mirror tracks the sun by means of a motor drive of a gimbal-mounting of the mirror. A fast-acting solenoid-actuated servo-system provides precision pointing of the mirror. To avoid stick-friction bearing noise, bent-hinge bearings are provided for the servo-system. An image of the sun is projected on an occulting disk which passes the outer 6″–20″ arc of the sun's disk to a rapidly spinning scanning wheel. Two apertures of slightly different size in the scanning wheel pass light to the main photocell. The error signal to the servo-system is derived from this photo-cell as the fundamental rotation frequency of the scanning wheel. The oblateness signal is derived from the 2nd harmonic of this frequency (reprinted courtesy Physical Review).

considered with the conclusion that, as yet, there is no explanation for the excess oblateness, other than the effect of a quadrupole moment.

Another paper, intended to be joint with P. J. E. Peebles, is no longer needed. The note in *Nature* contained the main result derived from our theory of the solar wind torque. This theory, based on Schatzman's (1962) original idea, has been independently developed by three other groups (Modiesette, 1967; Weber and Davis, 1967; and Alfonso-Faus, 1967). Unfortunately, through an error in placing a reference number, Modiesette's description of our calculation is inaccurate, having been meant for another reference.

I propose to organize the present article along lines similar to the *Nature* article, to discuss a number of important points in more detail, and to introduce a substantial amount of information new since 1964.

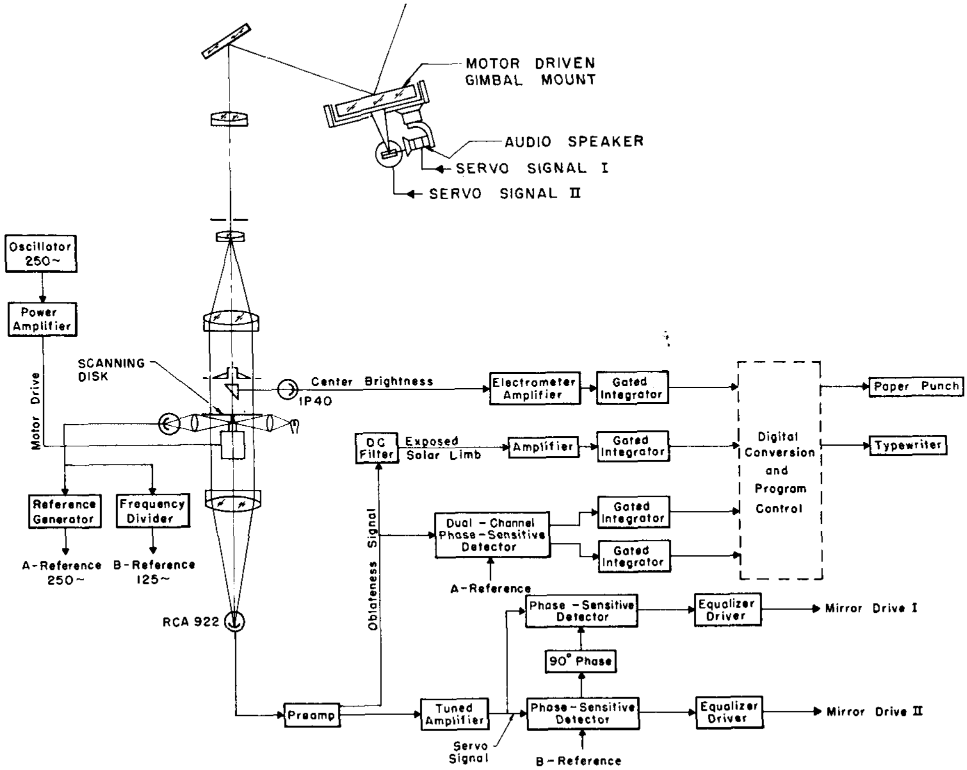

Fig. 3. A block diagram of the system showing the chief parts of the instrument (reprinted courtesy American Philosophical Society).

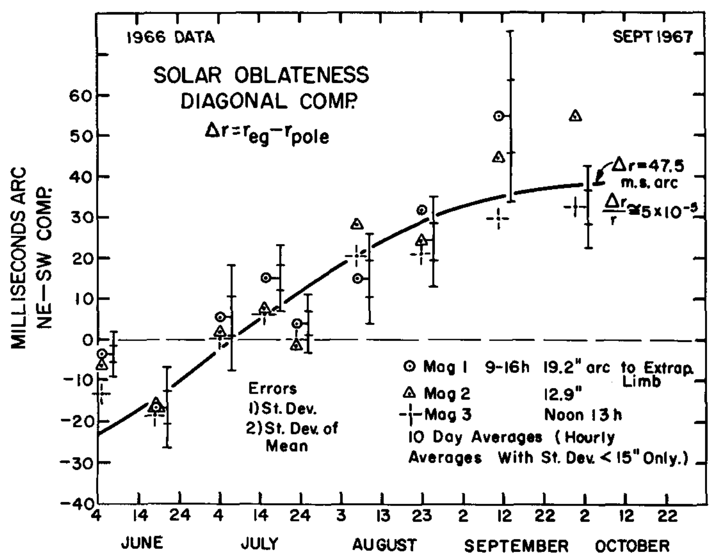

Fig. 4. The fit of the curve for the 'diagonal' component of the oblateness to the observed '10-day averages' of 1966. The exposed limb for magnification #3 (Mag 3) was 6.6″ arc. The 'diagonal' component is the part of the oblateness associated with a shortening of the sun's disk along the NE-SW line (reprinted courtesy American Philosophical Society).

In pre-relativity days the observed excess motion of Mercury's perihelion ($\sim 43''$ arc/century) was enigmatic and lead to several unsuccessful attempts to find a source of a gravitational field which could advance the perihelion. Among the sources considered were Vulcan, a hypothetical planet whose orbit was thought to lie between the sun and Mercury. It has not been discovered. To be large enough to be significant in perturbing Mercury's orbit Vulcan should have been visible against the sun's disk. Interplanetary debris is likewise an unpromising source of such a gravitational field (Chazy, 1928).

Of these old proposed sources, only a solar quadrupole moment (Newcomb, 1897) remains today as an interesting possibility, and it is unsuitable as a source of the full excess motion of Mercury's perihelion. Such a quadrupole moment would also induce a $43''$ arc/century regression of Mercury's node on the equatorial plane of the sun. Such a large error in orbital wobble can be excluded by the observations.

With the advent of Einstein's General Relativity, a relativistic explanation was available for the excess motion of Mercury's perihelion and the search for conventional sources ceased. But the scalar-tensor theory, an alternative general relativistic theory of gravitation, (Jordan, 1948, 1959; Thirry, 1948; Bergmann 1948; Brans and Dicke, 1961; Dicke, 1962) predicts a slightly smaller effect, $(1-\frac{4}{3}s)$ times Einstein's value. Here s is the fraction of a body's weight due to the scalar field under the Dicke (1962) version of the theory. Expressed in terms of ω, the coupling constant of the Brans-Dicke (1961) theory, $s=1/(2\omega+4)$. On various grounds ω had been estimated to fall in the range 4–7 (Brans and Dicke, 1961; Dicke and Peebles, 1965; Dicke, 1966).

The scalar-tensor theory with $\omega=5$ yields a relativistic rotation of the perihelion of $38\overset{''}{.}7$ arc/century. This is compatible with the observations, providing the sun has an oblateness of $\sim 5\times10^{-5}$ and the connection of this oblateness with a quadrupole moment has been properly interpreted. It had been noted (Dicke, 1964) that the resulting motion of the node of Mercury's orbit on the equator of the sun, $4.3''$ arc/century, becomes mainly a $0.21''$ arc/century decrease in the inclination, when referred to the ecliptic. This is a tricky point, that the coordinate transformation has such a large effect. It has been missed on several occasions and as often rediscussed by others (Shapiro, 1965; Gilvarry and Sturrock, 1967; O'Connell, 1968). The expected residual in the rate of increase of the inclination ($-0.21''$ arc/century with $\omega=5$) is to be compared with an observed residual of $-0.12''\pm0.16''$/century. It is evident that a quadrupole moment large enough to rotate the perihelion by $3''$–$4''$ arc/century can be tolerated but not one 3–4 times larger.

At the time that C. Brans and I published our paper on the scalar-tensor theory we considered the uncertainty in the indirectly determined mass of Venus large enough to permit a few percent correction to the 'observed' excess motion of Mercury's perihelion, but after the mass of Venus had been directly determined from the Mariner II orbit, this possibility was excluded.

