# KELVIN-HELMHOLTZ INSTABILITIES IN A MAGNETISED COMPRESSIBLE PLASMA (SHEARED FLOW).

T.P. Ray
Physics Dept., University College Dublin, Ireland
A.I. Ershkovich,
Dept. of Geophysics and Planetary Sciences,
Tel Aviv University, Israel.

## ABSTRACT.

We have investigated Kelvin-Helmholtz (K-H) instabilities for a homogeneous compressible plasma containing a uniform magnetic field and a linear velocity shear. A derivation of the relevant K-H dispersion equation and details regarding method of solution are given elsewhere (submitted to Mon. Not. R. Astr. Soc.). We present here an outline of our results.

## INTRODUCTION.

It is usual, when calculating K-H instability growth rates and phase velocities, to apply the vortex sheet approximation i.e. to assume the velocity shear is of zero thickness. In practice, viscosity (we use the word in its most general sense) introduces a velocity transition region of finite thickness. The effects of this finite shear layer only become apparent for K-H wavelengths comparable to the layer's width. Early results for incompressible fluids (Chandrasekhar, 1961; Michalke, 1964) showed that such a shear layer stabilizes all wavelengths shorter than its width. Blumen (1970), Blumen, Drazin & Billings (1975) and Drazin & Davey (1977) investigated a compressible fluid containing a hyperbolic tangent velocity profile. They showed that instability occurs at all Mach numbers. Ray (1982) examined the case of a compressible fluid with a linear profile. He found results which are analogous to those found in the hyperbolic tangent case. Ferrari, Trussoni & Zaninetti (1980) investigated the effects of magnetic fields on K-H modes but using the vortex sheet approximation. Our results, an outline of which follows, concern a MHD treatment of a layer of finite thickness. Let us assume that the equilibrium flow is in the x direction, the velocity shear is in the z direction and the uniform



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magnetic field is in the (x, y) plane. We use a linear profile:

$$V = \frac{U_o z}{2d} \quad -d < z < d$$

$$V = U_o/2, \quad z > d \qquad (1)$$

$$V = U_o/2, \quad z < -d$$

where $U_o$ is a constant. Thus the jump in velocity across the shear layer is $U_o$ and 2d is its width.

We can conveniently divide up our results as follows:

(i) Incompressible Case.

It is found that, irrespective of wavelength, K-H modes do not grow if $U_o \cos\theta < 2V_A \cos\beta$, where $V_A$, $\theta$ and $\beta$ are the Alfven speed and the angles the modes makes with the flow and magnetic field, respectively. This condition shows that the shear layer is unstable unless the flow is exactly along the field lines ($\theta = \beta$) and the velocity jump is less than twice the Alfven speed. All solutions are found to be stationary i.e. $\omega_R/k = 0$

where R denotes 'real part'. The complex frequency and real wavelength are denoted by $\omega$ and k respectively.

(ii) The Subsonic Regime. $M < 1$, $V_A > 0$.

Here M, the Mach number, is defined in terms of half the velocity jump across the layer

$$M = \frac{U_o}{2a} \qquad (2)$$

where a is the sound speed). Because of the large number of parameters involved, we shall only consider one magnetic field orientation, viz $B \parallel U_o$, in this and subsequent sections The case of $B_o \perp U_o$ is easily dealt with as the dispersion equation becomes identical in form to its $V_A = 0$ counterpart but with the sound speed replaced by $(a^2 + V_A^2)^{\frac{1}{2}}$. Consequently the results for $B_o \perp V_o$ are the same as the non magnetic results but with M replaced by the magnetosonic Mach number

$$M_m = \frac{U_o}{2(a^2 + V_A{}^2)^{\frac{1}{2}}} \qquad (3)$$

The interesting thing we find for $B \parallel U_o \parallel k$ is that for $V_A/U_o \sim 0.5$, the solutions cease to be stationary. Furthermore, one requires $V_A/U_o$ to be somewhat greater than 0.5 for stability. This compares with the incompressible case where

$$\frac{V_A}{U_o} = 0.5 \qquad (4)$$

is found to be sufficient.

(iii) The Supersonic Regime, $M > 1$, $V_A > 0$

For $B \parallel U_o \parallel k$ it is found that growth ceases ($\omega_I/k=0$) for $V_A > a$, the sound speed. This represents a generalization of the vortex sheet result due to Sen (1964). He found that for $B_o \parallel V_o$, modes parallel to the flow are stable for all values of $V_A/V_o$. We find this result applies, even when a shear layer is present, if $V_A \geq a$ ($a = 0$ in the work of Sen, 1964).

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