FLUCTUATIONS IN THE MICROWAVE BACKGROUND CAUSED BY ANISOTROPY OF THE UNIVERSE AND GRAVITATIONAL WAVES

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The problem described in the title has already been theoretically analysed several times (see for example Dautcourt, 1969 and Novikov, 1974). Recently, however some important new aspects of the problem have been discovered; they are discussed briefly in this report which is based upon the calculations of Doroshkevich, Lukash, Novikov and Polnarev. First consider the influence of primordial gravitational waves on the microwave background. It is natural to assume that in the Universe, in addition to acoustic (or adiabatic) density perturbations which result in galaxy formation and corresponding metric perturbations, there also exist metric perturbations in the form of gravitational waves with wavelengths of the same order of magnitude as the acoustic perturbations. The amplitude of such gravitational waves could in principle be quite arbitrary. Their amplitude can be estimated by comparing the theory of such waves in the expanding Universe with the observed fluctuations in the microwave background which are now available or will be in future.

Gravitional radiation - as well as any non-stationary metric perturbation - effect the microwave background radiation (we will refer to it as the relict radiation) and results in the observed temperature T of the relict radiation being different in different directions.

The amplitude h of the gravitational waves decreases in the course of the cosmological expansion. For this reason the major contribution to the fluctuations ΔT are made by h values at the earliest observable epoch, i.e. at the epoch of recombination. The calculations of Doroshkevich, Novikov and Polnarev which I report here include two important factors that were not taken into account accurately enough or in a consistent manner in the previous investigations. First, allowance is made for the fact that the recombination of the primordial plasma is not instantaneous, but rather it gradually becomes transparent over a time interval of the order of 0.1 t_{rec} where t_{rec} is the moment at which the recombination begins. This circumstance is important for the wavelengths noticeably smaller than the horizon at the moment t_{rec}. The depth of the plasma layer that contributes to the observed microwave

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background extends over a number of gravitational wavelengths, i.e., we see simultaneously a number of shells contributing with opposite signs to the distortion of the relict radiation. This effect strongly blurs fluctuations of the background.

Second, when calculating the expected mean square amplitude of ΔT and the correlation function, one should take account properly of the polar diagram of the radio antenna. Below results are given for a number of different polar diagrams.

The fluctuations in the electromagnetic relict radiation due to the primordial gravitational waves are quite different from perturbations of the acoustic (or adiabatic) type discussed in the paper by Sunyaev (this volume). The most important features that enable us to distinguish between them are as follows.

The fluctuations $\frac{\Delta T}{T}$ caused by adiabatic density perturbations are of the same order of magnitude as $\delta\varepsilon/\varepsilon$ (provided the wavelength of the perturbations is much less than the scale of the horizon, ct). The metric perturbations in this case are given by $\delta g = \left(\frac{\delta\varepsilon}{\varepsilon}\right)\left(\frac{\lambda}{ct}\right)^2$. The perturbations $\frac{\Delta T}{T}$ caused by δg can be neglected. For gravitational waves the quantity $\frac{\Delta T}{T}$ is of the order of δg, while $\delta\varepsilon$ vanishes identically in this case. Furthermore, when gravitational waves propagate through matter, the particles move perpendicular to the direction of wave propagation, while in acoustic waves the particles move parallel to the direction of propagation.

Finally, the velocity of gravitational waves is the velocity of light, which is not the case for acoustic waves. These are the main differences between the process discussed here and that considered in previous work.

The calculations include the solution of the kinetic equation for photons in a Friedmann metric with perturbations in the form of gravitational waves; both factors mentioned above were taken into account.

The results are as follows. For the amplitude of fluctuations we have

$$\overline{\left(\frac{\Delta T}{T}\right)^2} = \frac{\pi}{4}\int_0^1 d\mu \int_0^\infty \mathcal{D}_K^2(\mu) K^2 e^{-\alpha^2 K^2 \tau^2 (1-\mu^2)}\, dK \qquad (1)$$

were $K = \frac{2\pi a}{\lambda_{GW}}$, α - is the scale factor of the Universe, μ is the cosine of the angle of observation. Other quantities are defined below. To represent the observational data, observers use the function

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$$F(\Theta) = \overline{\left(\frac{T(\mu_1) - T(\mu_2)}{T}\right)^2} \qquad (2)$$

where the angle between the directions μ_1 and μ_2 is equal to θ. r is the angular diameter effective distance of the epoch for recombination. $F(\theta)$ we have

$$F(\Theta) = \overline{\left(\frac{\Delta T}{T}\right)^2} \left(1 - f(\Theta)\right) \qquad (3)$$

$$f(\Theta) = \frac{\pi}{4\overline{(\Delta T/T)^2}} \int_0^1 d\mu \int_0^\infty K^2 \mathcal{D}_K^2(\mu)\, J_0\left(\Theta K r (1-\mu^2)^{1/2}\right) e^{-\alpha^2 K^2 r^2 (1-\mu^2)} dK \qquad (4)$$

$$\mathcal{D}_K^2(\mu) = \frac{1}{4}(1-\mu^2)^2 \left[|A_+|^2 + |A_-|^2\right] h_K \qquad (5)$$

$$A_\pm = e^{-\frac{\Delta^2 K^2 (1\pm\mu)^2}{2}} \int_{\eta_{rec}}^\infty e^{iK\eta(1\pm\mu)} \frac{3 \pm 3iK\eta - K^2\eta^2}{\eta^4} d\eta \qquad (6)$$

J_0 is a Bessel function. It is assumed here that the spectrum of gravitational waves takes the form

$$h_K = h_0 K^n$$

for all relevant wavelengths, α is beam width of the radio antenna, $d\eta = \frac{dt}{a}$, and Δ is the duration of the process of recombinations in η-time. The results for different beam widths are shown in Fig. 1.