The mass of Venus is now known with even more precision and the classical perturbations (except for that of an oblate sun) must be assumed to be well known. The masses of the earth and Jupiter have long been accurately known. If the observations of Mercury's motion are as accurate as believed by the experts, the observed classical excess motion of Mercury's perihelion, 43″ arc/century, has an accuracy of 1%. Prudence would require that we permit an observational error as large as 5%, but this is not enough to admit the relativistic perihelion rotation of 39″ arc/century expected under the scalar-tensor theory. If the sun has an appropriate quadrupole moment, the scalar-tensor theory is favored. If not, it is excluded with $\omega < 10$.

It must be emphasized that a rapidly rotating solar core is not the only possible source of a substantial quadrupole moment in the sun. A strong, deeply buried field along the rotation axis could also generate such a distortion (Sturrock and Gilvarry, 1967). My reasons for rejecting this in 1963 still seem compelling. A magnetic field of the order of 10^6 gauss must stay buried or it will appear at the surface as a very strong permanent dipole. Such a strong dipolar field oriented along the rotation axis would be expected to diffuse to the surface. But a permanent 'general magnetic' field does not seem to exist at the solar surface.

2. Physical Requirements for a Rapidly Rotating Core

A rapidly rotating solar core is impossible unless two conditions are satisfied:

(a) The core must be able to spin with very little friction.

(b) A frictional drag on the solar surface must keep the surface rotating slowly.

The theory of the solar wind torque requires a knowledge of the magnetic field strength in the solar wind for a calculation of the torque and this information was not directly available in 1964. In lieu of an observed field strength, I estimate an equivalent field strength at the sun's surface and used it to calculate the torque. The basic conception of the solar wind torque is due to Schatzmann (1962).

The torque density at the equator on the solar surface is (Dicke, 1964; Modiesette, 1967; Weber and Davis, 1967; Alfonso-Faus, 1967)

$$K = Jr^2\omega_0 \tag{1}$$

where J is the mass flux density at the solar surface, expressed in gm/sec cm^2 lost to the solar wind. ω_0 is the angular velocity of the solar surface and r is the critical radius for which B^2 satisfies the equation

$$\varrho v^2 = \frac{1}{4\pi} B^2 \tag{2}$$

namely, the radius at which v, the radial component of the solar wind velocity equals the Alfvén velocity calculated from B, the radial component of the magnetic field.

The magnetic field is trapped in the solar wind and B falls off inversely as the square of the radial distance. Expressed in terms of the equivalent radial component of magnetic field strength at the sun's surface, B_0, the critical radius is such that the density at this radius satisfies

$$\varrho = 4\pi J^2 / B_0^2 . \tag{3}$$

In Equation (2) the square of magnetic field strength should not be interpreted as a mean squared value of the field over the whole surface but rather as an average over such weak unipolar regions as are pulled out by the solar wind. The r.m.s. of this field was estimated to be $\frac{3}{4}$ gauss. Subsequently, when the interplanetary field was measured at 1 AU using the Mariner II space probe (Coleman, 1966), the field was found to have a value consistent with this estimate. As expected, the field was found to be twisted into a spiral pattern. The radial component of B fluctuated some and the r.m.s. value of this component was 3.5×10^{-5} gauss. This is in good agreement with the value of B obtained from $B_0 = \frac{3}{4}$ gauss, namely $\frac{3}{4} \times \frac{1}{200}^2 = 2 \times 10^{-5}$ gauss assuming that the field falls off inversely as the square of the distance, a somewhat simplified assumption as it ignores centrifugal concentration to the equatorial plane.

From the same space probe, the rate of mass loss, or mass flux density at the solar surface, was determined to be $J = 1.7 \times 10^{-11}$ gm/cm^2 sec assuming that the flux density varies inversely with distance squared (Neugebauer and Snyder, 1966).

Substituting these results in (3) gives 3.9×10^3 for the proton number density at the critical radius. Making use of a standard model of the solar corona (e.g. Allen, 1963) gives $r = 20\, r_0$ for the critical radius and, from Equation (1), torque density of $K = 9.3 \times 10^7$ dyne/cm.

The close agreement between this value for the torque density and the value published in 1964 (1×10^8 dyne/cm) is fortuitous for the value of the effective solar field strength, $\frac{3}{4}$ gauss, was only an estimate. Also, because of concentration to the ecliptic, this 'observed' value may be much too large.

The basis for the estimate is worth some discussion, for, if correct, it can be used to extrapolate the solar wind torque into the past when the sun may have been much more active magnetically.

It is well known that a magnetic flux tube carrying a fluid moving along the field lines possesses a magnetic pressure $B^2/8\pi$, in excess of its gas pressure, and a tension along the field lines of $+(B^2/4\pi - \rho v^2)$. For a steady state of a cylindrical flux tube surrounded by field free gas of pressure P, the magnetic field must satisfy the condition

$$|B| < \sqrt{8\pi P}. \tag{4}$$

Equation (4) states that a negative gas pressure in the tube is impossible. The connection between the magnetic field and the gas properties inside the tube is actually stronger than that indicated above. For an arbitrary flux tube (not necessarily cylindrical) the distribution of *both* density and pressure inside the flux tube are uniquely determined (relative to their values outside) by the magnetic and velocity field distributions in the flux tube (Dicke, 1970b).

The above inequality must be imposed sufficiently low in the corona that the tension associated with the gas flow is small compared with the magnetic tension. From Allen's model of the corona at the equator the upper limit for B_0 given by Equation (4) takes on the values 0.8, 0.7, 0.5 and 0.35 gauss at $r/r_0 = 1.01$, 1.1, 1.4, and 2.0 respectively, and $r/r_0 = 2$ is probably too high in the corona. Apparently the strength of the coronal magnetic field is near its upper limit. Thus, assuming that the sun was more active magnetically in earlier times, the field B_0 was probably little greater than at present, perhaps by less than a factor of 2.

It can also be argued that the particle flux density in the solar wind has changed little during the past 4.5×10^9 years. It is now well recognized that this flux is determined by the rate of heating of the corona, in turn determined by the turbulence near the solar surface. But this turbulence is fixed by the luminosity which presumably has not changed drastically. Admittedly these arguments are crude, but they suggest that over the life-time of the sun the ratio of solar wind torque to angular velocity ω_0 can be assumed to have decreased little, by perhaps a factor of $\frac{1}{4}$. Furthermore for stars similar to the sun, the ratio of stellar wind torque to angular velocity would be expected to be approximately equal to that of the sun, providing the star is sufficiently active magnetically.

If the above equatorial torque density is correct, the total solar wind torque now is $(8\pi/3)r_0^2\,K = 3.8 \times 10^{30}$ dyne cm. If the whole sun is slowed by such a torque, proportional to ω_0, the e-folding time is 14×10^9 years. If only an outer shell is slowed in its rotation, the decay time is much shorter. Table I gives these times for a variety of assumptions about the inner radius of the outer, slowly rotating shell.

The second necessary condition requiring some discussion concerns the stability of, and frictional torque acting on, a rapidly rotating core. The outer convective zone cannot support a large angular velocity gradient as it is convectively unstable and turbulent.

While not certain, it seems likely that the observed latitude dependence of the surface angular velocity represents 'rotation on cylinders' in the convective zone, i.e.

TABLE I

Decay time in years and fractional moment of inertia as functions of the radius of an outer solar shell assuming that only the shell is slowed by a solar wind torque of 3.8×10^{30} dyne cm. The decay time for the sun rotating rigidly is 15×10^9 years

r/r_0	I_s/I	T (year)
0.86	0.0122	0.184×10^9
0.78	0.034	0.51×10^9
0.70	0.074	1.12×10^9
0.62	0.140	2.11×10^9
0.54	0.241	3.63×10^9

angular velocity a function of distance from the axis of rotation. If so, the observed variation with latitude implies that the angular velocity increases approximately linearly with distance from the rotational axis and that

$$\omega = \omega_0 + \omega_1 (r/r_0) \sin\theta \tag{5}$$

with

$$\omega_0 = 2.68 \times 10^{-3} (g/r_0)^{1/2}$$
$$\omega_1 = 1.94 \times 10^{-3} (g/r_0)^{1/2}.$$

Assuming that the magnetic and turbulent-viscous forces are small compared with the centrifugal force, we have for rotational motion approximately

$$0 = \nabla P + \varrho \nabla \varphi + \varrho \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}). \tag{6}$$

But in the convective zone, $P = P(\rho)$ implying that $\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ is derivable from a potential. This is possible if and only if ω is a function of $r \sin\theta$.

The turbulent-viscous stress is only 5% of that associated with the rotational velocity but, nonetheless, it is enormous. The internal stress of isotropic turbulence is many orders of magnitude greater than the solar wind stress. The source of the torque necessary to drive this differential rotation seems to be the forces derived from anisotropic turbulence. (Wasiutynski, 1946; Kippenhahn, 1963; Cocke, 1967.)

Below the convective zone, P and ρ are separately variable. Furthermore, the distributions over spherical surfaces of both pressure and density, and hence temperature, are uniquely determined by the distribution of angular velocity. The proof of this is similar to the discussion in Dicke (1967).

It should be emphasized that only the dynamical problem is being considered here. Thermally driven currents (Eddington-Sweet) may occur, as well as thermally driven instabilities (Goldreich and Schubert, 1967; 1968).

The temperature distribution forced by a particular distribution in angular velocity may not be compatible with the requirement that the heatflow be solenoidal beyond the nuclear-reaction-core. If not, circulation currents are induced to maintain a heat balance (Eddington-Sweet).

These thermally driven currents have velocities so low $< 10^{-6}$ cm/sec that their dynamical effects are negligible. It should be remarked that in a zone of differential rotation, the velocity of the thermally driven currents can be one or two orders of magnitude greater than that in a uniformly rotating core.

There are several aspects to the problem of stability in stars with rapidly rotating cores. First, it should be noted that some 2×10^9 cm below the bottom of the convective zone the sun becomes strongly density-stratified, the temperature gradient being substantially below the adiabatic level. Under these conditions the density stratification permits differential flow without turbulence, the velocity gradient being normal to level surfaces. Instead of the Reynolds number criterion for turbulence, that of Richardson namely

$$-r \frac{d\omega}{dr} < 2 \sqrt{\frac{g}{\gamma}} \left[(\gamma - 1) \frac{d \ln \varrho}{dr} - \frac{d \ln T}{dr} \right]^{1/2} \tag{7}$$

must be satisfied. This criterion permits very large angular velocity gradients. Adopting the Weymann (1957) solar model Equation (7) yields the limiting derivative angular velocity gradients shown in Table II.

TABLE II

Maximum angular velocity gradient under the Richardson criterion

r/r_0	$(r_0/\omega_0)\,(\mathrm{d}\omega/\mathrm{d}r)$
0.84	− 90.
0.80	− 870.
0.76	− 1230.
0.72	− 1390.
0.64	− 1730
0.52	− 2440

Dynamically driven turbulence is not the only possible source of instability in the sun. Howard *et al.* (1967) suggested that the existence of 'spin-down' would preclude the existence of a rapidly rotating core.

After a cup of tea is stirred to set it spinning, its rotation rapidly ceases, much more rapidly than would be expected from the diffusion of angular momentum to the walls of the cup. The slowing process, called 'spin-down' is due to the convection by Ekman currents of angular momentum to a thin layer at the bottom of the cup, the Ekman layer.

The reasons for these circulation currents are easily seen. The density of the tea is constant and, neglecting viscous forces, Equation (6) is applicable for purely rotational motion. But note that the implied rotation-on-cylinders is impossible if the boundary condition of zero rotation on the cup bottom is to be satisfied. Purely rotational motion cannot occur and furthermore the intrinsically weak viscous force becomes important because of a steep angular velocity of gradient near the bottom of the cup. The circulation current pumps the fluid into this thin layer (Ekman) at the bottom.

In a brief note (Dicke, 1967) it was remarked that the sun is 'no cup of tea', that the functional connection between pressure and density that forces Ekman currents does not occur deep in the sun, and that for each almost arbitrarily chosen angular velocity distribution there is a distribution of pressure and density compatible with purely rotational motion without dynamically driven circulation currents. Thus a density stratified interior which eliminates turbulence also permits separate variability of pressure and density and purely rotational motion without circulation (except for the thermally driven circulation currents discussed by Eddington and Sweet).

In a series of experiments, E. McDonald investigated the role of density stratification vis-à-vis the spin-down process (McDonald and Dicke, 1967). It was shown that a large angular velocity shear could be induced in a density stratified fluid without the fluid spinning down. For this to be true it was essential that the angular velocity gradient be induced gradually, by slowly changing the angular velocity of the cylindrical container of the fluid. A very small change in angular velocity of the container, if suddenly applied, would cause the fluid to spin-down.

The resulting spin-down was interesting to watch. The fluid would mix in two layers.

Circulation currents would separately mix the upper and lower halves of the contents of the container. This occurred by first establishing gravity waves of ever-increasing amplitude at mid-depths in the fluid. This wave behavior was followed by a period of turbulence in this thick layer and the establishment of circulation currents that separately mixed the upper and lower halves of the fluid. Meanwhile the layer of differential rotation became thinner. Finally there occurred the establishment of Ekman layers at the boundary of density change and at the container bottom causing the fluid to spin-down.

The reason for spin-down when the change in angular velocity is instantaneously established is easily seen. For purely rotational motion, a sudden change in the distribution of the angular velocity requires a sudden change in the density distribution. This is impossible to establish. Hence the actual density distribution is incompatible with such a sudden change in angular velocity, leading to the complicated set of motions described above.

If the angular velocity distribution is very slowly changed, very slow circulation currents (slow enough to be dynamically unimportant) permit the establishment of the new required density distribution.

Still another possible type of instability requires discussion. This is the thermally driven instability first discussed by Goldreich and Schubert (1967, 1968) and later by Fricke (1968). With this instability, angular momentum is transported toward the surface in outward and inward moving thin toruses, with a thickness in the θ direction of only a few km. The fluid must flow accurately parallel to these toruses if this instability is to develop. Thus circulation currents or oscillating motion in the θ direction would inhibit the instability and stabilize the rotation. Another means of stabilizing the rotation is provided by a gradient in mean molecular weight (Colgate, 1968; Goldreich and Schubert, 1968).

My overall impression of this instability is that it would occur if all the assumptions were satisfied, compositional gradients and magnetic fields being absent and the motion being accurately rotational. However, it is difficult to be certain that all these conditions would be satisfied in the deep solar interior.

One is reminded of the example of the thermo-haline instability discussed by Goldreich and Schubert (1967). Here, essentially the same analysis as that used by Goldreich, Schubert and Fricke can be employed to 'prove' that the salt concentration must be greater at the bottoms of all oceans than at the tops. If one's direct knowledge of the deep oceans were no better than that of the deep interior of the sun, he might be impressed by this 'proof'. Fortunately, we can observe the deep oceans, and the observations show quite the contrary, an increased salt concentration at the top in the tropics. The so-called 'proof' may be invalid because of oscillatory motions near the surface.

The viscosity of the solar medium below the convective zone is sufficiently low that a rapidly rotating core could exist for many times the solar age if diffusion controlled the flow of angular momentum. Over the depth range $r = 0.5$–0.85 the gaseous, or molecular, viscosity (Spitzer, 1962) is 4 to 10 times as great as that due to

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radiation (Thomas, 1930). These are tabulated in Table III, together with the kinematic viscosity η/ρ, of their sum $\nu=\eta/\rho$.

The effect of diffusion is one of widening the zone of differential rotation until it has a width of approximately 2.5 $(\nu T)^{\frac{1}{2}} \sim 3.8\times 10^9$ cm.

TABLE III

The molecular and radiative viscosities in gm/cm sec and the kinematic viscosity $\nu=(\eta_m+\eta_r)/\rho$ in cm^2/sec

r/r_0	η_m	η_r	ν
0.5	14.2	1.2	12.2
0.55	9.5	1.0	14.8
0.60	6.7	0.9	19.1
0.65	4.23	0.67	21.4
0.70	2.82	0.55	26.8
0.75	1.89	0.48	35.0
0.80	1.19	0.37	44.0
0.85	0.60	0.16	42.8

The diffusion equation can be written

$$\frac{\partial}{\partial r}\left(\nu\varrho r^4\frac{\partial\omega}{\partial r}\right)=r^4\varrho\frac{\partial\omega}{\partial t}. \tag{8}$$

The solution of this equation is simplified by treating ν as a constant and assuming that ϱr^4 varies with radial distance as $\exp(-kr)$. Based on Weyman's solar model, kr_0 has the values 2.7, 4.4, 5.7, 7.2 at $r/r_0=0.45$, 0.55, 0.65, 0.75 respectively. For these values 2.5 $(\nu T)^{\frac{1}{2}}k \ll 1$ and the variation of both ν and ϱr^4 can be neglected in a fairly good approximation.

In this approximation Equation (8) becomes

$$\nu\frac{\partial^2\omega}{\partial r^2}=\frac{\partial\omega}{\partial t}. \tag{9}$$

Equation (8), or (9) if applicable, can be used to calculate the present distribution of angular velocity in the sun and the present solar torque density, assuming that the sun arrived on the main sequence uniformly rotating. It is assumed that the rotation of an outer mixed layer, was quickly slowed, and that below this outer layer angular momentum is transported outward by diffusion.