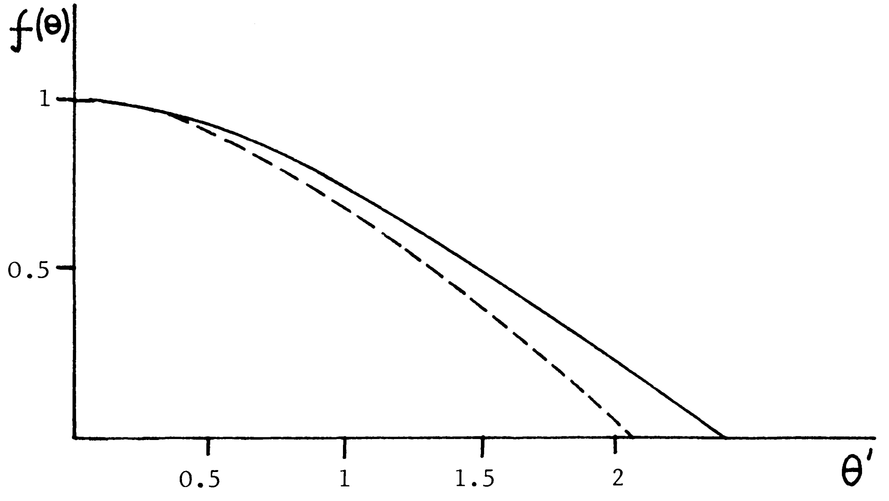

Fig. 1. The function f(θ) for n = 0 and for α = 1' (solid line) and for α = 2' (dotted line).

These formula should be used in analysing the implications of future observations.

Comparison with the observational data now available enables us to establish an upper limit for the energy density of long gravitational waves. This method is most sensitive for gravitational waves with $\lambda_{GW} \approx ct_{rec}$. The fluctuations $\frac{\Delta T}{T}$ due to these waves have scale $\approx$ 0.03 radian. If, according to modern observations, we take $\frac{\Delta T}{T} < 10^{-4}$, then $\varepsilon_{GW}/\varepsilon_{\gamma} < 10^{-8}$ for those gravitational waves which have $\lambda_{GW} = 5.10^{26}$ cm today where ε_{γ} is the energy density of relict radiation. The fluctuations $\frac{\Delta T}{T}$ due to long gravitational waves with $\lambda_{GW} = ct_{today} = 10^{28}$ cm, are quadrupole. For these fluctuations the observations give $\frac{\Delta T}{T} < 3.10^{-4}$ and so $\left[\frac{\varepsilon_{GW}(\lambda = 10^{28}\ \text{cm})}{\varepsilon_{\gamma}}\right] < 10^{-3}$.

I wish to emphasize that having constructed the spectral functions $\left(\frac{\Delta T}{T}\right)^2_K$ we can in principle make a clear distinction between fluctuations due to gravitational waves and fluctuations of the acoustic (or adiabatic) type which result in galaxy formation. This possibility is closely related to the facts mentioned above that (i) the velocity of propagation of gravitational waves is the velocity of light, and that (ii) the gravitational waves are transverse. As a result, if the spectrum

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of δg for adiabatic perturbations and gravitational waves are identical the spectral function $\left(\frac{\Delta T}{T}\right)_K^2$ at short wavelengths drops exponentially (due to blurring) in the case of adiabatic perturbations and only as a power law in the case of gravitational waves (the spectral index α_1 changes from α_1 to (α_1-1). This fact makes it possible to distinguish between these types of fluctuation.

I turn now to the anisotropy in the relict radiation T caused by anisotropy of the Universe as a whole. This problem had been extensively discussed by Novikov (1974), by Doroshkevich, Lukash and Novikov (1974) and by Zeldovich and Novikov (1975). I shall not reproduce the formulae and their derivations which are given in these works. I only recall that in the case of $\rho < \rho_{crit}$ there should be a spot on the sky in which $\Delta T/T$ greatly exceeds its value over the rest of the sky. The angular scale of the spot θ is of order of ρ/ρ_{crit}.

If one considers, for example, a perturbation that is a superposition of two homogeneous anisotropic models, say, of Bianchi type V, and each model differs from an isotropic Friedmannian model only slightly today, then one arrives at an inhomogeneous model with an inhomogeneity scale of the same order of magnitude as the curvature radius. The observational appearance of such a structure will be the superposition of two spots on the sky. This example demonstrates that we cannot say that anisotropic models of Bianchi type V is the limit when wavelength of perturbations tends to infinity. The analysis of perturbations with the wavelengths greater than the curvature radius in a curved Universe is a non-trivial problem that requires special consideration.

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