To simplify the diffusion problem, Equation (8) or (9) is solved with the initial condition $\omega=\omega_c$ for $r<r_m$, and with the boundary condition $\omega=\omega_0$ for $r=r_m$. Here r_m is the inner radius of the outer mixed zone which may be deeper than the convective zone of Weymann's model.

The solution to this equation is

$$\omega=\omega_0+(\omega_c-\omega_0)\,\mathrm{erf}\left[(r_m-r)/2\sqrt{\nu t}\right] \tag{10}$$

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where $\mathrm{erf}(x) = (2/\sqrt{\pi}) \int_0^x e^{-z^2}\,dz$. The equatorial torque density at the inner radius of the mixed zone is $\nu\varrho r_m^2\,(d\omega/dr)$. The equivalent equatorial torque density at the solar surface is

$$\begin{aligned} K &= \nu\varrho r_m^2 (r_m/r_0)^2\,(d\omega/dr) \\ &= \varrho r_0^2 (r_m/r_0)^4\,(\nu/\pi t)^{1/2}\,(\omega_c - \omega_0) \end{aligned} \tag{11}$$

Values of the relevant quantities are given in Table IV for 3 different choices of r_m/r_0. It should be noted that the resulting values for the torque density are nearly equal to that obtained for the solar wind.

To test the assumption that the variation with r of $\nu\varrho r^4$ could be neglected in Equation (8), this equation was integrated numerically for $kr_0 = 4$. For several different values of $t \leqslant 4.5 \times 10^9$ years the dependence of ω on r is closely approximated by Equation (10). See Figure 5. For the numerically analyzed example the computed surface torque agrees well with the approximate result given in Table IV.

TABLE IV

The equatorial torque density, K, at the solar surface calculated from a model for the diffusion of angular momentum (see Equation (11)). It is assumed that $\omega_c = 15\,\omega_0$, a value consistent with the observed solar oblateness. Note that the values given in the last column agree reasonably well with the torque density derived from the solar wind ($\sim 10^8$ dyne/cm)

r_m/r_0	ν cm²/sec	ϱ gm/cm³	K dyne/cm
0.8	35	0.035	2.5×10^7
0.7	22	0.124	4.2×10^7
0.6	15	0.404	6.1×10^7

When the above calculation was first made (Dicke, 1964), the surface torque density obtained was $K = 7 \times 10^7$ dyne/cm, the difference from the results obtained here being due to the use of a cruder approximation.

The boundary condition assumed above at $r = r_m$ is equivalent to the assumption that the angular velocity of the outer mixed shell is instantaneously reduced from ω_c to ω_0. But this is inconsistent with the results given in Table I, particularly for $r/r_0 \leqslant 0.7$. Nonetheless the above calculation is applicable as a good approximation providing the actual mixing depth is slightly greater than the value assumed. Consider the example plotted in Figure 5. For a cut taken at $r/r_0 = 0.587$, ω is observed to vary with t, for $t < 2$ b.y., in a manner similar to that expected for the whole mixed shell. Thus a correct calculation based on the proper boundary condition at this radius should give a solution very similar to that shown in Figure 5, and a torque nearly equal to that given by Equation (11).

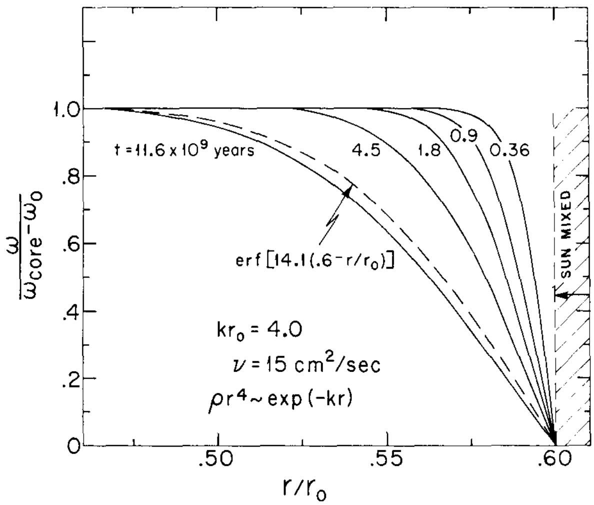

Fig. 5. Numerical integration of the diffusion of angular momentum from a rapidly rotating core, initially 0.6 in radius. The dashed curve shows an approximate analytic solution.

There is an additional way in which the above calculation may be an oversimplification. As mentioned earlier, the velocities of thermally driven circulation currents can be a couple of orders of magnitude greater in the zone of differential rotation than in the uniformly rotating core (Eddington-Sweet currents), and these velocities may be great enough to significantly transport angular momentum. The effect of non-uniform rotation on circulation has been previously considered by Baker and Kippenhahn (1959) and Mestel (1966).

In the absence of velocity and magnetic fields in the solar interior, surfaces of constant P, ϱ and φ (gravitational potential) coincide. If the mean molecular weight is a function of φ, temperature is also constant on these spherical surfaces and thermally driven circulation does not occur.

If the star is uniformly rotating, P, ϱ, and T are functions of the potential $\varphi - \frac{1}{2} \varrho r^2 \omega^2 \sin^2\theta$, which includes the centrifugal term (Von Zeipel, 1924). In general for any set of magnetic and velocity fields leading to a quasi-steady state and such that the velocity and magnetic force density has the form $\varrho \nabla W$, Von Zeipel's relation is applicable to the effective potential $\varphi - W$. It must be emphasized that this relation, P, ϱ, and T being functions of $(\phi - W)$, is applicable also over any fraction of the star, or surface in the star, for which the magnetic and velocity force density is of the above form. One such example is provided by the solar surface. Only the gravitational quadrupole moment and the stresses in the 'seen layers' of the sun can affect the oblateness. If Von Zeipel's relations hold in the 'seen layers', their implications for the interpretation of the oblateness are valid independent of conditions below the surface.

In the presence of magnetic and velocity fields, thermally driven currents occur because the temperature distribution forced by the presence of magnetic and velocity fields is generally incompatible with the vanishing of the divergence of the heat flux outside the energy generating core. To preserve the temperature distribution demanded by the magnetic and velocity field distribution, matter must flow and this flow represents the circulation current. Two separate effects contribute to the circulation current. The contribution arising in a non-spherical gravitational potential φ is global in origin, the whole of the distorted mass distribution generating the distorted potential. The contribution from W is local in origin. Over any spherical surface W, and hence the contribution of W to the radial component of circulation velocity, is given by local values of the fields.

Except for rotation on cylinders, the centrifugal force density per unit mass of non-uniform rotation is not derivable from a potential, and Von Zeipel's functional relations are not satisfied. Instead, over spherical surfaces the variation of P and ϱ are separately determined (Dicke, 1967). But once again the distribution of temperature over such a surface is determined by the rotational distribution. Here, even though the distortion of surfaces of constant gravitational potential may be small, the variation of the temperature induced locally can be relatively large and the violation of the divergence condition correspondingly large.

In general the angular velocity distribution obtained from the diffusion Equation (8) is such that circulation could occur and this circulation would modify the distribution of rotation in the diffusion zone. For rotational distributions without circulation, angular momentum is transported by molecular diffusion but the transport rate would be affected by the modification in the rotational distribution. This question requires an analysis.

3. A History of the Sun's Rotation

In my *Nature* article (Dicke, 1964) a possible history of the sun's rotation was assembled with the realization that the sun's past is even more hidden than its interior. In the intervening half decade new data have cast some light on this void, but these data do not seem to require any very fundamental change in the story. The one significant change that I would make appeared already in 1965 and was not forced by any new data, but rather by the realization that my model was probably defective at one point. I shall discuss this change in the last section.

We are considering the history of the sun with the view to asking whether this history could reasonably lead to a physical understanding of the presence of a rapidly rotating core in the sun. But in this connection, it is essential that we consider the sun to be a normal star, not an exception! If we are correct, all normal, solar type stars, young or old, will have rapidly rotating interiors and by observing solar type stars of various ages some evidence concerning the effect of such a rotation might conceivably be unearthed.

We visualize the solar system as having formed through gravitational collapse of a condensation in a gas cloud, the condensation having more than enough angular momentum to supply a rapidly rotating star. The resulting rotation is limited by the

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maximum angular momentum possible for the collapsing sun, and it may also be reduced to a value far below this limit by the Schatzman (1962) torque.

The twisting of magnetic field in the solar nebula may not have been as effective in slowing the proto-sun's rotation as had been previously thought. The stresses induced by a toroidal magnetic field tend to make a rotating star prolate, also unstable. This instability, to be discussed elsewhere, occurs when roughly half of the star's kinetic energy is converted to magnetic energy. It results in a precession of the star relative to the rotation axis, putting the magnetic axis perpendicular to the rotation axis. In the perpendicular position the star's magnetic field becomes cut-off from the outside, greatly reducing the torque. If this picture is correct it may provide an explanation for stars which are magnetic variables as 'oblique rotators' (see Preston, 1967). Mestel (1968) has discussed this same problem showing that stellar wind torques may make the perpendicular orientation of the magnetic axis the preferred position.

Whatever the physical explanation for the rather low value for a star's surface rotation relative to the limiting value, after most of the star's mass has been accumulated in a central concentration, a large effective radius seems to be required for mass loss or gain.

If the angular momentum cut-off occurs during formation for all stars at the same fractional collapse (or average density), the angular momentum upon first arrival on the main sequence should vary as $M^{5/3}$. This can be best seen from a crude argument based on homology. For the radius at which centrifugal and gravitational forces balance,

$$GM = \omega^2 r^3$$

or $\omega \sim \varrho^{1/2}$. But then

$$J \sim M r^2 \omega \sim M^{5/3}. \tag{12}$$

It is found that, up to 10 solar masses, stars bluer than F0 show this dependence on mass. (See Figure 6.) But the reduction to lower values of J/M for $M/M_0 > 10$ suggests that this simple argument is inadequate. Also another significant change may occur for $M/M_0 < 1.6$. However, the situation is complicated for stars of lower mass, redder than F5. As discussed by Kraft (1968), a stellar wind torque, i.e. slowing on the main sequence, is expected for such stars. Thus the observations may not reflect the initial rotation on the main sequence.

The extension of the dotted curve to the left in Figure 6 may provide an appropriate value for the initial solar rotation on the main sequence. However, it will be shown below that a somewhat lower initial value is more likely. From this viewpoint all stars on the main sequence, 1 solar mass or greater, are rotators. They probably arrive on the main sequence uniformly rotating having passed through a state with a deep convective envelope. For stars redder then F5 the stellar wind driven by surface turbulence provides a torque to slow down the rotation of an outer shell (or the whole star). If our picture is correct only an outer shell is slowed. But the angular momentum content of such a shell is small and the angular momentum remains deeply buried in these stars.

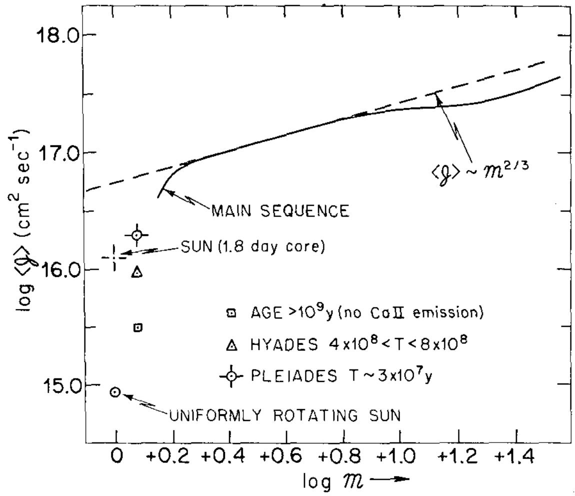

Fig. 6. The rotation of stars of various masses (expressed as angular momentum per unit mass). Stars bluer than F5 are believed to be free of a stellar wind torque. The surface rotations of stars redder than F5 decrease with age. The three points for these stars and the 'uniformly rotating sun' assume uniform rotation with the observed surface velocity. The other point for the sun (1.8 day core) is consistent with the observed solar oblateness. (Based on Kraft (1968), Fig. 17.)

As the discussion in the next section will bring out, there is a non-trivial amount of support for this viewpoint. It will be shown that the sun's initial rotation on the main sequence was probably somewhat below the dotted extension of Figure 6, similar to that seen in the Pleiades. It will also be shown that the outer mixed shell, slowed by the solar wind, may go as deep as $r/r_0 = 0.55$.

I shall now briefly summarize our picture of the history of the sun's rotation. The sun is viewed as having approached the main sequence rotating at a rate of about 30 km/sec on the equator, similar to that seen in the Pleiades. Having passed through a convective state and containing magnetic fields, the sun was probably initially uniformly rotating. The solar wind torque slowed an outer mixed shell. The beginning solar wind is believed to have had a ratio of torque to angular velocity somewhat larger than today's value. How rapidly the outer shell was slowed depends upon both this torque and the shell thickness. In the next section it will be shown that the shell was probably fairly thick and that the e-folding time for slowing this shell may have been as great as 10^9 years.

Classical discussions of the formation of the solar system were always plagued with a persistent problem, how to remove the original angular momentum from the sun. If we are correct, this problem has vanished, for the angular momentum is still in the sun, but deeply buried.

4. The Sun as a Rotating Star

If the sun has a rapidly rotating core this is to be regarded as a normal condition for stars of about 1 solar mass, not as an exception. If the sun was originally rotating with a substantial equatorial velocity of 30 km/sec, this rotation should be seen in young stars. If the outer layers of the sun were slowed in 10^9 years, this slowing should be seen in stars as a function of age.

Figure 6 and Table V are based on R. Kraft's work, in particular on Figure 17 and

TABLE V[a]

	log (J/M)
$M^{2/3}$ extrapolation	16.8
Pleiades $\sim 3 \times 10^7$ years	16.3
Hyades $\sim 6 \times 10^8$ years	16.0
Old $t > 10^9$ years	15.5
Initial main-sequence	16.32
Sun, uniformly rotating	14.9
Sun, 1.8 day core	16.1

[a] J/M (in cm^2/sec) is the angular momentum per unit mass of rotating stars with $M/M_\odot = 1.2$ for rows 1–4. (Based on Kraft, 1968.) See Figure 6.

Table III of Kraft (1968). The solid curve is Kraft's, but the dotted line is mine. In Kraft's Figure 17 the point marked 'Dicke Sun' seems to have been incorrectly positioned. It has been relocated in Figure 6. The data for the three points representing stars of 1.2 solar masses were taken from Table III of Kraft's article.

There are several possible interpretations of the Pleiades point in Figure 6:

(a) These stars are so young that their surface angular velocities have not been appreciably affected by the stellar wind torque.

(b) The original angular velocity was consistent with the angular momentum given by the dotted curve, but the outer 10% of the star (by radius) was slowed to $\frac{1}{3}$ of its original rotation by a stellar wind like the present solar wind.

(c) The whole star was slowed by the factor of $\frac{1}{3}$ by a stellar wind torque 10^4 times as strong as the present solar wind, i.e. with a ratio of torque to angular velocity 500 times as great as the standard (solar wind) ratio.

(d) The outer 40% of the star, by radius, was slowed by a torque 10^3 times as great as that of the solar wind (50 times as great in ratio of torque to angular velocity).

The hypotheses (b) and (c) are unlikely for reasons, to be discussed below, connected with the depletion of lithium in young stars. These observations suggest that the outer 40% by radius, but only the outer 40%, is slowed by a stellar wind. If the argument given above for a standard ratio of stellar-wind torque to angular velocity is correct (for solar type stars), the hypothesis (d) is also unlikely.

That the interpretation (a) is most likely is seen by considering the Hyades point. If the Pleiades point represents approximately the initial main-sequence rotation at 1.2 solar masses, the e-folding time for slowing rotation in the Hyades is roughly 9×10^8 years. This would require a ratio of stellar wind torque to angular velocity consistent with the present solar wind torque if the outer 30% of the star, by radius, is being decelerated. If both the Pleiades and Hyades are assumed to be acted on by a stellar wind of the same torque ratio and have the same thickness shell decelerated, the point representing the initial value for the logarithm of angular momentum per unit mass would fall on Figure 6 only 0.015 above the Pleiades point. If the point marked 'sun, 1.8 day core' represents the solar rotation approaching the main sequence, this point is consistent with the interpretation of the Pleiades point as initial rotation on the main sequence.

Reasons were given earlier for believing that the initial solar wind torque, and stellar wind torque of other young solar-type stars, have a somewhat greater ratio of torque to angular velocity than that found associated with the present solar wind. In the case of the Hyades the whole star could be slowed by a torque 15 times as great. This possibility cannot be excluded by the above argument, but the argument to be discussed in the next section seems convincing.

5. The Rotation of the Sun and the Depletion of Lithium

Goldreich and Schubert (1967) have noted that the Howard *et al.* (1967) spin-down of the sun, initially rapidly rotating, would have transported lithium and beryllium, from the outer parts of the sun below the radii of $r/r_0 = 0.6$ and 0.5, respectively, at which these elements would have been rapidly destroyed. They also note that diffusion associated with their thermally driven turbulence would have had a similar effect.

The argument that angular momentum transport and depletion of lithium and berylium should be connected seems convincing. It has been shown that angular momentum cannot be transported by molecular diffusion more than $0.05\, r_0$ during the lifetime of the sun. Thus, to remove angular momentum from the deep solar interior to the surface requires the transport to the surface of depleted solar material.

If the transport were to occur via the spin-down process, circulation currents outside the core would erode its surface, transporting its contents to the convective zone. Thermally driven currents inside the uniformly rotating core would be slow. Thus circulation currents would penetrate ever deeper, first destroying the lithium and then the berylium. After the depth of burning is reached for a given isotope, its destruction proceeds rapidly.

In similar fashion the Goldreich-Schubert instability, if it should occur, would transport angular momentum to the convective shell by a type of turbulent diffusion. The thin ring of high angular momentum material ejected from the core would not float all the way to the surface. It would be quickly destroyed by instabilities. The successive formation and destruction of these rings would lead to diffusion of angular momentum out from the surface of the steadily shrinking core and of lithium and berylium down to their zones of burning. Thus this process couples the loss of angular momentum from the interior to the depletion of lithium and berylium.

Goldreich and Schubert interpret the presence of lithium and berylium in the sun to mean that the sun was not rapidly rotating initially. But if this argument were valid, it should also be applicable to other stars. But solar type stars in very young clusters are observed to be rotating at about 400 km/sec whereas only 6 km/sec rotation is seen in old stars of the same mass. As in the case of the sun, lithium is observed to have suffered depletion, but not berylium.

We shall consider the following questions:

(a) What do the observations tell us about the depletion of lithium and berylium in stars?

(b) How much should lithium and berylium be decreased if the whole star is slowed by spin-down, or by turbulent diffusion of angular momentum?

(c) What seems to be the implications of the depletion of lithium for internal rotation in solar-type stars?

We consider first the observational material regarding the abundance of lithium and berylium.

Lithium is believed to be depleted in the sun by almost 3 orders of magnitude relative to that in chondritic meteorites. (See survey article by Wallerstein and Conti, 1969, for references.) The berylium abundance in the sun seems to be in good agreement with that seen in these meteorites. Herbig (1965) has noted a correlation between lithium abundance and stellar age of main-sequence stars of spectral type G. This correlation was studied by Wallerstein, Danziger, and Conti, and most recently by Danziger (1969) who found that the data for T Tauri stars, the Pleiades, the Coma Cluster, the Hyades, and the sun are consistent with the assumption that the lithium abundance was initially that found in chondritic meteorites but has since decreased exponentially with an e-folding time of 7×10^8 years.

It is questionable whether there is any appreciable amount of Li^6 in the sun or solar-type stars. Schmall and Schröter (1965) find that the profile of a lithium resonance line observed in sunspots agrees well with a Li^7 profile but that the fit is slightly improved by adding a small amount of the Li^6 profile. If the improved fit implies the presence of Li^6, the abundance ratio of Li^6 to Li^7 should be roughly 0.05 in reasonable agreement with the terrestrial value of 0.08.

The abundance of berylium in the sun appears to be essentially the same as that found in chondritic meteorites (Wallerstein and Conti, 1969). This also seems to be true for substantially all main-sequence field stars redder than F7. It seems clear from Danziger (1969), and Wallerstein and Conti (1969), that late F and early G stars arrive on the main sequence with the meteoritic abundance of both berylium and lithium. The lithium is depleted as the star ages, but not the berylium. Li^7 and Be^9 are burned at radii differing by only 10% of the solar radius.

A reasonable interpretation of the above observations is that the slowing on the main sequence of the rotation of the sun, and of other solar type stars, was accompanied by a mixing of the star down to a depth of about $r/r_0 = 0.6$. The depth of 0.6 lies well below the presumed bottom of the convective zone (0.85 for Weymann's model), but the angular momentum could have been transported outward to the convective zone by thermally driven circulation currents, lithium being carried inward.

TABLE VI

Fractional depth, r_b/r_0, at which burning takes place with indicated mean life (from Fowler *et al.*, 1967)

Mean life	3×10^6 years	3×10^7 years	3×10^8 years
Li^6	0.57	0.6	0.63
Li^7	0.51	0.55	0.58
Be^9	0.42	0.45	0.47

TABLE VII

Minimum depletions of Li^6, Li^7, and Be^9. For the sun, the decrease in the logarithmic abundance relative to that in chondritic meteorites is given by $[X]_m - [X]_s = (1/2.3)(\Delta M/(1 - M_b))$ and is tabulated in the last two columns. ΔM_1 and ΔM_2 are the mass fractions of the sun outside the rapidly spinning core of adopted radii $r_1 = 0.55$ and $r_2 = 0.40$ respectively but inside the radii of burning, r_b. Relative to calcium the observed depletion for Li^7 is $[Li/Ca]_m - [Li/Ca]_s \simeq +2.8$. No depletion of Be^9 is observed.

Isotope	r_b/r_0	$M_b(r)$	ΔM_1	ΔM_2	$(\Delta M/2.3)(1 - M_b)$ #1	$(\Delta M/2.3)(1 - M_b)$ #2
Li^6	0.63	0.976	0.028	0.158	0.51	2.86
Li^7	0.58	0.961	0.013	0.143	0.01	1.59
Be^9	0.47	0.896	—	0.078	—	0.33

Another possibility is that turbulent diffusion driven by the Goldreich-Schubert-Fricke effect could result in the diffusion of angular momentum outward from a zone of molecular diffusion within which this effect is for some reason inoperative. The turbulent diffusion of angular momentum should be accompanied by a diffusion of lithium inward to the zone of burning. Both of these possibilities for transporting lithium inward and angular momentum outward will be discussed.

To simplify the discussion it will be assumed that a sharp boundary marks the zone of rapid burning of each of the isotopes Li^6, Li^7 and Be^9. The values adopted for the radii of these boundaries are given in Table VII. As there is no observable depletion of beryllium, the boundary of the rapidly rotating core (or minimum radius from which angular momentum is convected to the surface) is assumed to be greater than 0.47.

It is possible to calculate a minimum value for lithium depletion using a simple argument. Define two concentric shells for the sun, one ranging from the core radius r_c to the outer boundary for burning a lithium isotope, the second lying between this boundary and the solar surface. The material containing angular momentum but without lithium originally in the inner shell is assumed to be transported to the convective zone, where it is mixed with the original material. This results in a dilution of the original lithium content. A lower value for the dilution factor is obtained if the zone of mixing is assumed to be the whole of the outer zone instead of just the convective layer. For the latter case, the depletion of lithium relative to calcium, in comparison with meteorites is given by,

$$[\mathrm{Li}^7/\mathrm{Ca}]_s - [\mathrm{Li}^7/\mathrm{Ca}]_m = - \Delta M/(1 - M_b) \ln 10 \tag{13}$$

where the brackets represent logarithms of abundance ratios, ΔM is the mass transferred from the lithium burning shell, and $1-M_b$ is the mass fraction lying outside r_b, (i.e. the mass of the mixed zone). The relevant numbers are given in Table VII. It is evident that this lower bound for the depletion factor is much smaller than the observed depletion.

If the sun were originally rotating as rapidly as the G stars in the Pleiades, $\omega_p \sim 5\omega_0$ and were slowed down to its presently observed rotation, ω_0, by the Goldreich-Schubert-Fricke instability, or by the spin-down process of Howard *et al.* (1967), the decrease in the logarithmic abundance of berylium would be expected to be at least

$$\begin{aligned} - [\mathrm{Be/Ca}]_s + [\mathrm{Be/Ca}]_m &= \\ &+ \left(1 - \frac{\omega_0}{\omega_p}\right) M_b/2.3(1 - M_b) \\ &= + 0.8 \times 0.896/2.3 \times 0.104 = + 3.00\,. \end{aligned} \tag{14}$$

By contrast, no appreciable depletion is observed.

Instead of computing a lower bound for lithium depletion one might attempt to make a calculation from a model based on an assumed transport means for angular momentum. It will be assumed that outside the core of radius r_c, rotating uniformly with an angular velocity ω_c, there is a differentially rotating shell whose outer surface has a radius r_m and angular velocity ω_m. It is assumed that through some unspecified means the Goldreich-Schubert-Fricke instability is inoperative in this shell of differential rotation and the flow of angular momentum is limited by molecular diffusion. Thus the angular velocity distribution is of the type shown in Figure 5. (See the last section for a discussion of one of several possible means for stabilizing the flow in this shell.) It is assumed that outside of this shell to the bottom of the convective zone at r_v angular momentum is transported by the Goldreich-Schubert-Fricke thermally-driven turbulent diffusion. Because of the effectiveness of this diffusion process it must operate near threshold conditions to provide the low angular momentum flow rate given by molecular diffusion in the inner shell. Thus the angular velocity in the intermediate shell varies inversely as the square of the radius and

$$\omega_m/\omega_0 = (r_v/r_m)^2\,.$$

(See Goldreich and Schubert, 1967). Again ω_0 designates the angular velocity of the outer convective zone.

In the intermediate shell the turbulent diffusion of angular momentum is governed by Equation (8) with the kinematic viscosity ν automatically adjusted to provide the correct angular momentum flux and $\omega \sim r^{-2}$. These conditions require $2\varrho\nu = kr$, where k is the angular momentum flux density.

The diffusion of lithium from the bottom of the convective zone to r_b, the radius of

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burning is governed by the diffusion equation

$$\frac{\partial}{\partial r}\left(\nu \varrho r^2 \frac{\partial}{\partial r} F\right) = r^2 \cdot \varrho \frac{\partial F}{\partial t} \tag{15}$$

where $F\varrho$ is the mass density of the isotope in question. Because the diffusion is turbulent, the diffusivity ν is the same for both angular momentum and matter. Integrating Equation (15) outward from a spherical surface just inside the convective zone gives

$$\left(\nu \varrho r^2 \frac{\partial F}{\partial r}\right)_{\text{v}} = \frac{1}{4\pi} \frac{M_{\text{v}} F}{\tau_F} \tag{16}$$

where the quantities on the left side are to be evaluated below but near r_{v}. M_{v} is the mass of the convective zone. τ_F is the mean life for the decay of the isotope. $(\partial F/\partial r)$ is evaluated from a numerical integration of the lowest normal decay mode of Equation (15). In similar fashion, Equation (8) is integrated to yield the analogue of (16) where τ_ω would represent the e-folding time for increasing ω_0 if the solar wind torque were to be suddenly switched off. Combining these equations yields

$$\tau_F = \frac{2\tau_\omega F_{\text{v}}}{r_{\text{v}} (\partial F/\partial r)_{\text{v}}}. \tag{17}$$

There is considerable uncertainty about the depth of the convective layer in the sun and Weymann's (1957) value for $r_{\text{v}} = 0.85$ may be too great. It has been suggested by Sears and Weymann (1965) that convection may exist down to $r_{\text{v}} = 0.7$. Equation

TABLE VIII

The mean decay times τ_6 and τ_7 for the destruction of Li^6 and Li^7 by turbulent diffusion. Three different radii are chosen for r_{v}, the bottom of the convective zone. τ_ω, from Table I, is the decay time for slowing of the convective zone based on the torque derived from solar wind measurements. The radii adopted for rapid burning of Li^6 and Li^7 are $r_b/r_0 = 0.63$ and 0.58 respectively. (See Table VI.) In order to avoid burning berylium while permitting the burning of Li^7, r_m should satisfy $0.5 < r_m/r_0 < 0.58$. The radius of the rapidly spinning core is approximately $r_c/r_0 \sim 0.5$. For the sun, the 'observed' decay time of Li^7 is $\tau_7 = 7 \times 10^8$ years (Danziger, 1969). The calculated values are based on Equation (17).

r_{v}/r_0	τ_ω (years)	τ_6 (years)	τ_7 (years)
0.86	1.84×10^8	5.3×10^8	10.2×10^8
0.78	5.1×10^8	4.5×10^8	8.8×10^8
0.70	11.2×10^8	3.1×10^8	7.1×10^8

(17) has been used to determine τ_F for both isotopes of lithium with three choices for r_{v}; see Table VIII. For each of these choices τ_ω is derived from the values given in Table I. For Li^7 to be depleted but not berylium, $0.5 < r_m/r_0 < 0.58$. Thus the radius of the rapidly rotating core should be approximately $r_c/r_0 \sim 0.5$.

The dimensionless ratio $(r_v/F_v)(\partial F/\partial r)_v$ is determined from a numerical integration of the eigen value equation

$$\frac{\partial}{\partial r}\left(v\varrho r^2 \frac{\partial}{\partial r} F\right) + \Lambda r^2 \varrho F = 0, \tag{18}$$

obtained from (15). The lowest decay mode is found for the appropriate boundary condition at r_v. The higher decay modes disappear in a few hundred million years. In (18), $r\varrho v = \frac{1}{2} r^2 k$ is assumed to be constant.

From Table VIII, the mean decay time of Li^7 is in good agreement with the value observed by Danziger (1969). The calculated decay time of Li^6 is less by a factor of $\frac{1}{2}$. This implies a greater depletion in the sun of Li^6 relative to Li^7 by 2 or 3 orders of magnitude. As was noted above there is not yet conclusive evidence for the presence of Li^6 in the sun.

Another possible model yielding a depletion of lithium employs thermally driven circulation currents outside the zone of molecular diffusion of angular momentum. These currents would transport angular momentum upward and lithium isotopes downward into their burning zones. It is assumed that the currents do not extend deep enough to cause a depletion of beryllium.

It is assumed that the currents carry both of the isotopes so deep as to destroy them completely. Upward currents are then devoid of these isotopes.

The calculation leading to Equation (13) is now applicable to this problem providing the zone of mixing is assumed to be the convective zone. Angular momentum is conserved along a stream line of the circulation and many cycles of circulation are required to transport the angular momentum. ΔM of Equation (13) must contain the number of cycles as a multiplier. For reasonable values of this number, ΔM, and the mass fraction in the convective zone, the depletion is far too great. It is concluded that of these two models, the turbulent diffusion is best.

6. The Solar Magnetic Field

One aspect of the formation of the solar system requires some additional discussion. This concerns the magnetic field trapped in the gas when the proto-sun is first formed. In 1964 I pictured this field as stirred by convection during the Hayashi phase. The long lived magnetic modes were assumed to be completely converted into short lived ones by the convection. The short lived modes would rapidly decay after the magnetic field became frozen into the radiative core upon termination of the Hayashi phase.

There can be little doubt that this mechanism would work if the sun was properly stirred, but, I now doubt that the stirring would be sufficiently thorough and of the right type to completely destroy the long lived modes. The destruction of long wavelength modes must be extremely thorough for a poloidal field with axial symmetry characterized by a magnetic mode with long wavelengths would generate toroidal magnetic modes of ever increasing strength under the influence of differential rotation.

I now consider it unlikely that the destruction of such modes (long wavelength) could be so thorough. This is the point mentioned earlier where I have had second thoughts about my 1964 model.

Two other possibilities now seem more likely. The first involves magnetic flux exclusion from the radiative core by the 'beer foam process'. The sun is pictured as initially completely convective, and the magnetic field is assumed to be thoroughly twisted. This is similar to the previous picture. However the low magnetic modes are not destroyed at this stage. Instead the twisted field configuration is assumed to contain null-field surfaces, where regions of oppositely oriented magnetic fields meet. The fluid in these null-field regions is free of magnetic pressure and hence has a greater density. It is conceivable, but by no means certain, that a radiative core would slowly grow by developing a central spherical cavity filled with this dense fluid. The field-free fluid is pictured as filling the cavity by flowing along null-field channels. As these channels close, the diffusion of magnetic field would generate more field-free fluid. It must be emphasized that this picture is highly conjectural.

The second view of the magnetic field is also conjectural, but it may be correct as there is a non-trivial amount of observational support for the picture. According to this viewpoint, the sun arrived on the main sequence rotating with its magnetic field of a few thousand gauss trapped in a perpendicular axis configuration, this configuration having been induced by the instability mentioned earlier. This field is assumed to be deeply buried at this stage, cut off by differential rotation. Thus the interior of the sun is pictured to be something like a perpendicular rotator model of a magnetic A star.

If this picture is valid for the sun, it should also be applicable to other stars arriving on the main-sequence to the right of F5. From this viewpoint solar type stars are both magnetic and rapid rotators but most of the angular momentum and magnetic field is deeply buried.

The work on this magnetic model is being published elsewhere. Here I shall only summarize the central conclusions. This model is too complicated to permit a complete theoretical study and all that has been accomplished is to analyze parts of the problem using approximation methods.

The most interesting feature of the model is the torsional oscillation of the rapidly rotating core. The completely embedded approximately perpendicular dipolar-field gives the core an elasticity for torsional modes. The lowest frequency mode is a torsional mode for which the north and south magnetic poles move on ellipses, oscillating back and forth between the northern and southern hemispheres. If the core magnetic field strength is of the order of magnitude of 10^5 gauss the oscillation period can be made to agree with the 22 year sunspot cycle.

Another feature of the torsional oscillation is the generation of toroidal magnetic fields of alternating polarity. This toroidal field is generated by differential rotation acting on the (oscillating) axial component of the dipolar field. The toroidal field appears in the differentially rotating shell in the vicinity of $\pm 45°$ latitudes with opposite polarities in the two hemispheres. Owing to magnetic buoyancy, strands of this toroidal field slowly float up to the solar surface where magnetic field breaking through the surface could be the source of the sunspot phenomena.

The various observational tests of this hypothesis are being published elsewhere, and only a single example will be considered here.

One obvious implication of this magnetic model is that, owing to the extremely high Q of the torsional oscillation, the sunspot cycle should be timed by a very precise central clock. Owing to the delayed arrival of the torsional magnetic field at the surface there could occur random phase errors, i.e. delays in the time of arrival of the sunspot maximum. But such a phase error would not be cumulative. It should later disappear.

Under the sunspot theory of Babcock (1961) and Leighton (1969), the magnetic field of a new sunspot cycle is regenerated from the old. This model does not contain a tuned oscillator, and phase shifts are cumulative. If for a couple of cycles the time of sunspot maximum were to be delayed a year or two, this delayed phase would be expected to continue indefinitely into the future.

A least squares fit of a straight line has been made to the epoch of sunspot maximum plotted against sunspot cycle number N for the period of 1615 to 1958, the following relation is obtained for the years of sunspot maximum (see Allen (1963) for a table of sunspot maxima):

$$y = 1749.41 + 11.0814\,N\,. \tag{19}$$

With few exceptions the times of observed sunspot maxima agree with this formula to within a year. One pronounced anomaly occurs for two successive maxima, in 1778 and 1788, when the time of maximum sunspot eruption appeared $4\frac{1}{4}$ and $5\frac{2}{3}$

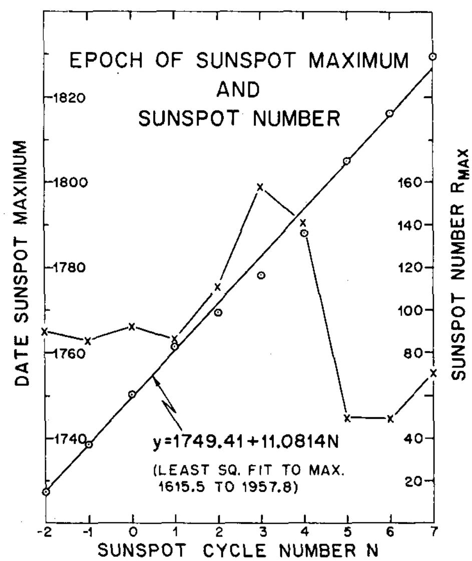

Fig. 7. The phase shift in sunspot maximum observed for cycle numbers 2–4. Note the recovery of the correct phase by #5 contrary to expectation under the Babcock-Leighton theory. The correction of the phase error by 1805 suggests that the sunspot cycle is controlled by a tuned oscillator, perhaps the torsional oscillation of the core suggested in the text.

years too early respectively. This anomaly was accompanied by unusually intense sunspot activity, the average number of sunspots being approximately twice as great as normal. (See Figure 7.) It should be noted that this large phase shift disappeared permanently at the next sunspot maximum. Apparently some internal clock remembered the old phase throughout this massive disturbance. Under the Babcock-Leighton theory this phase shift would be expected to continue indefinitely into the future.

The presence of an oscillation in a rapidly rotating core could be significant for the Goldreich-Schubert-Fricke instability. Such an oscillation would induce a meridional component of velocity in the differentially rotating shell. This could be as great as 1 m/sec, eliminating the instability (see Goldreich and Schubert, 1967).

7. Summary and Conclusions

In my 1964 article in *Nature* it was suggested that the sun might have a rapidly rotating core and consequently an appreciable quadrupole moment that could advance the perihelion of Mercury's orbit about 4″ arc/century to bring the observations in agreement with the scalar-tensor theory of gravitation. The solar wind torque density was estimated to be 1×10^8 dyne/cm. Whereas the torque density necessary to keep the outside of the sun rotating slowly while angular momentum diffused out of the core was evaluated from a solution of the diffusion equation and found to be comparable with this value.

In 1966 and 1967 the oblateness of the sun was measured and found to be consistent with the presence of such a gravitational quadrupole moment, and a value of $\omega = 5$ for the coupling constant of the scalar-tensor theory. The Mariner II space probe measurements of the solar wind flux and magnetic field strengths in the solar wind give a solar wind torque density consistent with the earlier estimate.

Other information not available in 1964 tends to support the original conjecture providing the sun is believed to be a typical star and not a special case. Kraft has shown that surfaces of very young solar type stars are rotating with substantially the same angular velocity postulated for the rapidly rotating solar core. Hyades solar-type stars show a loss of angular momentum of the amount expected if the stellar wind torques were equal to the solar wind torque and the stars were mixed down to a radius of 0.7. Reasons are given supporting the contention that stellar-wind torques should be roughly equal for main-sequence stars of the same mass redder than F5.

For angular momentum to be transported from the deep stellar interior in the absence of magnetic stresses requires mass transport and the destruction of berylium carried below $r = 0.5$. Thus to slow the rotation of the deep interior of a star by convection or the Goldreich-Schubert-Fricke effect implies a loss of almost all of its berylium. For solar-type stars of all ages, including the sun, berylium appears to be present with the meteoritic abundance. However, for such stars, lithium which burns at $r \sim 0.6$ appears to be depleted relative to the meteoritic value, the abundance of lithium decreasing exponentially with time with an e-folding time of 7×10^8 years.

These observations suggest that in the sun an outer mixed slowly rotating shell has an inner radius $0.5 < r < 0.6$. The moment of inertia of the shell implied by this value implies an e-folding time for slowing reasonably consistent with the present solar wind torque and the observed rotation in the Hyades.

The radius assumed in 1964 for the rapidly rotating core was about $r = 0.75$ whereas the lithium data seem to force a value of approximately 0.5. The observations seem to support a picture of turbulent mixing outside a molecular diffusion shell about 0.05 in thickness. Such turbulence could be driven by the Goldreich-Schubert-Fricke instability which for some reason is inoperative below this. A torsional oscillation of the rapidly rotating core, if it contains a crossed dipolar magnetic field, provides at least one means for stabilizing rotation against this instability in the molecular diffusion zone. Reasons are given for eliminating 'spin-down' as a source of instability.

In conclusion, if the sun is a normal main-sequence star, the notion that it has a rapidly rotating core receives support from the observations of rotation in young solar-type stars, the observations of the depletion of lithium in such stars, and the lack of berylium depletion. All of these observations are consistent with the core rotation needed to account for the solar oblateness.

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Discussion

Roxburgh: You already stated that a small non-conservative distortion produces a large temperature variation. This shows that only a very small distorting effect produces a large oblateness on temperature constant surface. If it is such surfaces that are effectively being measured then the oblateness is to be expected from the magnitude of the disturbing forces.

Of course one should construct model photospheres and integrate the energy flux coming to an observer; have you done this?

Dicke: The shapes of isophotes are not measured. Rather the integrated flux passing the occulting disk is used to determine both the oblateness of the extreme limb and the equatorial polar difference in surface brightness. By making measurements with several different amounts of exposed limb, the two effects are separated. The answer to your question is "yes". The significance of the atmosphere on the above interpretation has been considered, also the effect on the model atmosphere of the observed latitude independence of surface brightness. In the absence of surface magnetic and velocity fields (other than rotation), the position of the limb is determined by a surface of constant potential to within 3 meters and the limb brightness is remarkably latitude independent. Surface stresses induce both an oblateness and a latitude dependence in surface brightness. But the observations show that the sun's brightness is latitude independent and this implies a latitude independence of the atmospheric model for the normal photosphere.

Deutsch: Consider a solar-type star of the same age as the sun, which formed with the angular velocity in the core double that in the sun. Can you say how you would expect the rotational velocity of the hydrogen convection zone to compare with that we see in the sun?

Dicke: The surface angular velocity would be expected to be doubled if the stellar wind torque per unit angular velocity were the same as that of the sun and convective transport of angular momentum is unimportant, both of which are doubtful.

Abt: How did you allow for differential refraction?

Dicke: The refractive correction was computed from temperature and barometric pressure at the observatory. This was subtracted from the data. The residual showed no significant variation with time of the form characteristic of the computed refractive term.